DISCRETE-TIME MODEL FOR LINEAR CONTINUOUS-TIME REPETITIVE SYSTEM

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Abstract. A discrete-time model is proposed for continuous-time repetitive process. Model is calculated based on the zero order hold on input approach and trapezoidal approximation of output signal on input to the state equation. Numerical examples are presented.

Keywords: repetitive systems, iterative learning control, discrete-time model, linear systems, continuous-time systems.

INTRODUCTION

Repetitive or multipass processes play important role in industry. We say that the process is repetitive if the same action is repeated many times and the time interval of this action is constant and finite. The process can be illustrated by considering machining operations where the material or workpiece involved is processed by a sequence of passes of the processing tool. Classical examples of repetitive process are coal miner in strip mine or metal rolling operations (Rogers and Owens, 1992). Other examples one can find in paper and steel industry, agriculture, etc.

We will deal with the problem of calculation of discrete-time model for continuous-time linear repetitive system. In the literature one can find references to the so-called single step, trapezoidal and single step higher order discretization methods for calculation of the model, see for instance (Gałkowski *et al*., 1999; Gałkowski, 2000; Gramacki, 2000). There are also some references to calculation of discrete-time model for continuous-time 2-D Roesser system with application of step-wise approximation of state signal (Chen et. all., 1999). In this note we present an exact calculation of the discrete-time model under assumption of the step-wise and trapezoidal-wise approximation of the input signals to the differential state equation of the system. The model is easy for calculation and seems to be better than the other known models. We will also discuss stability problem of the calculated model.

PROBLEM FORMULATION

Mathematical model of continuous-time repetitive process can be given by the following equation (Rogers and Owens, 1992)

$$\begin{aligned} \dot{x}_{l+1} &= A_c x_{l+1} + B_c u_{l+1} + E_c y_l \\ y_{l+1} &= C_c x_{l+1} + D_c u_{l+1} + F_c y_l \end{aligned} \tag{1}$$

where $x \in R^n$ is a state vector, $u \in R^m$ is an input signal, $y \in R^p$ is an output signal and l denotes a repetition; A_c, B_c, E_c, C_c, D_c, F_c are real matrices of appropriate dimensions. It is assumed that every repetition is carried out in finite constant time $t \in [0,T]$. Initial conditions for the system are as follows

$$x_l(0) = x_{0l}, \quad l = 1,2... \text{ and } y_0 = y_0(t), \quad t \in [0,T] \tag{2}$$

The problem can be now formulated the following way: given repetitive process (1), find discrete-time model for the system in the form

$$\begin{aligned} x_{l+1}(k+1) &= Ax_{l+1}(k) + Bu_{l+1}(k) + Ey_l(k) \\ y_{l+1}(k) &= Cx_{l+1}(k) + Du_{l+1}(k) + Fy_l(k) \end{aligned} \tag{3}$$

It is expected that model (3) will have the same properties as system (1), e.g. concerning system stability. Moreover, we are interested to find a model

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that retains system properties with bigger sampling time. This model will be more useful in engineering applications.

STEP-WISE APPROXIMATION MODEL

It is well known that solution x to equation (1) has the following form

$$x_{l+1}(t+T)=e^{A_cT}x_{l+1}(t)+\int_0^T e^{A_c\tau}B_c u_{l+1}(t+\tau)d\tau + \int_0^T e^{A_c\tau}E_c y_l(t+\tau)d\tau \tag{4}$$

For calculation of the integrals one can assume that signals u and y can be approximated by step-wise functions: $u(t+\tau)=u(t)$ and $y(t+\tau)=y(t)$ for $\tau\in[0,T)$. This model we will call DSS (Direct-Step-Step).

Then, one obtains from (4)

$$x_{l+1}(t+T)=e^{A_cT}x_{l+1}(t)+\int_0^T e^{A_c\tau}B_c u_{l+1}(t)d\tau + \int_0^T e^{A_c\tau}E_c y_l(t)d\tau \tag{5}$$

Thus, assuming that T_p denotes sampling time, the discrete-time model (3) has the following matrices

$$A=e^{A_cT_p},\quad B=\int_0^{T_p} e^{A_c\tau}B_c d\tau,\quad E=\int_0^{T_p} e^{A_c\tau}E_c d\tau,$$
$$C=C_c,\quad D=D_c,\quad F=F_c$$

For comparison, assuming that integral of state x can be approximated by trapezoid one obtains the following matrices for model (3) (Gramacki, 2000)

$$A=\left(I+A_c\frac{T_p}{2}\right)\left(I-A_c\frac{T_p}{2}\right)^{-1}$$
$$B=(I+A_c)B_cT_p,\quad E=(I+A_c)E_cT_p \tag{6}$$
$$C=C_c\left(I-A_c\frac{T_p}{2}\right)^{-1},\quad D=D_c,\quad F=F_c$$

This model we will call TSS (Trapezoidal-Step-Step).

