LEARNING CONTROL OF CURRENT-FED INDUCTION MOTOR WITH MECHANICAL UNCERTAINTIES

M. Montanari * **A. Tilli** *

* *Center for Research on Complex Automated Systems*
"G. Evangelisti" (CASY)
Dept. of Electronics, Computer Science and Systems (DEIS)
University of Bologna
Viale Risorgimento 2, 40136 Bologna, Italy
e–mail: {mmontanari, atilli}@deis.unibo.it

Abstract: In this paper repetitive learning control technique has been applied to the position/flux tracking control of an Induction Motor (IM) under hypothesis of periodic reference trajectory and uncertainties on the mechanical model. The electro-magnetic IM model has been directly taken into account in the control development. Indirect Field Oriented approach has been exploited and combined with control actions derived from Lyapunov-like design. In order to compensate the periodic disturbances, the model of a generic periodic signal with known period has been embedded in the controller with a suitable update rule. The convergence properties of the overall solution proposed have been formally proven. Simulation results confirm the validity of the approach presented.

Keywords: Induction motor, learning control, position control, system uncertainties, tracking error convergence

1. INTRODUCTION

Periodic reference trajectories and mechanical uncertainties cause unknown periodic disturbances in servo-drives dynamics.
The *Internal Model Principle* represents the basic idea to solve the control problem of asymptotic tracking under condition of unknown disturbances/trajectories with known dynamic model (exosystem), without using high gain/large bandwidth approaches. The *Repetitive Learning Control* (RLC) can be interpreted as a formalization of the above-mentioned principle in case of generic periodic references/disturbances with known period. In particular, as reported in (Hara *et al.*, 1988), the adopted internal model is a closed-loop time-delay system with delay T (in the continuous-time framework) which is able to generate any periodic signal with period T.
The RLC approach (or similar solutions as Betterment Learning Control (Arimoto *et al.*, 1984) or Iterative Learning Control (Moore, 1999), (Ham *et al.*, 2001), (Xu and Tan, 2002)) has been widely used in robotic applications to cope with mechanical uncertainties leading to periodic disturbances (Horowitz *et al.*, 1991), (Dixon *et al.*, 2001). Nevertheless, the electromagnetic dynamics of the adopted servo-drive has been usually neglected.
In this paper, the case of Induction Motor (IM) servo drives with mechanical periodic disturbances is considered. The IM electromagnetic dynamics has been taken into account in the control design and the standard Indirect Field Oriented (IFO) solution has been adopted as starting point (see for instance (Taylor, 1994), (Novotny and Lipo, 1996), (Leonhard, 2001) as references on speed/flux control techniques for IM). A position controller designed according to backstepping and robust control techniques has been de-

signed to ensure asymptotic tracking for the uncertain dynamics without time-dependent disturbance. Lyapunov-like design has been adopted to guarantee stability in presence of state-dependent uncertainties. A closed-loop time delay system with known delay has been embedded in the controller with a suitable update rule to cope with the periodic disturbances. A current-fed IM has been assumed in this work.
The paper is organized as follows. In Section 2 the IM model is recalled and the control problem is stated. In Section 3 the proposed controller is reported and the stability and convergence proofs are presented. Simulation results are reported in Section 4. Concluding remarks are discussed in Section 5.

2. INDUCTION MOTOR MODEL AND PROBLEM FORMULATION

Under assumptions of linear magnetic circuits and balanced operating condition, the equivalent two-phase electromagnetic model of the current-fed IM, expressed in the generic rotating reference frame (d-q), is (Leonhard, 2001)

$$\begin{aligned}\dot{\psi}_d &= -\alpha\psi_d + (\omega_0 - \omega)\psi_q + \alpha L_m i_d \\ \dot{\psi}_q &= -\alpha\psi_q - (\omega_0 - \omega)\psi_d + \alpha L_m i_q.\end{aligned} \quad (1)$$

The mechanical model of the IM rotor shaft connected to a generic 1-degree of freedom mechanical system is expressed by the Euler-Lagrange equation (Ortega *et al.*, 1998)

$$J(\theta)\dot{\omega} = -\frac{1}{2}\frac{\partial J}{\partial \theta}(\theta)\omega^2 - b(\theta)\omega + \mu(\psi_d i_q - \psi_q i_d) \quad (2)$$

$$\dot{\theta} = \omega. \quad (3)$$

In (1)-(3), $\psi_d, \psi_q, \omega, \theta$ denote rotor fluxes, rotor shaft angular speed and position respectively. According to indirect field orientation approach (Novotny and Lipo, 1996) for current-fed IMs, control inputs are stator currents i_d, i_q and the reference frame angular speed ω_0. Subscripts d, q stand for vector components along the d and q reference frame axis respectively. Angular position of the (d,q) reference frame with respect to a fixed stator reference frame is ε_0, with $\dot{\varepsilon}_0 = \omega_0$. Slip frequency is defined as $\omega_2 = \omega_0 - \omega$. Positive constants related to IM parameters are defined as $\alpha = R_r/L_r$, $\mu = (3L_m)/(2L_r)$, where R_r is the rotor resistance and L_r, L_m are the rotor and magnetizing inductances respectively. The equivalent total inertia of the mechanical load is given by $J(\theta)$, the equivalent total linear viscous friction torque acting on rotor shaft is $b(\theta)\omega$, IM torque is expressed by $\mu(\psi_d i_q - \psi_q i_d)$ and no load torque is assumed to be applied.

