AN LMI-BASED METHOD FOR H_2/H_∞ CONTROL DESIGN UNDER SPARSITY CONSTRAINTS

M. Yagoubi and P. Chevrel

IRCCyN, UMR CNRS 6597, 1 rue de la Noë, BP 92101, 44321 Nantes Cedex 3, France
Ecole des Mines de Nantes, 4 rue Alfred Kastler, La Chantrerie, 44307 Nantes, France
Fax:+(33)02-40-37-69-30, ☎:+(33)02-51-85-82-13, e-mail: mohamed.yagoubi@emn.fr
Fax:+(33)02-51-85-83-49, ☎:+(33)02-51-85-83-40, e-mail: philippe.chevrel@emn.fr

Abstract: H_2 and H_∞ based control design methodologies are widely used to deal with multivariable control problems. In general, however, finding a norm-minimizing controller subject to a structural constraint is not a convex optimization problem, and in many cases is intractable. In this paper we propose an LMI based solution to some H_2/H_∞ specific design cases where the controller satisfies a quadratic invariance property with respect to the controlled system. The proposed method can be applied indifferently on stable or unstable systems. The problem of Lower Block Triangular (LBT) H_2/H_∞ controller design is considered as a particular case. Finally, an application to the control of a platoon of vehicles problem and a comparison with some existing methods is given.

Keywords: H_2/H_∞ Control, Structured control, Youla parametrization, LMI.

1. INTRODUCTION

An important problem in control consists in considering a specific structure on the overall control scheme especially when dealing with complex or distributed systems (Šiljak *et al.*, 1991).

This structure often depends on the own structure of the global system when subdivided in subsystems. It also depends on the accessible measured signals and the authority on actuation variables of each separated controller included in the global desired controller. In general, finding an H_2/H_∞ controller under structural constraints is considered as a non-convex optimization problem (see (Yagoubi and Chevrel 2001) and references therein).

In this paper we consider a special class of structured controller design problems with sparsity structural constraints. The constraints on the controller, considered in this paper, satisfy the so called *quadratic invariance* property (Voulgaris 2001, Rotkowitz and Lall 2002) with respect to the considered system. In this case, a convex optimization of the Youla parameter (Youla *et al.*, 1976) is possible to find an optimal H_2/H_∞ controller under such constraints and a method is proposed to realize that. This new method can be considered as an alternative to the one proposed in (Qi *et al.*, 2003). An application to a LBT system (Stanković *et al.*, 2000, Claveau *et al.*, 2003) shows the efficiency of the method.
The paper is organized as follows: The principal

notations and the position of the problem are first presented in section 2. Section 3 introduces the method based on an optimization of the Youla parameter in a special basis. In section 4, the platoon of vehicles control problem taken as a benchmark (Stanković *et al.*, 2000, Claveau *et al.*, 2003) is considered in order to test the method and compare it with other ones.

2. PROBLEM STATEMENT

2.1 Notations

The standard scheme of Fig. 1, where P and K are both linear time-invariant systems, is considered throughout this paper.

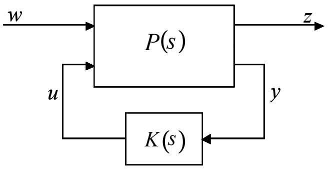

Fig. 1 Standard scheme

P is the standard model associated to the process model $G = P_{22}$ (1), and K is the controller to be designed (2).
$P(s)$ is defined by

$$P(s) = \begin{bmatrix} P_{11}(s) & P_{12}(s) \\ P_{21}(s) & P_{22}(s) \end{bmatrix} := \left[\begin{array}{c|cc} A & B_1 & B_2 \\ \hline C_1 & D_{11} & D_{12} \\ C_2 & D_{21} & D_{22} \end{array}\right] \tag{1}$$

$$\Leftrightarrow \begin{bmatrix} \dot{x} \\ z \\ y \end{bmatrix} = \left[\begin{array}{c|cc} A & B_1 & B_2 \\ \hline C_1 & D_{11} & D_{12} \\ C_2 & D_{21} & D_{22} \end{array}\right] \begin{bmatrix} x \\ w \\ u \end{bmatrix}$$

and $K(s)$ by

$$K(s) := \left[\begin{array}{c|c} A_K & B_K \\ \hline C_K & D_K \end{array}\right] \overset{\Delta}{=} \bar{K} \tag{2}$$

