INFINITE-BUS-EQUIVALENT ESTIMATION FROM LOCAL MEASUREMENTS ON A SYNCHRONOUS MACHINE

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Abstract: The present paper treats the rotor angle estimation problem in a single-machine infinite bus setup. The components of the infinite bus voltage phasor are estimated in a coordinate frame that turns with the rotor by means of a Kalman-like filter from exclusively local measurements. Therefore, in addition to the rotor angle, the observer also provides an estimate of the infinite bus voltage. The proposed methodology is tested in simulation. *Copyright ©2005 IFAC*

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1. INTRODUCTION

A power system is basically a collection of nonlinearly coupled systems – the generators – that supply electric power to some charges. Nonlinear behavior is not exclusively specific to generators; charges and some elements of the interconnecting network (e.g. FACTS devices) could also exhibit nonlinear dynamics. However, a power system is always required to operate at a specific point, which usually changes slowly on a future load prediction or planning basis. For this reason, relatively simple linear controllers were employed in practice in order to maintain transient stability and damp oscillations as far as small disturbances are concerned. As regards large disturbances, or those occurring when the system is loaded up to its stability limit, the performance of linear controllers can no longer match the performance of nonlinear ones.

This fact was recognized as soon as new developments in nonlinear control theory, such as linearization techniques, were applied to power systems. The first important result based on external feedback linearization (Marino, 1984), was further developed by several authors, for example Mak (1992), Mielczarski and Zajaczkowski (1994), King *et al.* (1994), Jain *et al.* (1994), Akhrif *et al.* (1999), Jiang *et al.* (2001). An alternative to this linearization technique is given by the direct feedback linearization, recent results being available for instance in (Guo *et al.*, 2000; Guo *et al.*, 2001). Somewhat better results can be obtained with the backstepping method, which in opposition to standard linearization techniques, preserves in closed loop the useful nonlinearities of the system. One of its applications to power systems can be found in (Roosta *et al.*, 2001).

A common feature of these methods is that they use the state of the system in order to achieve the desired control action. One important state variable of a synchronous generator is the *rotor angle*, defined as the phase difference between the position of the rotor and a reference that turns at rated speed ω_0. This reference could be the rotor of a specific generator in the power

system, or a conveniently chosen voltage phasor. Unlike the *internal rotor angle*, defined as the phase difference between the position of the rotor and the terminal voltage phasor, which can be accurately computed from local measurements (de Mello, 1994), the rotor angle can only be computed if remote information is used.

Most authors admit indeed that the rotor angle is not readily available, but there are few solution proposals to overcome this problem. For instance, in (Mielczarski and Zajaczkowski, 1994) the authors propose an observer derived from the linearized model, while in (Wang *et al.*, 1993), the angle is obtained during the transitory by simply integrating the rotor speed deviation with respect to ω_0 under the assumption that the error upon the previous steady state value is within reasonable limits.

Since the vast majority of papers on nonlinear control in power systems involve a synchronous machine connected to an infinite bus (a concept that will be clarified in Section 2), the present paper proposes an observer that estimates the rotor angle along with the magnitude of the infinite bus voltage. The observer uses exclusively local measurements, namely the mechanical input torque, exciter output voltage and rotor speed deviation.

The transformations of the model that allow the design of such an observer are presented in Section 3. The corresponding simulation results are shown in Section 4 and then in Section 5 some conclusions are drawn.

2. SINGLE-MACHINE INFINITE BUS SYSTEM MODEL

As in the case of other electrical drives, the equations of the synchronous machine are written in a **d-q** reference frame, which for this particular drive is chosen to revolve with the rotor. The complete set of equations and the significance of the **d-q** coordinate system can be found for instance in (Kundur, 1994). Several degrees of simplification are possible, starting from the complete model, in order to obtain a model that reduces the computational effort in numerical simulations and preserves properties demanded by the analysis level. Generally, stator dynamics are neglected, such that interconnections between elements of the network are described through pure algebraical equations. At rotor side, the subtransient behavior, associated with the damper circuits, may be simplified or even eliminated from the model – see (Ilić and Zaborszky, 2000) for the most encountered approximations of the complete model. With the damper circuits neglected, one

(a)