The following assumptions are introduced:

A1. Rotor position and speed are measured.
A2. IM electromagnetic parameters are constant and known.
A3. Reference trajectories for position θ and flux amplitude $|\psi| = \sqrt{\psi_d^2 + \psi_q^2}$ are known functions $\theta^*(t) : \mathbb{R}^+ \to \mathbb{R}$ and $\psi^*(t) : \mathbb{R}^+ \to \mathbb{R}$, with bounded and known time derivatives $\dot{\theta}^*, \ddot{\theta}^*, \dot{\psi}^*$. It holds $\psi^*(t) > 0, \forall t$.
A4. θ^*, ψ^* are T-periodic functions, i.e. $\theta^*(t) = \theta^*(t+T), \psi^*(t) = \psi^*(t+T), \forall t$.
A5. Inertia $J(\theta)$ and its time derivatives are *unknown* functions with the following known bounds: there exist positive constants $J_m, J_M, k_A, k_B > 0$ such that $J(\theta) \in [J_m, J_M]$, $\left|\frac{\partial J}{\partial \theta}(\theta)\right| \le k_A$, $\left|\frac{\partial^2 J}{\partial \theta^2}(\theta)\right| \le k_B, \forall \theta$.
A6. Friction torque $b(\theta)\omega$ is an *unknown* function with known positive bounds k_C, $k_D \ge 0$ such that $|b(\theta^*)\omega^* - b(\theta)\omega| \le k_C|\omega^* - \omega| + k_D|\omega^*||\theta^* - \theta|, \forall\, \omega^*, \omega, \theta^*, \theta$.

In learning control framework, time interval $[0, \infty)$ can be partitioned into a sequence of finite intervals $[iT + 0, iT + T]$, $i = 0, 1, \dots$, called learning trials, over which the reference trajectory is periodic. In the following, subscript i refers to evaluation of variables over the i-th learning trail, i.e. with abuse of notation for the generic variable x it holds $x_i(t) = x(iT + t)$, with $t \in [0, T], i \in [0, \infty)$. Note that $x_i(t - T) = x_{i-1}(t)$. When no confusion arises, subscript i and time-dependence are omitted for the sake of brevity.

Remark 1. Assumption A4 deals with the case of continuous repetition of the position reference trajectory, with no stop of the process, which is a very common task in motion control applications. Note that no reset of state variables is applied at the beginning of learning trials.

Remark 2. The mechanical model is supposed to be not perfectly known, which is a quite standard situation for complex mechanical kinematisms. Boundedness and Lipschitz-like assumptions are introduced with A5 and A6. Note that assumption A6 is satisfied in the case of linear friction torques/forces acting on the mechanical system.
Due to mechanical uncertainties, feed-forward actions in the q-axis (torque) current cannot be introduced to compensate for the desired torque obtained through mechanical model inversion. On the other hand, process repeatability can be exploited to design a learning adaptation law for the compensation of the unknown reference-dependent terms at each learning trial. With respect to online parameter identification techniques, the RLC approach allows to deal with more general cases of unstructured uncertainties (e.g. for friction forces, unmodelled nonlinearities).

Under assumptions A1-A6, the control objective is to design a learning controller for the IM model (1)-(3) which guarantees perfect tracking of position/flux reference trajectories as learning trial tends to infinity, i.e. $\lim_{i\to\infty} \theta_i(t) = \theta^*(t)$, $\lim_{i\to\infty} |\psi_i|(t) = \psi^*(t)$, $\forall t \in [0, T]$.

Remark 3. Applying field orientation strategy for vector flux control, requirement of flux amplitude tracking is equivalent to require that $\lim_{i\to\infty}\psi_{d,i}(t) = \psi^*(t)$ and $\lim_{i\to\infty}\psi_{q,i}(t) = 0, \forall t \in [0,T]$ (perfect field orientation).

3. POSITION/FLUX TRACKING LEARNING CONTROLLER

The controller is composed by three main parts: an improved indirect field oriented (I-IFO) controller for the flux subsystem, a position tracking controller based on backstepping and robust control techniques for the mechanical subsystem and a learning-based adaptation law for compensation of unknown and periodic trajectory-dependent feed-forward terms. Based on direct Lyapunov method, the I-IFO and position controllers are designed in order to guarantee global exponential stability of the "nominal" tracking error model, i.e. considering state-dependent uncertainties but supposing perfect compensation of reference-dependent terms. Learning controller compensates for the periodic time-dependent disturbance.