where $x \in \mathbb{R}^{n}$, $w \in \mathbb{R}^{n_w}$, $u \in \mathbb{R}^{n_u}$, $z \in \mathbb{R}^{n_z}$, $y \in \mathbb{R}^{n_y}$. T_{zw} denotes the closed-loop transfer matrix between the exogenous inputs w and the weighted output z

$$T_{zw} = F_l(P, K) = P_{11} + P_{12} K (I - GK) P_{21} \tag{3}$$

Let us consider, without loss of generality, that

$$T_{zw} := \left[\begin{array}{c|cc} A_{cl} & B_{1cl} & B_{2cl} \\ \hline C_{1cl} & 0 & D_{12cl} \\ C_{2cl} & D_{12cl} & 0 \end{array}\right] \tag{4}$$

where A_{cl}, B_{1cl}, B_{2cl}, C_{1cl}, C_{2cl}, D_{12cl} and D_{21cl} depends affinelly on $\bar{K}$.

Let $\Re_p^{m\times p}$ (resp. $\Re_{sp}^{m\times p}$) be the set of matrix-valued real-rational proper (resp. strictly proper) transfer functions. The structure constraint on the controller is defined as

$$\bar{K} = \Lambda_K \otimes \bar{K}, \;\; \Lambda_K \in \{0,1\}^{(n_K+n_u)\times(n_K+n_y)} \tag{5}$$

where $\Lambda_K \in \{0,1\}^{(n_K+n_u)\times(n_K+n_y)}$ and × denotes the direct product of matrices (n_K is the order of the controller).

Note that it is straightforward to prove the convexity of the set
$\Omega_{\Lambda_K} := \left\{ K \in \Re_p^{n_u \times n_y} \,/\, \bar{K} = \Lambda_K \otimes \bar{K}, \;\; \Lambda_K \in \{0,1\}^{(n_K+n_u)\times(n_K+n_y)} \right\}$

2.2 Structured H_2 / H_∞ control problems

In this paper, H_2 and H_∞ control problems under the structure constraint $K \in \Omega_{\Lambda_K}$ are considered. A bilinear matrix inequality formulation is given in both cases.

Pb 1. Structured optimal H_2 control problem

It consists in finding the optimal controller K^* such that

$$K^* = \arg\min_{K \in \Omega_{\Lambda_K}} \|T_{zw}\|_2$$

K^* may be obtained by solving the BMI optimization problem (6)

$$\begin{cases} \min\limits_{X_2, Y, \bar{K}} trace(Y) \\ \begin{bmatrix} A_{cl}^T X_2 + X_2 A_{cl} & C_{2cl}^T & \multicolumn{2}{c}{0} \\ C_{2cl} & -I & & \\ \multicolumn{2}{c}{0} & -Y & B_{2cl}^T X_2 \\ & & X_2 B_{2cl} & -X_2 \end{bmatrix} < 0 \\ \bar{K} = \Lambda_K \otimes \bar{K} \end{cases} \tag{6}$$

Pb 2. Structured optimal H_∞ control problem

It consists in finding the optimal controller K^* such that

$$K^* = \arg\min_{K \in \Omega_{\Lambda_K}} \|T_{zw}\|_\infty$$

K^* may be obtained by solving the BMI optimization problem (7)

$$\begin{cases} \min\limits_{X_\infty, \gamma, \bar{K}} \gamma \\ \begin{bmatrix} A_{cl}^T X_\infty + X_\infty A_{cl} & X_\infty B_{1cl} & C_{1cl}^T \\ B_{1cl}^T X_\infty & -\gamma I & 0 \\ C_{1cl} & 0 & -\gamma I \end{bmatrix} < 0 \\ \bar{K} = \Lambda_K \otimes \bar{K} \end{cases} \tag{7}$$

Thus, in the case where a structure constraint is imposed on the controller the H_2 and H_∞ control problems are both formulated as BMI optimization problems that can not be reduced to an LMI.

This paper shows, however, that for a large class of structured H_2 and H_∞ control problems where the controller and the system verify a special *quadratic invariance* property, it is possible to reduce problems (6) and (7) to some LMI optimization problems thanks to the use of a structured Youla parameter.