(b)
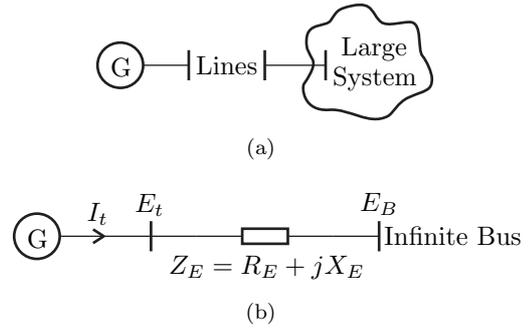
Fig. 1. Generator connected to a large system.

obtains the *one-axis* model, where the electrical characteristics of the machine are represented by a single differential equation, which describes the field circuit dynamics:

$$\frac{\mathrm{d}E_q'}{\mathrm{d}t} = \frac{\omega_0}{T_{d0}'} \left[E_{fd} - E_q' - (X_d - X_d')\, i_d \right]. \quad (1)$$

In the above equation, E_q' represents a voltage proportional to the field flux, E_{fd} is the exciter output voltage, X_d and X_d' represent the **d**-axis synchronous and transient reactance respectively, T_{d0}' represents the **d**-axis open-circuit transient time constant and i_d represents the **d**-axis component of the stator terminal current.

The mechanical characteristics of the machine are described in terms of the rotor speed deviation $\Delta\omega_r$ with respect to the rated speed value ω_0, and the angle δ between the position of the rotor and a reference that turns at rated speed:

$$\frac{\mathrm{d}\Delta\omega_r}{\mathrm{d}t} = \frac{1}{2H}\left(T_m - T_e - K_D\Delta\omega_r\right) \quad (2)$$

$$\frac{\mathrm{d}\delta}{\mathrm{d}t} = \omega_0\Delta\omega_r, \quad (3)$$

with

$$T_e = E_q' i_q + (X_q - X_d') i_d i_q. \quad (4)$$

In these equations, T_m and T_e denote the mechanical and electromagnetic torque respectively, H denotes the inertia constant, K_D denotes the damping factor, X_q denotes the **q**-axis synchronous reactance and i_q denotes the **q**-axis component of the stator terminal current.

Consider now a synchronous machine connected to a large power system through transmission lines. By means of Thévenin's theorem, the general setup of Figure 1(a) can be transformed in the form of Figure 1(b), where the transmission network and the external power system have been replaced by an equivalent impedance, Z_E, and an equivalent voltage, E_B. Since the machine is connected to a large system, it is reasonable to assume that its dynamics do not alter the voltage nor the frequency of the equivalent source E_B; it's the reason why this source is referred to as an *infinite bus*. However, the infinite bus voltage might change as a result of a change in the operating point of the external network.
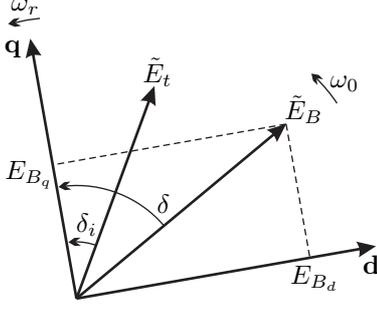
Fig. 2. Rotor angle illustration.

Under the assumption that the infinite bus voltage phasor $\tilde{E}_B$ always turns at speed ω_0, it is natural to consider this phasor as the reference leaded by the **q**-axis with the rotor angle δ, as shown in Figure 2. The choice of the **q**-axis as a means to identify the rotor position with respect to $\tilde{E}_B$ is due to the fact that the same axis is used to identify the internal rotor angle δ_i with respect to the stator terminal voltage phasor $\tilde{E}_t$. As a matter of fact, under no-load conditions, $\tilde{E}_t$ lies on the **q**-axis, thus $\delta_i = 0$, whereas under loaded conditions $\delta_i > 0$.