3.1 Flux control

The I-IFO controller is defined as

$$\begin{aligned} i_d &= \frac{1}{\alpha L_m}\left(\dot{\psi}^* + \alpha\psi^* + \nu_\psi\right) \\ \omega_0 &= \omega + \frac{\alpha L_m i_q}{\psi^*} - \frac{\nu_0}{\psi^*}, \end{aligned} \tag{4}$$

where ν_ψ, ν_0 are auxiliary signals to be designed according to Lyapunov-like technique in order to compensate for torque/flux coupling terms. Defining flux tracking errors as $\tilde{\psi}_d = \psi_d - \psi^*$, $\tilde{\psi}_q = \psi_q$, from (1) and (4) the flux error dynamics becomes

$$\begin{aligned} \dot{\tilde{\psi}}_d &= -\alpha\tilde{\psi}_d + \omega_2\tilde{\psi}_q + \nu_\psi \\ \dot{\tilde{\psi}}_q &= -\alpha\tilde{\psi}_q - \omega_2\tilde{\psi}_d + \nu_0. \end{aligned} \tag{5}$$

3.2 Position control

Defining the position and speed tracking errors as $\tilde{\theta} = \theta - \theta^*$, $\tilde{\omega} = \omega - \omega^*$, where ω^* is the fictitious control input for the position control loop, the backstepping robust position tracking controller is designed as

$$\begin{aligned} \omega^* &= \dot{\theta}^* - k_1\tilde{\theta} \\ i_q &= \frac{1}{\mu\psi^*}\left(-k_2\tilde{\omega} - k_3\tilde{\theta} + \eta_{q1} + \eta_{q2}\right), \end{aligned} \tag{6}$$

where $k_1, k_2, k_3 > 0$ are constant gains, η_{q1} is the additional robust control term and η_{q2} is the learning-based control law. From (3), (2) and (6), the mechanical error dynamics can be written as

$$\begin{aligned} \dot{\tilde{\theta}} &= -k_1\tilde{\theta} + \tilde{\omega} \\ J(\theta)\dot{\tilde{\omega}} &= -k_2\tilde{\omega} - k_3\tilde{\theta} + \mu(i_q\tilde{\psi}_d - i_d\tilde{\psi}_q) + \\ &\quad + \xi(\tilde{\theta}, \tilde{\omega}, t) + \eta_{q1} - d(t) + \eta_{q2} \end{aligned} \tag{7}$$

with

$$\begin{aligned} \xi &= (J(\theta^*) - J(\theta))\,\ddot{\theta}^* + \left(\tfrac{1}{2}\tfrac{\partial J}{\partial\theta}(\theta^*)\dot{\theta}^{*2} - \tfrac{1}{2}\tfrac{\partial J}{\partial\theta}(\theta)\omega^2\right) + \\ &\quad + \left(b(\theta^*)\dot{\theta}^* - b(\theta)\omega\right) + k_1 J(\theta)(-k_1\tilde{\theta} + \tilde{\omega}) \\ d &= J(\theta^*)\ddot{\theta}^* + \frac{1}{2}\frac{\partial J}{\partial\theta}(\theta^*)\dot{\theta}^{*2} + b(\theta^*)\dot{\theta}^* \end{aligned}$$

where $\xi(\tilde{\theta}, \tilde{\omega}, t)$ is the unknown state-dependent term to be compensated by η_{q1} and $d(t)$ represents the T-periodic unknown disturbance, which is dependent on the reference trajectory only (and not on state variables) and will be compensated by η_{q2}.

Now, robust control η_{q1} is designed.
Let $\mathbf{e} = (\tilde{\psi}_d, \tilde{\psi}_q, \tilde{\omega}, \tilde{\theta})^{\mathrm{T}}$ and define the candidate Lyapunov function

$$V(\mathbf{e}, t) = \frac{1}{2}\left[\gamma(\tilde{\psi}_d^2 + \tilde{\psi}_q^2) + J(\theta)\tilde{\omega}^2 + k_3\tilde{\theta}^2\right] \tag{8}$$

with constant parameter $\gamma > 0$. Designing auxiliary terms of the I-IFO control as

$$\nu_\psi = \frac{1}{\gamma}(-\mu i_q\tilde{\omega}), \qquad \nu_0 = \frac{1}{\gamma}(\mu i_d\tilde{\omega}), \tag{9}$$

the time derivative of V along trajectories of (5), (7) is

$$\begin{aligned} \dot{V} &= -\gamma\alpha\left(\tilde{\psi}_d^2 + \tilde{\psi}_q^2\right) - k_2\tilde{\omega}^2 - k_1k_3\tilde{\theta}^2 + \\ &\quad + \tilde{\omega}\left(\tfrac{1}{2}\tfrac{\partial J}{\partial\theta}(\theta)\omega\tilde{\omega} + \xi + \eta_{q1} - d + \eta_{q2}\right). \end{aligned} \tag{10}$$