2.3 Problem formulation

Given $G \in \Re_{sp}^{n_y \times n_u}$, let us define the map $h : \Re_p^{n_u \times n_y} \to \Re_p^{n_y \times n_u}$ by $h(K) = K(I - GK)^{-1}$ for all $K \in \Re_p^{n_u \times n_y}$. Note that, if $G \in \Re_{sp}^{n_y \times n_u}$ then $(I - GK)$ is invertible for all $K \in \Re_p^{n_u \times n_y}$.

Definition: Quadratic invariance(Rotkowitz and Lall 2002)

The subspace $S \subset \Re_p^{n_u \times n_y}$ is said quadratically invariant under $G \in \Re_{sp}^{n_y \times n_u}$ if $KGK \in S$ for all $K \in S$.

In order to introduce a characterization for the quadratic invariance property we need some more notations.

Let $H^{bin} \in \{0,1\}^{m \times p}$ be a binary matrix. The subspace $Sp(H^{bin})$ is then defined as

$$Sp(H^{bin}) = \left\{ \bar{H} \in \Re^{m \times p} ; \bar{H}_{ij} = 0 \ \forall (i,j) \text{ such that } H_{ij} = 0 \right\}$$

Also, if $\bar{H} \in \Re^{m \times p}$ is a transfer matrix then $Pa(\bar{H})$ denotes the binary matrix

$$Pa(\bar{H}) = H^{bin} \text{ with } H_{ij}^{bin} = \begin{cases} 0 \text{ if } \bar{H}_{ij} = 0 \\ 1 \text{ otherwise} \end{cases}.$$

Suppose that the subspace $S \subset \Re_p^{n_u \times n_y}$ is quadratically invariant under $G = P_{22} \in \Re_{sp}^{n_y \times n_u}$. This paper deals with the following problem: "Find the optimal controller K^* such that

$$K^* = \arg\min_{K \in S} \left\| F_l(P,K) \right\|_{2,\infty} \qquad (8)$$

Theorem 1 : (Rotkowitz and Lall 2002)

Let $K^{bin} \in \{0,1\}^{n_u \times n_y}$, $S \overset{\Delta}{=} Sp\left(K^{bin}\right)$ and $G^{bin} = Pa(G)$. Then the subspace S is said *quadratically invariant* under G if and only if $K_{ki}^{bin} G_{ij}^{bin} K_{jl}^{bin} (1 - K_{kl}^{bin}) = 0$, for all $i, l = 1, \ldots, n_y$ and $j, k = 1, \ldots, n_u$.

Proof : (cf. (Rotkowitz and Lall 2002))

Remarks :

A particular case of such subspace is obtained when K^{bin} and G are both LBT (Lower Block Triangular). In this case $S \overset{\Delta}{=} Sp\left(K^{bin}\right)$ is quadratically invariant under G. It is important to notice that G and S having the same sparsity structure is not in general sufficient to imply that S is quadratically invariant under G.

Theorem 2 :

Suppose that the subspace $S \subset \Re_p^{n_u \times n_y}$ is quadratically invariant under $G = P_{22} \in \Re_{sp}^{n_y \times n_u}$. The optimal structured problem $\min_{K \in S} \left\| F_l(P,K) \right\|_{2,\infty}$ (named H_2 / H_∞ *control problems under sparsity constraints*) admits a solution if and only if there exists $Q^* \in S$ such that

$K^* = h(Q^*) = Q^*(I - GQ^*)^{-1}$ and

$$Q^* = \arg\min_{Q \in S} \left\| P_{11} + P_{12} Q P_{21} \right\|_{2,\infty} \qquad (9)$$

Proof :

This result is a direct consequence of the Youla parameterization (Youla *et al.*, 1976) and the fact that if S is quadratically invariant under G then $h(S) = S$ i.e. $h(Q) = Q(I - GQ)^{-1} = Q$, $\forall Q \in S$.

3. AN LMI APPROACH BASED ON THE PROJECTION OF THE YOULA PARAMETER

Consider the general feedback system shown in Fig. 1. The set of achievable stable closed loop maps is given by $\Phi = \left\{ T_{zw} = P_{11} + P_{12} Q P_{21} \ / \ Q \in \Re_p^{n_u \times n_y} \right\}$ where Q is a free parameter and the transfer matrices P_{11}, P_{12}, Q and P_{21} are all stable. If the system is not stable a stabilization step under structure constraint has to be considered first.