STEP-WISE AND RAMP-WISE APPROXIMATION MODEL

For calculation of the first integral in (4) one can assume that input signal u can be approximated by step-wise function as above. This approximation is exact in the case when computer generates control input signal. Unfortunately, this approximation is not appropriate for calculation of the second integral, y_k is not a step-wise signal since it is output of the continuous-time system from previous operation. Therefore, we propose ramp-wise approximation for signal $y_l(k+\tau)$, $\tau\in[0,T)$, in the second integral in (4), namely

$$y_l(t+\tau)=y_l(t)+[y_l(t+T)-y_l(t)]\frac{\tau}{T}\quad \text{for}\quad \tau\in[0,T).$$

This model we will call DST (Direct-Step-Trapezoidal).

Then, one obtains from (4)

$$x_{l+1}(t+T)=e^{A_cT}x_{l+1}(t)+\int_0^T e^{A_c\tau}B_c d\tau u_{l+1}(t) + \int_0^T e^{A_c\tau}E_c d\tau y_l(t) + \int_0^T e^{A_c\tau}E_c\tau d\tau\frac{y_l(t+T)-y_l(t)}{T} \tag{7}$$

Calculating the third integral one gets

$$\int_0^T e^{A_c\tau}E_c\tau d\tau=\left[\int e^{A_c\tau}d\tau E_c\tau-\int\int e^{A_c\tau}d\tau E_c d\tau - C_2\right]_{\tau=0}^{\tau=T}$$
$$=\int e^{A_c\tau}d\tau\Big|_{\tau=T}E_c-\int_0^T\int e^{A_c\tau}d\tau d\tau E_c$$

Henceforth, one can present (7) in the following form

$$x_{l+1}(t+T)=\tilde{A}x_{l+1}(t)+\tilde{B}u_{l+1}(t)+\tilde{E}y_l(t) + (\tilde{E}_1-\tilde{E}_2)[y_l(t+T)-y_l(t)] \tag{8}$$

where

$$\tilde{A}=e^{A_cT},\quad \tilde{B}=\int_0^T e^{A_c\tau}B_c d\tau,\quad \tilde{E}=\int_0^T e^{A_c\tau}E_c d\tau$$
$$\tilde{E}_1=\int e^{A_c\tau}E_c d\tau\Big|_{\tau=T},\quad \tilde{E}_2=\frac{1}{T}\int_0^T\int e^{A_c\tau}E_c d\tau d\tau$$

Next, using the series expansion of the e^{At} one obtains

$$\int e^{At}dt=\int\left(I+\frac{t}{1!}A+\frac{t^2}{2!}A^2+\ldots\right)dt$$
$$=It+\frac{1}{2!}At^2+\frac{1}{3!}A^2t^3+\frac{1}{4!}A^3t^4+\ldots$$

Thus, we have $\int e^{At}Bdt\Big|_{t=0}=0$. Similarly one obtains $\int\int e^{At}Bdtdt\Big|_{t=0}=0$. From this it follows that $\tilde{E}=\tilde{E}_1$. Hence, we have from (8)

$$x_{l+1}(t+T)=\breve{A}x_{l+1}(t)+\breve{B}u_{l+1}(t)+\breve{E}y_l(t) + \breve{E}_T y_l(t+T) \tag{9}$$

where

$$\breve{A}=\tilde{A}=e^{A_cT},\quad \breve{B}=\tilde{B},\quad \breve{E}=\tilde{E}-\tilde{E}_1+\tilde{E}_2=\tilde{E}_2$$
$$\breve{E}_T=\tilde{E}_1-\tilde{E}_2=\tilde{E}-\tilde{E}_2$$

Let $T=T_p$ where T_p is a sampling time and $t=kT_p$. Based on the above one can write the following discrete-time model for system (1)

$$x_{l+1}(k+1)=\hat{A}x_{l+1}(k)+\hat{B}u_{l+1}(k)+\hat{E}y_l(k) + \hat{E}_T y_l(k+1) \tag{10}$$
$$y_{l+1}(k)=C_cx_{l+1}(k)+D_cu_{l+1}(k)+F_cy_l(k)$$

where

$$\hat{A}=\tilde{A}\Big|_{T=T_p},\hat{B}=\tilde{B}\Big|_{T=T_p},\hat{E}=\tilde{E}_2\Big|_{T=T_p},\hat{E}_T=(\tilde{E}-\tilde{E}_2)\Big|_{T=T_p}$$