Note that from A5 it follows that $|J(\theta^*) - J(\theta)| \leq k_A|\theta^* - \theta|$, $\left|\frac{\partial J}{\partial\theta}(\theta^*) - \frac{\partial J}{\partial\theta}(\theta)\right| \leq k_B|\theta^* - \theta|, \forall\,\theta^*, \theta$. From assumptions A5 and A6 it follows that

$$\begin{aligned} &\left(\tfrac{1}{2}\tfrac{\partial J}{\partial\theta}(\theta)\omega\tilde{\omega} + \xi(\tilde{\theta}, \tilde{\omega}, t)\right)\tilde{\omega} \leq \tfrac{1}{2}k_A|\omega|\tilde{\omega}^2 + \\ &\quad + \left(k_A|\ddot{\theta}^*| + \tfrac{1}{2}k_B\dot{\theta}^{*2} + k_D|\dot{\theta}^*|\right)|\tilde{\theta}|\,|\tilde{\omega}| + \\ &\quad + \Big(\tfrac{1}{2}k_A|\tilde{\omega} - k_1\tilde{\theta}| + k_A|\dot{\theta}^*| + \\ &\quad + (k_C + k_1 J_M)\Big)|\tilde{\omega}|\,|\tilde{\omega} - k_1\tilde{\theta}|. \end{aligned} \tag{11}$$

Considering inequality in (11), the robust control law η_{q1} is designed according to nonlinear damping argument (Khalil, 1996, Section 13.1.2) as

$$\begin{aligned} \eta_{q1} = -\Bigg[&\frac{1}{\varepsilon_1}\left(\frac{k_A\omega}{4}\right)^2 + \frac{1}{\varepsilon_2}\left(\frac{k_A\ddot{\theta}^*}{2}\right)^2 + \\ &+ \frac{1}{\varepsilon_2}\left(\frac{k_B\dot{\theta}^{*2}}{4}\right)^2 + \frac{1}{\varepsilon_2}\left(\frac{k_D\dot{\theta}^*}{2}\right)^2 + \\ &+ \frac{1}{\varepsilon_3}\left(\frac{k_A(\tilde{\omega} - k_1\tilde{\theta})}{4}\right)^2 + \\ &+ \frac{1}{\varepsilon_3}\left(\frac{k_A\dot{\theta}^*}{2}\right)^2 + \frac{1}{\varepsilon_3}\left(\frac{k_C + k_1 J_M}{2}\right)^2\Bigg]\tilde{\omega}, \end{aligned} \tag{12}$$

where $\varepsilon_1, \varepsilon_2, \varepsilon_3$ are constant positive gains. Applying Young's inequality $2xy \leq \epsilon x^2 + y^2/\epsilon, \forall\,x, y \in \mathbb{R}, \forall\,\epsilon > 0$ to terms in (11), from (10) and (12) it follows that

$$\dot{V} \le -\gamma\alpha\left(\tilde{\psi}_d^2+\tilde{\psi}_q^2\right) - k_2\tilde{\omega}^2 - k_1k_3\tilde{\theta}^2 + \varepsilon_1\tilde{\omega}^2 + \\ + 3\varepsilon_2\tilde{\theta}^2 + 3\varepsilon_3(\tilde{\omega}-k_1\tilde{\theta})^2 + \tilde{\omega}\left(-d+\eta_{q2}\right). \quad (13)$$

Choosing $k_1, k_2, k_3, \varepsilon_1, \varepsilon_2, \varepsilon_3$ such that the matrix

$$\begin{bmatrix} k_2-\varepsilon_1-3\varepsilon_3 & 3\varepsilon_3k_1 \\ 3\varepsilon_3k_1 & k_1k_3-3\varepsilon_2-3\varepsilon_3k_1^2 \end{bmatrix}$$

is positive definite, (13) can be compactly rewritten as

$$\dot{V} \le -\mathbf{e}^{\mathrm{T}}\mathbf{Q}\mathbf{e} + \tilde{\omega}\left(-d(t)+\eta_{q2}\right) \quad (14)$$

with matrix $\mathbf{Q} = \mathbf{Q}^{\mathrm{T}} > 0$, hence the controller ensures global exponential stability of the origin of the "nominal" error model, i.e. with $d(t)=0$.

3.3 Learning control law

Referring to the i-th learning trial, the learning control law $\eta_{q2}(t) = \hat{d}_i(t)$ is defined as

$$\hat{d}_i(t) = \hat{d}_{i-1}(t) - \lambda\tilde{\omega}_i(t),\ t\in[0,T],\ i=0,1,\ldots \quad (15)$$

where $\lambda > 0$ is a constant tuning gain. Initial zero conditions of the learning control law are assumed, i.e. $\hat{d}_{-1}(t) = 0, \forall\, t\in[0,T]$. Note that an equivalent expression of the learning control law is

$$\hat{d}(t) = \hat{d}(t-T) - \lambda\tilde{\omega}(t), \quad \forall t\in[0,\infty).$$

The learning error is defined as $\tilde{d}(t) = \hat{d}(t) - d(t)$. The following theorem states that the I-IFO control combined with the backstepping robust mechanical control and the learning-based adaptation law ensure perfect tracking of position/flux trajectories over the i-th period T as the repetitions tend to infinity.