Let $Q := \left[\begin{array}{c|c} A_Q & B_Q \\ \hline C_Q & D_Q \end{array} \right]$. Assume that D_{11} and D_{22} are null matrices. It is then clear that a realization for $T_{zw} = F_l(P,K) = P_{11} + P_{12} K(I - GK) P_{21}$ can be given by

$$T_{zw} = \left[\begin{array}{cc:c} \bar{A}_1 & \bar{B}_2 M_Q \bar{C}_2 & \bar{B}_1 + \bar{B}_2 M_Q \bar{D}_2 \\ 0 & \bar{A}_2 & \bar{B}_3 \\ \hdashline \bar{C}_1 & \bar{D}_1 M_Q \bar{C}_2 & \bar{D}_1 M_Q \bar{D}_2 \end{array} \right] \qquad (10)$$

with :

$$\overline{A}_1 = \begin{bmatrix} A & 0 \\ 0 & A \end{bmatrix}, \overline{A}_2 = \begin{bmatrix} A_Q & B_Q C_2 \\ 0 & A \end{bmatrix}, \overline{B}_1 = \begin{bmatrix} B_1 \\ 0 \end{bmatrix}, \overline{B}_2 = \begin{bmatrix} 0 \\ B_2 \end{bmatrix}$$

$$\overline{B}_3 = \begin{bmatrix} B_Q D_{21} \\ B_1 \end{bmatrix}, \overline{C}_1 = [C_1 \quad C_1], \overline{C}_2 = \begin{bmatrix} I & 0 \\ 0 & C_2 \end{bmatrix}$$

$$\overline{D}_1 = D_{12}, \overline{D}_2 = \begin{bmatrix} 0 \\ D_{21} \end{bmatrix}, M_Q = \begin{bmatrix} C_Q & D_Q \end{bmatrix}$$

In the following, the Q-parameter will be chosen either static or defined in an orthonormal basis (e.g. the Niness orthonormal basis (Niness and Gustafsson 1997) :

$$Q(s) = \sum_{i=1}^{N} \theta_i Q_i(s) \tag{11}$$

$$\text{with : } Q_i(s) = \frac{\sqrt{2\operatorname{Re}(a_i)}}{s + a_i} \prod_{k=1}^{i-1} \frac{s - \overline{a}_k}{s + a_k} \tag{12}$$

The poles of the elements of the basis have to be chosen accordingly to the problem considered. (This point will be discussed later).
Note that A_Q and B_Q are fixed from (12) (the poles of the Q-basis are chosen *a priori*). n_q is the assumed order of A_Q.

If Q is restricted to be a static parameter the following matrices have to be used in (10)

$$\overline{A}_1 = \begin{bmatrix} A & 0 \\ 0 & A \end{bmatrix}, \overline{A}_2 = A, \overline{B}_1 = \begin{bmatrix} B_1 \\ 0 \end{bmatrix}$$

$$\overline{B}_2 = \begin{bmatrix} 0 \\ B_2 \end{bmatrix}, \overline{B}_3 = B_1, \overline{C}_1 = [C_1 \quad C_1], \overline{C}_2 = C_2$$

$$\overline{D}_1 = D_{12}, \overline{D}_2 = D_{21}, M_Q = Q$$

The $H_{2,\infty}$ control problems consist now in finding a (static or a dynamic) parameter Q solution of (9). Using the state space formulation (10), the only free parameter is M_Q.

The structure constraint on the Q-parameter can be defined as

$$M_Q = \Lambda \otimes M_Q, \;\; \Lambda \in \{0,1\}^{n_u \times (n_y + n_q)} \tag{13}$$

where $M_Q \in \mathbb{R}^{n_u \times (n_y + n_q)}$, $\Lambda \in \{0,1\}^{n_u \times (n_y + n_q)}$.

Note that $\Lambda \in \{0,1\}^{n_u \times (n_y + n_q)}$ is chosen such that the Q-parameter has the same structure as K. The convex structure constraint (13) reduces the number of decision variables.

Theorem 3 :
For given matrices A_Q and B_Q, the H_2 (resp. H_∞) control problem under sparsity constraints is equivalent to the LMI problem (14) (resp. (15)).