Next, introducing the new vector χ as follows

$$\chi_{l+1}(k+1) = x_{l+1}(k+1) - \widehat{E}_T y_l(k+1) \tag{11}$$

one obtains from (10) a discrete-time model for system (1)

$$\begin{aligned} \chi_{l+1}(k+1) &= A\chi_{l+1}(k) + Bu_{l+1}(k) + Ey_l(k) \\ y_{l+1}(k) &= C\chi_{l+1}(k) + Du_{l+1}(k) + Fy_l(k) \end{aligned} \tag{12}$$

where

$$A = \widehat{A} = e^{A_c T_p},\ B = \widehat{B} = \int_0^{T_p} e^{A_c\tau} B_c d\tau,\ C = C_c,\ D = D_c$$

$$\begin{aligned} E &= \widehat{E} + \widehat{A}\widehat{E}_T \\ &= \frac{1}{T_p}(I - e^{A_c T_p}) \int_0^{T_p}\!\!\int e^{A_c\tau} E_c d\tau d\tau + e^{A_c T_p} \int_0^{T_p} e^{A_c\tau} E_c d\tau \end{aligned}$$

$$F = F_c + C_c \widehat{E}_T = F_c + C_c \int_0^{T_p} e^{A_c\tau} E_c d\tau$$

Based on (2) one easily finds initial conditions for this system

$$\chi_l(0) = x_{0l} - \widehat{E}_T y_{l-1}(0), \quad l = 1,2,...$$

and

$$y_0 = y_0(t), \quad t \in [0, T] \tag{13}$$

Considering stability of system (12) along the repetitions we find that the system is asymptotically stable if and only if all eigenvalues of F are inside the unit circle, e.g. (Kurek and Zaremba, 1993). From series expansion of e^{At} one has

$$\begin{aligned} \widehat{E}_T &= \int_0^{T_p} e^{A_c\tau} E_c d\tau - \frac{1}{T_p} \int_0^{T_p}\!\!\int e^{A_c\tau} E_c d\tau d\tau \\ &= \left(\frac{1}{2!} IT_p + \frac{2}{3!} A_c T_p^2 + \frac{3}{4!} A_c^2 T_p^3 + \frac{4}{5!} A_c^3 T_p^4 + \dots \right) E_c \end{aligned}$$

Thus, it follows from (12) and the above that eigenvalues of matrix F depend on T_p and E_c. However, for $T_p \to 0$ we have $\widehat{E}_T \to 0$ and $F \to F_c$. Thus, the discrete time model retains stability properties of continuous time system for small enough sampling time T_p, i.e. placement of eigenvalues of F is similar to the placement of eigenvalues of F_c. System dynamics along the time is properly modeled.

Summarizing, in order to obtain proper model the sampling time T_p for the system should be chosen in such a way that:

1. Matrix F has to have eigenvalues in the same region as matrix F_c, i.e. inside or outside the unit circle in order to retain stability properties of the continuous-time system (1).
2. System dynamics along the time is properly modeled; according to the practical suggestions one should presume for asymptotically stable processes $T_p \le T_{95}/6$, where T_{95} is a setting time of the 95% step response, e.g. (Iserman, 1989).

For trapezoidal approximation of integrals of state x and output y, and step-wise approximation of integral of input u in (1) one obtains the following matrices for model (3), (Kurek, 1995):

$$\begin{aligned} A &= \left(I + A_c \frac{T_p}{2} \right)\left(I - A_c \frac{T_p}{2} \right)^{-1} \\ B &= (I + A_c) B_c T_p, \quad E = (I + A_c) E_c \frac{T_p}{2} \\ C &= C_c \left(I - A_c \frac{T_p}{2} \right)^{-1}, \quad D = D_c \\ F &= F_c + CE_c \frac{T_p}{2} = F_c + C_c \left(I - A_c \frac{T_p}{2} \right)^{-1} E_c \frac{T_p}{2} \end{aligned} \tag{14}$$

The model we call TST (Trapezoidal-Step-Trapezoidal). Clearly, for $T_p \to 0$ we have $F \to F_c$, too.