Theorem 4. The controller given by (4), (6), (9), (12) with learning control law (15) and appropriate gain selection (see Section 3.2) guarantees global exponential position/flux tracking with bounded tracking errors $\mathbf{e}\in L_\infty$ and L_2-norm bounded learning control law $\hat{d}_i \in L_2$ over each learning trial.

PROOF. Tracking convergence properties are analyzed in three steps: first, boundedness of errors $\mathbf{e}(t)$ and $\tilde{d}(t)$ over the first and second learning trials is proven. Then, boundedness of tracking error $\mathbf{e}(t)$ for each learning trial and its convergence to zero as learning trials tend to infinity is shown. Finally, it is shown that the learning control law $\hat{d}_i(t)$ is bounded in L_2 norm over the i-th learning trial $[0,T]$, $\forall i$.

Boundedness of $\mathbf{e}_0(t), \tilde{d}_0(t), \mathbf{e}_1(t), \tilde{d}_1(t)\ \forall t\in[0,T]$. Consider the following candidate-Lyapunov function, composed by the Lyapunov function $V(\mathbf{e},t)$ for tracking error $\mathbf{e}$ and the L_2 norm of learning error $\tilde{d}$ at the i-th learning trial:

$$U_i(\mathbf{e},\tilde{d},t) = V_i(\mathbf{e},t) + \frac{1}{2\lambda}\int_0^t \tilde{d}_i^2(\tau)\mathrm{d}\tau,\ i=0,1,\ldots. \quad (16)$$

Recalling (15) and initial conditions for $\hat{d}$, it holds $\hat{d}_0(t) = -\lambda\tilde{\omega}_0(t)$. After substitutions in (14), time derivative of $U_0(t)$ at the first learning trial ($i=0$) is

$$\dot{U}_0 = \dot{V}_0 + \frac{1}{2\lambda}\tilde{d}_0^2 \le -\mathbf{e}_0{}^{\mathrm{T}}\mathbf{Q}\mathbf{e}_0 - \frac{\lambda}{2}\tilde{\omega}_0^2 + \frac{1}{2\lambda}d_0^2. \quad (17)$$

Integrating (17) over the interval $[0,t]$, with $t\in[0,T]$, it holds $U_0(t) \le U_0(0) + \frac{1}{2\lambda}d_M^2 t$, where d_M is an upper-bound of $d(t)$, i.e. $|d(t)| \le d_M$, $\forall t$. Since $d(t)$ is bounded and $U_0(0)$ is bounded, $U_0(t)$ is bounded $\forall t\in[0,T]$. Hence $\mathbf{e}_0(t)$ and $\tilde{d}_0(t) = -d_0(t) - \lambda\tilde{\omega}_0(t)$ are bounded $\forall t\in[0,T]$.

For next developments, boundedness of $\mathbf{e}_1(t)$, $\tilde{d}_1(t)$, $\forall t\in[0,T]$ is proven. Noting that $\tilde{d}_1(t) = \tilde{d}_0(t) - \lambda\tilde{\omega}_1(t)$, following same considerations of (17) it follows that

$$\dot{U}_1 \le -\mathbf{e}_1{}^{\mathrm{T}}\mathbf{Q}\mathbf{e}_1 - \frac{\lambda}{2}\tilde{\omega}_1^2 + \frac{1}{2\lambda}\tilde{d}_0^2 \quad (18)$$

and, since $\tilde{d}_0$ is bounded, it can be proven that $U_1(t)$, $\mathbf{e}_1(t)$, $\tilde{d}_1(t)$ are bounded, $\forall t\in[0,T]$.

Convergence of tracking error $\mathbf{e}_i(t)$. In order to prove boundedness of $\mathbf{e}_i(t)$, $\forall i$, $\forall t\in[0,T]$ and its asymptotic convergence to zero, the following Lyapunov function is defined at the i-th learning trial, $\forall i = 1,2,\ldots$:

$$W_i(\mathbf{e},\tilde{d},t) = V_i(\mathbf{e},t) + \frac{1}{2\lambda}\int_{t-T}^t \tilde{d}_i^2(\tau)\mathrm{d}\tau. \quad (19)$$

With respect to U_i, a different interval of integration, i.e. a shifting interval of length T, is considered. Note that since $\mathbf{e}_1(t)$, $\tilde{d}_1(t)$ are bounded, it follows that $W_1(t)$ is bounded $\forall t\in[0,T]$.