Proof :
Let us consider the Lyapunov function $X_{2,\infty}$ partitioned into $X_{2,\infty} = \begin{bmatrix} W & Z \\ Z^T & Y \end{bmatrix}$ according to $\begin{bmatrix} \overline{A}_1 & \overline{B}_2 M_Q \overline{C}_2 \\ 0 & \overline{A}_2 \end{bmatrix}$.

The matrix inequalities (6) and (7) corresponding to $H_{2,\infty}$ control problems applied to the closed loop system are non linear in the decision variables M_Q and $X_{2,\infty}$. They can, however, be transformed by a change of variable and a congruence transformation.

$$\min_{R,S,T,M_Q} \; trace(Y)$$

$$\begin{bmatrix} \overline{A}_1 R + R\overline{A}_1^T & \overline{A}_1 S - S\overline{A}_2 + \overline{B}_2 M_Q \overline{C}_2 & R\overline{C}_1^T & 0 & 0 & 0 \\ (.)^T & T\overline{A}_1 + \overline{A}_1^T T & S^T \overline{C}_1 + \overline{C}_2^T M_Q^T \overline{D}_1 & 0 & 0 & 0 \\ (.)^T & (.)^T & -I & 0 & 0 & 0 \\ 0 & 0 & 0 & -Y & \overline{B}_3^T S^T - \overline{D}_2^T M_Q^T \overline{B}_2^T - \overline{B}_1^T & -\overline{B}_2^T T \\ 0 & 0 & 0 & (.)^T & -R & 0 \\ 0 & 0 & 0 & (.)^T & 0 & -T \end{bmatrix} < 0 \tag{14}$$

$$\min_{R,S,T,M_Q,\gamma} \gamma$$

$$\begin{bmatrix} \overline{A}_1 R + R\overline{A}_1^T & \overline{A}_1 S - S\overline{A}_2 + \overline{B}_2 M_Q \overline{C}_2 & \overline{B}_3^T S^T - \overline{D}_2^T M_Q^T \overline{B}_2^T - \overline{B}_1^T & R\overline{C}_1^T \\ (.)^T & T\overline{A}_1 + \overline{A}_1^T T & -R & S^T \overline{C}_1 + \overline{C}_2^T M_Q^T \overline{D}_1 \\ (.)^T & (.)^T & -\gamma I & 0 \\ (.)^T & (.)^T & 0 & -\gamma I \end{bmatrix} < 0 \tag{15}$$

Let us consider the change of variable (see (Khargonekar and Rotea 1991):

$$\begin{bmatrix} W & Z \\ Z^T & Y \end{bmatrix} \to \begin{bmatrix} R & S \\ S^T & T \end{bmatrix} = \begin{bmatrix} W^{-1} & -W^{-1}Z \\ -Z^T W^{-1} & Y - Z^T W^{-1} Z \end{bmatrix} \quad (16)$$

and define

$$N = \begin{bmatrix} R & 0 \\ S^T & I \end{bmatrix} \quad (17)$$

Let $\Theta_1 = diag(N, I, I, N)$ (resp. $\Theta_2 = diag(N, I, I)$), then pre-multiplying and post-multiplying the matrix inequality involved in (5) (resp. in (6)) by Θ_1^T and Θ_1 (resp. by Θ_2^T and Θ_2) yields to the LMI problem (14) (resp. (15)).

Remark :
The sub-optimal H_2 (resp. H_∞) control problem under sparsity constraints can be formulated as the LMI optimization problems (14) (resp. (15)) which depends affinely on the variables R, S, T (resp. R, S, T, γ) and M_Q.

4. AN LMI-BASED METHOD

4.1 Control of a platoon of vehicles

In order to appreciate the efficiency of the proposed algorithm, the speed control of a platoon of vehicles is now considered. Many papers consider this problem (see *e.g.* (Özgüner and Perkins 1978, Stanković *et al.*, 2000, Levine and Athans 1966, Rajamani and Shladover 2001) and references therein). Briefly stated, the problem consists in keeping the platoon of vehicles with a constant velocity and constant intra platoon separations. A platoon of vehicles is indeed a potential large-scale system being composed of interconnected subsystems. A structured controller is a natural solution to meet the technological constraints (local embedded controllers). Moreover, limitations on information sharing between vehicles will restrict the admissible structures for the control.