Define now

$$x_d \triangleq X_d + X_E$$
$$x'_d \triangleq X'_d + X_E$$
$$x_q \triangleq X_q + X_E.$$

With stator and line resistances neglected, the components of the terminal current can be written for the setup of Figure 1(b) as

$$i_d = \frac{E'_q - E_{B_q}}{x'_d} \tag{5}$$

$$i_q = \frac{E_{B_d}}{x_q} \tag{6}$$

with

$$E_{B_d} = E_B \sin \delta \tag{7}$$
$$E_{B_q} = E_B \cos \delta. \tag{8}$$

Consequently, Equations (1) and (4) become respectively

$$\frac{dE'_q}{dt} = \frac{\omega_0}{T'_{d0}} \left(E_{fd} - \frac{x_d}{x'_d} E'_q + \frac{x_d - x'_d}{x'_d} E_{B_q} \right) \tag{9}$$

and

$$T_e = \frac{1}{x'_d} E'_q E_{B_d} - \frac{x_q - x'_d}{x_q x'_d} E_{B_d} E_{B_q}. \tag{10}$$

Finally, taking the time derivatives of E_{B_d} and E_{B_q} and using Equation (3), one gets

$$\frac{dE_{B_d}}{dt} = \omega_0 \Delta\omega_r E_{B_q} \tag{11}$$

$$\frac{dE_{B_q}}{dt} = -\omega_0 \Delta\omega_r E_{B_d}. \tag{12}$$

The differential equation of $\Delta\omega_r$ (2) along with Equations (9)-(12) completely describe the model of the generator connected to the infinite bus.

3. BUS VOLTAGE AND ROTOR ANGLE ESTIMATION

This section presents a solution for the estimation of E_{B_d} and E_{B_q} from available measurements of the output $\Delta\omega_r$ and inputs T_m and E_{fd}.

The necessary step in order to design an observer for a nonlinear system is to put the system in a convenient form. Here, in a similar manner as in (Besançon and Ţiclea, 2003) for induction motors, the nonlinearities will be replaced by some auxiliary state variables, which will yield an affine representation with respect to the variables that are to be estimated.

The input-out map of the original system has to be preserved, so in general the construction starts by taking the output as the first state variable. Then, state variables are created conveniently so that an affine structure is obtained. The construction stops when the derivative of the last created variable only depends on already defined variables.

Starting from Equation (2), one solution is to replace T_e by using (10) and then take $E'_q E_{B_d}$ and $E_{B_d} E_{B_q}$ as state variables. An alternative is to directly take T_e as state variable, which is the solution adopted herein. Both solutions yield in the end the same number of state variables, and apart from the possibility of estimating the electromagnetic torque directly, the second solution has no advantage over the first one.

By proceeding with the differentiation of T_e with respect to time, one obtains

$$\begin{aligned}
\frac{dT_e}{dt} =& \Delta\omega_r \omega_0 (a_1 E'_q E_{B_q} - a_2 E_{B_d}^2 + a_2 E_{B_q}^2) \\
&+ a_3 E_{fd} E_{B_d} - a_4 T_e + a_5 E_{B_d} E_{B_q}
\end{aligned} \tag{13}$$

with the following definitions of constants:

$$a_1 \triangleq \frac{1}{x'_d}$$

$$a_2 \triangleq -\frac{x_q - x'_d}{x_q x'_d}$$

$$a_3 \triangleq a_1 \frac{\omega_0}{T'_{d0}}$$

$$a_4 \triangleq \frac{\omega_0}{T'_{d0}} \frac{x_d}{x'_d}$$

$$a_5 \triangleq a_2 a_4 + a_3 \frac{x_d - x'_d}{x'_d}.$$

In Equation (13), E_{B_d}, $E_{B_d} E_{B_q}$, $E_{B_d}^2$, $E_{B_q}^2$ and $E'_q E_{B_q}$ have to be taken as state variables. The time derivative of E_{B_d} (11) requires that E_{B_q} be a state variable, for whom the time derivative is given by (12). As for the rest of variables, one has

$$\frac{\mathrm{d}E_{B_d}E_{B_q}}{\mathrm{d}t} = \Delta\omega_r\omega_0(E_{B_q}^2 - E_{B_d}^2) \qquad (14)$$

$$\frac{\mathrm{d}E_{B_d}^2}{\mathrm{d}t} = 2\Delta\omega_r\omega_0 E_{B_d}E_{B_q} \qquad (15)$$