Computing the difference of $W_i(t)$ over two consecutive periods it holds

$$\Delta W_i(t) = W_i(t) - W_{i-1}(t) = \\ = V_i(\mathbf{e},t) - V_{i-1}(\mathbf{e},t) + \frac{1}{2\lambda}\int_{t-T}^t\left(\tilde{d}_i^2(\tau) - \tilde{d}_{i-1}^2(\tau)\right)\mathrm{d}\tau.$$

Since $\tilde{d}_i(t) = -d(t) + \hat{d}_{i-1}(t) - \lambda\tilde{\omega}_i(t)$, from (14) it holds

$$V_i(\mathbf{e},t) - V_{i-1}(\mathbf{e},t) \le -\int_{t-T}^t\left(\mathbf{e}_i{}^{\mathrm{T}}\mathbf{Q}\mathbf{e}_i + \lambda\tilde{\omega}_i^2\right)\mathrm{d}\tau + \\ + \int_{t-T}^t \tilde{\omega}_i(\tau)(-d(\tau) + \hat{d}_{i-1}(\tau))\mathrm{d}\tau$$

and from (15) it holds

$$\frac{1}{2\lambda}\int_{t-T}^t\left(\tilde{d}_i^2(\tau) - \tilde{d}_{i-1}^2(\tau)\right)\mathrm{d}\tau \le \\ \le \frac{\lambda}{2}\int_{t-T}^t \tilde{\omega}_i^2(\tau)\mathrm{d}\tau - \int_{t-T}^t \tilde{\omega}_i(\tau)(-d(\tau) + \hat{d}_{i-1}(\tau))\mathrm{d}\tau,$$

hence it follows that

$$\Delta W_i(t) \le -\int_{t-T}^t\left(\mathbf{e}_i{}^{\mathrm{T}}\mathbf{Q}\mathbf{e}_i + \tfrac{\lambda}{2}\tilde{\omega}_i^2\right)\mathrm{d}\tau \le 0. \quad (20)$$

The Lyapunov function $W_k(t)$, $t \in [0,T]$, $k \geq 2$ at the k-th learning trial can be expressed as

$$W_k(t) = W_1(t) + \sum_{i=2}^{k} \Delta W_i(t) \leq$$
$$\leq W_1(t) - \sum_{i=2}^{k} \int_{t-T}^{t} \left(\mathbf{e}_i{}^{\mathrm{T}} \mathbf{Q} \mathbf{e}_i + \tfrac{\lambda}{2} \tilde{\omega}_i^2 \right) \mathrm{d}\tau. \quad (21)$$

Since $W_1(t)$ is bounded, $W_i(t) \geq 0$ and $\Delta W_i(t) \leq 0$, $\forall t \in [0,T]$, $\forall i$, it follows that $W_k(t)$ is bounded over each learning cycle and $W_\infty(t) = \lim_{k\to\infty} W_k(t)$ exists and is finite. Hence, tracking error $\mathbf{e}(t)$, $\forall t$ is bounded for each learning trial.

Taking the limit for $k \to \infty$ of (21) it follows that

$$\lim_{k\to\infty} \sum_{i=2}^{k} \int_{t-T}^{t} \mathbf{e}_i{}^{\mathrm{T}} \mathbf{Q} \mathbf{e}_i \mathrm{d}\tau \leq W_1(t) - \lim_{k\to\infty} W_k(t),$$

hence, since $\mathbf{e}_i{}^{\mathrm{T}} \mathbf{Q} \mathbf{e}_i \geq 0$, $\sum_{i=2}^{\infty} \int_{t-T}^{t} \mathbf{e}_i{}^{\mathrm{T}} \mathbf{Q} \mathbf{e}_i \mathrm{d}\tau$ converges and

$$\lim_{i\to\infty} \int_{t-T}^{t} \mathbf{e}_i{}^{\mathrm{T}} \mathbf{Q} \mathbf{e}_i \mathrm{d}\tau = 0, \; \forall t \in [0,T],$$

which implies that $\mathbf{e}_i(t) \longrightarrow 0$, $\forall t \in [0,T]$ as the learning trial i tends to infinity.

Learning control law $\hat{d}_i \in L_2$. Learning control law $\hat{d}_k(t)$ is bounded over finite learning trials since it can be expressed as $\hat{d}_k(t) = \sum_{i=0}^{k} (-\lambda \tilde{\omega}_i(t))$, i.e. as the sum of bounded functions $\tilde{\omega}_i(t)$. Hence, finite escape time of all internal variables is avoided. From (19) and (21) with $t = T$ and taking the limit for $i \to \infty$ it holds

$$0 \leq \lim_{i\to\infty} \frac{1}{2\lambda} \int_0^T \tilde{d}_i^2(\tau) \mathrm{d}\tau \leq W_\infty(T) \leq W_1(T).$$

Since $W_1(T)$ is bounded, $\lim_{i\to\infty} \int_0^T \tilde{d}_i^2(\tau)\mathrm{d}\tau$ is bounded. Hence, defining $\tilde{d}_\infty = \lim_{i\to\infty} \tilde{d}_i$, it follows that $\tilde{d}_\infty \in L_2$ over the interval $[0,T]$. Since $\tilde{d}_\infty, d \in L_2$, it follows that $\hat{d}_\infty \in L_2$.