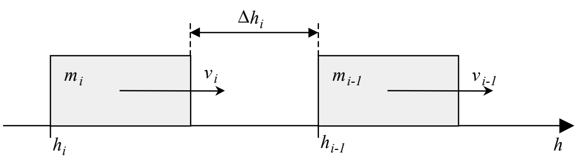

Fig. 2. Vehicles moving in string

As in (Stanković et al., 2000), the very simple model of platoon of vehicles given by (18) will be used. Consider a set of N vehicles moving in a straight line as illustrated in Figure 2. The variables parameterizing the model of the i^{th} vehicle are $h_i(t)$, $v_i(t)$, m_i, $u_i(t)$, $g_i[v_i(t)] = -\alpha_i v_i(t)$ which are respectively the position of the i^{th} vehicle at time t, its velocity, its mass, the force applied, and lastly the drag force action, assumed to be locally linearly dependent of $v_i(t)$. All the states are assumed to be measured. By considering the following deviations

$$\begin{cases} x_1 = v_d - v_1 \\ x_i^1 = v_d - v_i \\ x_i^2 = \Delta h_d - \Delta h_i,\ i = 2, \ldots, N \end{cases} \quad (18)$$

where v_d and Δh_d are respectively the desired velocity and intra platoon separation, a state-space representation of a platoon of N vehicles can be defined as (19).

$$\begin{cases} \begin{bmatrix} \dot{x}_1 \\ \dot{x}_2^1 \\ \dot{x}_2^2 \\ \vdots \\ \dot{x}_N^1 \\ \dot{x}_N^2 \end{bmatrix} = \begin{bmatrix} -\alpha_1/m_1 & 0 & 0 & \ldots & 0 & 0 \\ 0 & -\alpha_2/m_2 & 0 & \ldots & 0 & 0 \\ 1 & -1 & 0 & \ldots & 0 & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\ 0 & 0 & 0 & \ldots & -\alpha_N/m_N & 0 \\ 0 & 1 & 0 & \ldots & -1 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2^1 \\ x_2^2 \\ \vdots \\ x_N^1 \\ x_N^2 \end{bmatrix} + \begin{bmatrix} 1/m_1 & 0 & \ldots & 0 \\ 0 & 1/m_2 & \ldots & 0 \\ 0 & 0 & \ldots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \ldots & 1/m_N \\ 0 & 0 & \ldots & 0 \end{bmatrix} \begin{bmatrix} u_1 \\ u_2 \\ \vdots \\ u_N \end{bmatrix} \\ \begin{bmatrix} y_1 \\ y_2^1 \\ y_2^2 \\ \vdots \\ y_N^1 \\ y_N^2 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & \ldots & 0 & 0 \\ 0 & 1 & 0 & \ldots & 0 & 0 \\ 0 & 0 & 1 & \ldots & 0 & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\ 0 & 0 & 0 & \ldots & 1 & 0 \\ 0 & 0 & 0 & \ldots & 0 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2^1 \\ x_2^2 \\ \vdots \\ x_N^1 \\ x_N^2 \end{bmatrix} \end{cases} \quad (19)$$

To focus on the methodology, it is assumed from now (as in (Özgüner and Perkins 1978)) that $\alpha_i = 1$, $m_i = 1$, and $N = 3$.

4.2 The H_2 criterion

The standard model $P(s)$ (1) for H_2 control is defined accordingly to (Stanković et al., 2000) with $B_1^T = [10 \ \ 0 \ \ 0 \ \ 0 \ \ 0]$,

$$C_1^T = \begin{bmatrix} \sqrt{3} & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \end{bmatrix}, D_{11} = 0, D_{21} = 0$$

and $D_{12}^T = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}$.

Remark :
The choice for B_1 comes from the initial value $x_1(t = 0) = 10$ considered in (Stanković et al., 2000).

4.3 Results

The following structured controllers have been compared: the static controller K_1 obtained by the local / sequential *H_2* procedure in (Özgüner and Perkins 1978), the static controller K_2 optimized thanks to a BMI algorithm (the heuristic proposed in

(Yagoubi and Chevrel 2001) for solving H_2 structured problems was used), a dynamic controller K_3 obtained thanks to the method proposed in section 3 with a static Q parameter and a dynamic controller K_4 obtained thanks to the method proposed in section 3 with a dynamic Q parameter.