$$\frac{\mathrm{d}E_{B_q}^2}{\mathrm{d}t} = -2\Delta\omega_r\omega_0 E_{B_d}E_{B_q} \qquad (16)$$

$$\frac{\mathrm{d}E_q'E_{B_q}}{\mathrm{d}t} = \Delta\omega_r\frac{\omega_0}{a_1}(T_e - a_2 E_{B_d}E_{B_q}) \\ - a_4 E_q'E_{B_q} + a_6 E_{fd}E_{B_q} \qquad (17) \\ + a_7 E_{B_q}^2$$

where

$$a_6 \triangleq \frac{\omega_0}{T_{d0}'}$$

$$a_7 \triangleq a_6 \frac{x_d - x_d'}{x_d'}.$$

The set of equations (11...17) can be put in the form

$$\frac{\mathrm{d}x}{\mathrm{d}t} = A(u,y)x + Bu \qquad (18)$$

$$y = Cx, \qquad (19)$$

with

$$x = \begin{bmatrix} \Delta\omega_r & T_e & E_{B_d} & E_{B_q} & E_{B_d}E_{B_q} & E_{B_d}^2 & E_{B_q}^2 & E_q'E_{B_q} \end{bmatrix}^T$$

$$u = \begin{bmatrix} T_m & E_{fd} \end{bmatrix}^T \qquad y = \Delta\omega_r$$

and the definitions of matrices A, B and C from Appendix A.

Further on, a Kalman-like observer can be designed as in Equations (20) and (21), which offers arbitrarily fast exponential convergence provided the existence of sufficient excitation (Besançon *et al.*, 1996).

$$\frac{\mathrm{d}\hat{x}}{\mathrm{d}t} = A(u,y)\hat{x} + Bu - S^{-1}C^T(C\hat{x} - y) \qquad (20)$$

$$\frac{\mathrm{d}S}{\mathrm{d}t} = -\lambda S - A^T(u,y)S - SA(u,y) + C^TC \qquad (21)$$

4. SIMULATION RESULTS

The above estimation technique was tested in simulation for the system given by Equations (2), (3), (9) and (10) with the following values of parameters:

$$X_d = 1.81 \text{ pu}, X_d' = 0.30 \text{ pu}, X_q = 1.76 \text{ pu},$$
$$X_E = 0.65 \text{ pu}, H = 3.5 \text{ pu}, K_D = 10 \text{ pu},$$
$$\omega_0 = 377 \text{ rad/s}, T_{d0}' = 8 \text{ s}.$$

The initial values of the infinite bus voltage components E_{B_d} and E_{B_q} were such that at rated speed

$$E_t = 1 \text{ pu}, P = 0.5 \text{ pu}, Q = 0.3 \text{ pu},$$

that is

$$E_{B_d} = 0.68 \text{ pu}, E_{B_q} = 0.53 \text{ pu},$$

or equivalently

$$E_B = 0.86 \text{ pu}, \delta = 51.87 \text{ degrees}.$$

The observer was initialized with values computed from erroneous initial guesses of E_{B_d} and E_{B_q} that contained 50% and 100% errors respectively. The tuning parameter λ was set at 500.

With mechanical input torque and exciter output voltage kept constant at values corresponding to the above equilibrium point but with very low magnitude excitation added to the exciter output, the evolution of the estimation errors is presented by Figure 3.
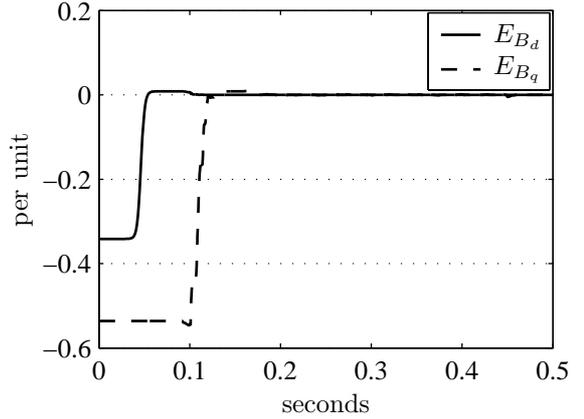
Fig. 3. Estimation errors at steady state.