Remark 5. It has been shown that tracking error $\mathbf{e}$ is point-wise bounded and tends to zero, guaranteeing asymptotic position/flux tracking. Moreover, learning control law $\hat{d}_i \in L_\infty$ for the i-th finite learning trial and $\hat{d}_\infty \in L_2$, from which it comes out that control inputs $i_{d,i}, i_{q,i}, \omega_{0,i} \in L_2$ over each learning trial $[0,T]$. From a formal viewpoint, it is known that there exist technical issues related to proof of L_∞-boundedness of control law with learning control (see also (Xu and Tan, 2002) and successive comments).

4. SIMULATION RESULTS

As an example, simulations of the proposed learning controller have been performed for motion control of the mechanism shown in Fig.1, in which the IM

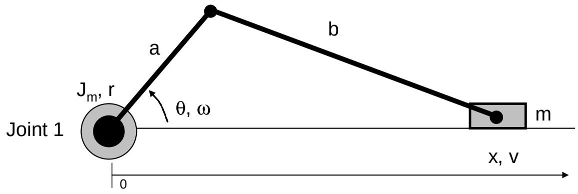

Fig. 1. Scheme of the mechanism.

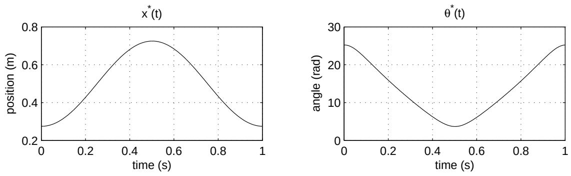

Fig. 2. Reference trajectories.

rotor shaft is connected to the rotational joint 1 with gear ratio r, IM inertia is J_m, slider mass is m, other masses and inertias are supposed to be null, a, b are link lenghts, with $b > a$. Kinematic relation between slider position x and IM rotor angle θ is $x(\theta) = a\cos(\theta/r) + \sqrt{b^2 - a^2 \sin^2(\theta/r)}$. Total inertia is $J(\theta) = J_m + m \frac{\partial x}{\partial \theta}(\theta)^2$. Linear friction torque $b_\omega \omega$ and force $b_v \dot{x}$ act on the rotor shaft and on the slider respectively. Total friction torque on the IM rotor shaft is expressed as $b(\omega,\theta) = \left(b_\omega + b_v (\partial x/\partial\theta)^2\right)\omega$. Known parameters are $a = 0.25$ m, $b = 0.50$ m, $r = 10$, $\alpha = 10$, $L_m = 0.5$ H, $\mu = 1.4$. Unknown mechanical parameters are $m = 16$ Kg, $J_m = 0.005$ Kg m^2, $b_\omega = 0.025$ Nm/(rad/s), $b_v = 16$ N/(m/s). Uncertainty equal to $\pm 20\%$ on each parameter is supposed for tuning of controller gains.
After initial flux excitation phase, flux reference is maintained constant at $\psi^*(t) = 1$ Wb. The slider position reference trajectory $x^* = b - 0.9a\cos(2\pi t)$, with period $T = 1$s, is required to be tracked. To this aim, the IM motor is required to track the periodic trajectory $\theta^*(t)$ obtained through kinematic inversion of $x^*(t)$, assuming $\theta^*(t) > 0$, $\forall t$. In Fig. 2 reference trajectories are reported, while unknown disturbance $d(t)$ is shown in Fig. 3. Initial conditions of the IM state variables are $\psi_d(0) = 0.96$ Wb, $\psi_q(0) = 0$ Wb, $\omega(0) = 0$ rad/s, $\theta(0) = 26.2$ rad, i.e. $\tilde{\theta}(0) = 1$ rad.
Flux and backstepping controller gains are selected as $\gamma = 20$, $k_1 = 5$, $k_2 = 0.5$, $k_3 = 5$, while robust control η_{q1} is designed based on bounds on uncertainties on $J(\theta)$ and $b(\omega,\theta)$ and according to stability criterion given in Section 3.2. Learning gain is set at $\lambda = 0.1$.

In Fig. 3 tracking errors, learning control law $\hat{d}_i(t)$ and control inputs are shown for learning trials $i = 0$ and $i = 30$ respectively. While during the first period large position and flux errors are present, tracking errors converge to zero as the learning trial increase, thanks to the correct compensation of the unknown term $d(t)$. In order to evaluate the tracking performance, the L_2 norm of the position tracking error over the i-th period T, namely $\Upsilon_i = \int_0^T \tilde{\theta}_i(\tau)^2 \mathrm{d}\tau$, is used. In Fig. 4 the performance index Υ_i is reported both in the case

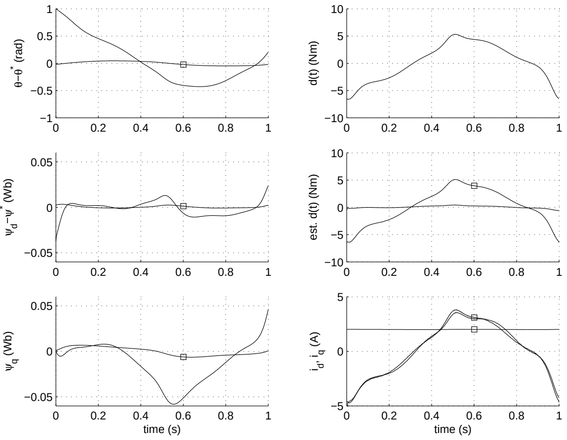

Fig. 3. Tracking errors, learning control $\hat{d}_i(t)$ and control inputs i_d, i_q for learning trial $i = 0$ (solid line) and $i = 30$ (marked line).