The global criterions $\|T_{zw}\|_2$ obtained for each of them are compared. The results are summed up in Tab. 1. The differences observed between K_2 and K_3 or K_4 comes from the fact that if K_2 is probably the best *static* controller, K_3 and K_4 are *dynamic* controllers.

Remarks:

Note that the system of a platoon of N vehicles (19) is unstable. Before applying the proposed method based on the optimization of a static or a dynamic Q-parameter, a stabilization step was performed by a classical sequential stabilization procedure (see for example (Stanković *et al.*, 2000, Özgüner and Perkins 1978)).

Tab. 1. Criterions for several structured controllers

Controllers	K_1	K_2	K_3	K_4
$\|T_{zw}\|_2$	12,135	11,676	11,602	11,450
Order	0	0	5	17

5. CONCLUSION

In general, finding an H_2/H_∞-norm-minimizing controller under structural constraints is considered as a non-convex optimization problem (see. (Yagoubi and Chevrel 2001) and references therein). In this paper we consider a special class of structured control design problems with sparsity structural constraints.

In that case, a simple condition is recalled to test the *quadratic invariance* property (Voulgaris 2001, Rotkowitz and Lall 2002). The proposed LMI-based method finds an H_2/H_∞-norm-minimizing controller under sparsity constraints by optimizing a Q-parameter. This parameter is either static or defined on an orthonormal base.

The comparison between the BMI, sequential H_2 and the proposed method has shown the interest of this last one when applied to a LBT system. In fact, the LMI-based proposed method has some advantages when applied to such systems.

First, this method optimizes a global criterion on the contrary of some sequential methods (see (Stanković *et al.*, 2000, Özgüner and Perkins 1978)). Secondly, the proposed method makes it possible to deal with a multi-objective or multi-criterion structured design. Finally, the static Q-parameter gives a satisfactory controller with the same order of the system in an relatively short time.

Model reduction can be included as a part of the method to prevent a rapid increase of the controller degree when a dynamic Q-parameter is used.

REFERENCES

Claveau F., P. Chevrel (2003). A sequential design methodlogy for large-scale LBT systems. *Mediteranean Control Conference*, Greece.

Khargonekar P., M. Rotea (1991). Mixed H_2/H_∞ control: a convexe optimization approach. *IEEE Trans. Autom. Control.*, vol. 36, pp. 824-837.

Levine S., and M. Athans (1966). On the optimal error regulation of a string of moving vehicles. *IEEE Trans. Autom. Contr.*, vol. 11, pp. 355-361.

Niness B., F. Gustafsson (1997). A Unifying Construction of Orthonormal Bases for System Identification. *IEEE Trans. Autom. Control.*, vol. 42, n° 4, pp. 515-521.

Özgüner U., and W.R. Perkins (1978). Optimal control of multilevel large-scale systems. *Int. J. Contr.*, vol. 28, pp. 967-980.

Qi X., M. V. Salapaka, P. G. Voulgaris and M. Khammash (2003). Structured optimal control with applications to network flow coordination. *American Control Conference.*

Rajamani R., S.E. Shladover (2001). An experimental comparative study of autonomous and co-operative vehicle-follower control systems. *Transportation Research*, vol. C-9, pp. 15-31.

Rotkowitz M., S. Lall (2002). Decentralized Control Information Strutures Preserved Under Feedback. *IEEE Conference on Decision And Control.*

Šiljak D.D. (1991). Decentralized control of complex systems. New-York, Academic Press, volume 184 of *Mathematics in Science Engineering*,.

Stanković S.S., M.J. Stanojević, and D.D. Šiljak (2000). Decentralized overlapping control of a platoon of vehicles. *IEEE Trans. Contr. Syst. Tech.*, vol. 8, n°8, pp. 816-832.

Voulgaris P. G. (2001). A Convex Characterization of classes of problems in control with specific Interaction and communication structures. *American Control Conference.*

Yagoubi M., and Ph. Chevrel (2001). An I.L.M.I. approach to structure constrained LTI control design. *Proc. Europ. Control Conf.*, Porto, Portugal.

Youla D. C., H. A. Jabr and J. J. Bongiorno (1976). Modern Wiener-Hopf Desin of optimal controllers-part 2: The multivariable case. *IEEE Trans. Autom. Control*, vol. 21.