With the system operating at the same point as above, a sudden increase of 0.04 pu is now considered in the mechanical torque. As a result, the rotor oscillates until the new equilibrium is reached. Note that this kind of behavior is shared by another situation in which the mechanical power exceeds the electrical power, namely the sudden loss of a part of the load. For our example, the oscillations are illustrated in Figure 4 through the evolution of the infinite bus voltage components in rotor coordinates. The corresponding estimation errors are presented in Figure 5.
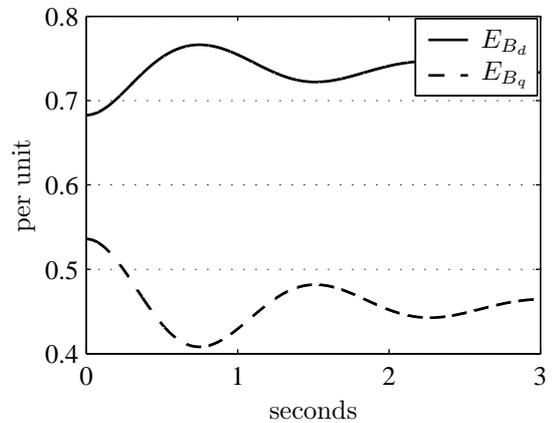
Fig. 4. Infinite bus voltage components in rotor coordinates during rotor swings.
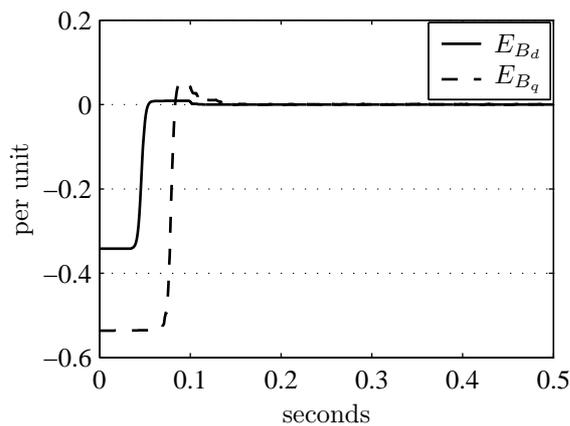
Fig. 5. Estimation errors during rotor swings.

5. CONCLUSIONS

The simulation results show that the proposed methodology is effective if sufficiently rich signals are supplied to the observer. Moreover, with this condition fulfilled the observer provides global exponential convergence, which is an important feature of the method.

Additional signals were added to the DC excitation voltage in order to ensure a sufficient frequency content, but it is worth noticing that in practice, the exciter's output is delivered by a commutation-based converter, which already provides a certain level of excitation. However, investigations through experiments on physical machines should check whether this is enough to ensure observer convergence.

From the power system point of view, the advantage is the possibility to estimate, in a configuration involving a generator and a large network, the magnitude of the infinite bus equivalent phasor and the angle between this phasor and the position of the machine's rotor. This information can be useful for both control and monitoring purposes.

Further improvements of this method, such as the estimation of line parameters and the validation of the estimator on more complex setups are the object of under way studies.

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Appendix A. DEFINITION OF MATRICES IN
EQUATIONS (18)-(19)

$$A = \begin{bmatrix} -\dfrac{K_D}{2H} & -\dfrac{1}{2H} & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & -a_4 & a_3 u(2) & 0 & a_5 & -a_2\omega_0 y & a_2\omega_0 y & a_1\omega_0 y \\ 0 & 0 & 0 & \omega_0 y & 0 & 0 & 0 & 0 \\ 0 & 0 & -\omega_0 y & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & -\omega_0 y & \omega_0 y & 0 \\ 0 & 0 & 0 & 0 & 2\omega_0 y & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & -2\omega_0 y & 0 & 0 & 0 \\ 0 & \dfrac{\omega_0}{a_1}y & 0 & \omega_0 u(2) & -\dfrac{a_2}{a_1}\omega_0 y & 0 & a_7 & -a_4 \end{bmatrix}$$

$$B = \begin{bmatrix} \dfrac{1}{2H} & 0 \\ 0 & 0 \\ \cdots\cdots \\ 0 & 0 \end{bmatrix}$$

$$C = \begin{bmatrix} 1 & 0 & \cdots & 0 \end{bmatrix}$$