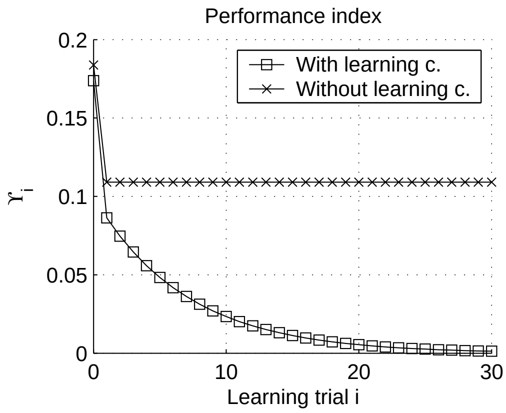

Fig. 4. Performance index Υ_i during learning trials with learning controller (15) and with $\eta_{q2} = 0$.

of the learning controller and without compensation of $d(t)$, i.e. with $\eta_{q2} = 0$. With the learning controller, tracking error $\tilde{\theta}_i(t)$ tends to 0 after about 30 periods. On the contrary, with $\eta_{q2} = 0$ the error model is an exponentially stable dynamics with bounded disturbance, hence only ultimately boundedness of state error variables is ensured thanks to feedback controller (see also (14)). Large initial tracking error (for $i = 0$) is due both to non-null initial conditions and to unperfect compensation of $d(t)$ during learning transient.

From a practical viewpoint, the robust control η_{q1} defined in (12) can be modified as $\eta_{q1} = -k_{q1}\tilde{\omega}$, with sufficiently large constant gain $k_{q1} > 0$, in order to reduce the computational burden and without impairing tracking performances.

5. CONCLUSIONS

Position/flux tracking control of current-fed IM under hypothesis of periodic position reference and mechanical uncertainties has been considered. The solution proposed is based on: a) an improved IFO control for torque/flux decoupling, b) a backstepping position controller with additional robust terms to guarantee stability in presence of state-dependent uncertainties, c) a learning based adaptation law exploiting repeatability of the position/flux trajectory in order to compensate for unknown disturbance.

This work represents a preliminary result for more complex and sophisticated controllers. In particular, further researches will focus on learning control based on backstepping technique for the voltage-fed IM. Moreover, implementation issues, such as discrete-time version of the controller, noise sensitivity and robustness, will be tackled.

REFERENCES

Arimoto, S., S. Kawamura and F. Miyazaki (1984). Bettering operation of robots by learning. *Journal of Robotics Systems* **1**(2), 123–140.

Dixon, W. E., E. Zergeroglu, D. M. Dawson and B. T. Costic (2001). Repetitive learning control: a Lyapunov-based approach. In: *Proc. of the 2001 IEEE Int. Conf. on Control Applications*. Mexico City, Mexico.

Ham, C., Z. Qu and J. Kaloust (2001). Nonlinear learning control for a class of nonlinear systems. *Automatica* **37**, 419–428.

Hara, S., Y. Yamamoto, T. Omata and M. Nakano (1988). Repetitive control system: a new type servo system for periodic exogenous signals. *IEEE Trans. on Automatic Control* **33**(7), 659–668.

Horowitz, R., W. Messner and J. B. Moore (1991). Exponential convergence of a learning controller for robot manipulators. *IEEE Trans. on Automatic Control* **36**(7), 890–894.

Khalil, H. K. (1996). *Nonlinear Systems*. 2nd ed.. Prentice Hall. Upper Saddle River (NJ).

Leonhard, W. (2001). *Control of Electrical Drives*. 3rd ed.. Springer-Verlag. Berlin.

Moore, K. L. (1999). Iterative learning control - an expository overview. *Appl. Comput. Controls, Signal Proc., Circuits* **1**, 151–214.

Novotny, D.W. and T.A. Lipo (1996). *Vector control and dynamics of AC drives*. Oxford Univ. Press, Oxford, U.K.

Ortega, R., A. Loria, P. J. Nicklasson and H. Sira-Ramirez (1998). *Passivity-based control of Euler-Lagrange Systems. Mechanical, electrical and electromechanical applications*. Springer-Verlag, London.

Taylor, D. (1994). Nonlinear control of electric machines: an overview. *IEEE Control Systems Magazine* **14**(6), 41–51.

Xu, J. and Y. Tan (2002). A composite energy function-based learning control approach for nonlinear systems with time-varying parametric uncertainties. *IEEE Trans. on Automatic Control* **47**(11), 1940–1945. Comments to paper on IEEE TAC, **48**(9),1671-1674.