RAMP-WISE APPROXIMATION MODEL

The proposed model (12) is appropriate for modeling of control system where u is a step-wise control signal, e.g. computer generated control input. However, in the case of a system where u is a continuous-time signal it could be suitable to calculate the first integral in (4) using ramp-wise approximation: $u_l(t+\tau) = u_l(t) + [u_l(t+T) - y_l(t)]\frac{\tau}{T}$ for $\tau \in [0,T)$. This model we will denote as DTT (Direct-Trapezoidal-Trapezoidal). Then, one obtains similarly to (7)

$$\begin{aligned} x_{l+1}(t+T) &= e^{A_c T} x_{l+1}(t) + \int_0^T e^{A_c\tau} B_c d\tau u_{l+1}(t) \\ &+ \int_0^T e^{A_c\tau} B_c \tau d\tau \frac{u_{l+1}(t+T) - u_{l+1}(t)}{T} \\ &+ \int_0^T e^{A_c\tau} E_c d\tau y_l(t) \\ &+ \int_0^T e^{A_c\tau} E_c \tau d\tau \frac{y_l(t+T) - y_l(t)}{T} \end{aligned}$$

Next, similarly to (8) we have

$$\begin{aligned} x_{l+1}(t+T) &= \widetilde{A} x_{l+1}(t) + \widetilde{B} u_{l+1}(t) \\ &+ (\widetilde{B}_1 - \widetilde{B}_2)[u_l(t+T) - u_{l+1}(t)] + \widetilde{E} y_l(t) \\ &+ (\widetilde{E}_1 - \widetilde{E}_2)[y_l(t+T) - y_l(t)] \end{aligned}$$

where

$$\widetilde{B}_1 = \int e^{A_c\tau} B_c d\tau \Big|_{\tau=T}, \quad \widetilde{B}_2 = \frac{1}{T} \int_0^T\!\!\int e^{A_c\tau} B_c d\tau d\tau$$

It can be easily shown, as in the case of $\widetilde{E}_1$, that $\widetilde{B} = \widetilde{B}_1$.

Finally, we have form similar to (9)

$$\begin{aligned} x_{l+1}(t+T) &= \breve{A} x_{l+1}(t) + \breve{B} u_{l+1}(t) + \breve{B}_T u_{l+1}(t+T) \\ &+ \breve{E} y_l(t) + \breve{E}_T y_l(t+T) \end{aligned}$$

where

$$\breve{B} = \widetilde{B} - \widetilde{B}_1 + \widetilde{B}_2 = \widetilde{B}_2, \quad \breve{B}_T = \widetilde{B}_1 - \widetilde{B}_2 = \widetilde{B} - \widetilde{B}_2$$

Then, for $T=T_p$ one obtains

$$\begin{aligned} x_{l+1}(k+1) &= \hat{A}x_{l+1}(k) + \hat{B}u_{l+1}(k) + \hat{B}_T u_{l+1}(k+1) \\ &\quad + \hat{E}y_l(k) + \hat{E}_T y_l(k+1) \\ y_{l+1}(k) &= C_c x_{l+1}(k) + D_c u_{l+1}(k) + F_c y_l(k) \end{aligned} \tag{15}$$

where

$$\hat{B} = \tilde{B}_2\Big|_{T=T_p}, \quad \hat{B}_T = (\tilde{B} - \tilde{B}_2)\Big|_{T=T_p}$$

Using notation similar to (11)

$$\eta_{l+1}(k+1) = x_{l+1}(k+1) - \hat{B}_T u_{l+1}(k+1) - \hat{E}_T y_l(k+1)$$

one obtains from (15) the following discrete-time model for system (1)

$$\begin{aligned} \eta_{l+1}(k+1) &= A\eta_{l+1}(k) + Bu_{l+1}(k) + Ey_l(k) \\ y_{l+1}(k) &= C\eta_{l+1}(k) + Du_{l+1}(k) + Fy_l(k) \end{aligned} \tag{16}$$

where

$$A = \hat{A} = e^{A_c T_p}, \qquad C = C_c$$

$$\begin{aligned} B &= \hat{B} + \hat{A}\hat{B}_T \\ &= (I + e^{A_c T_p})\int_0^{T_p} e^{A_c\tau}B_c d\tau - \frac{1}{T_p}e^{A_c T_p}\int_0^{T_p}\int e^{A_c\tau}B_c d\tau d\tau \\ E &= \hat{E} + \hat{A}\hat{E}_T \\ &= \frac{1}{T_p}(I - e^{A_c T_p})\int_0^{T_p}\int e^{A_c\tau}E_c d\tau d\tau + e^{A_c T_p}\int_0^{T_p} e^{A_c\tau}E_c d\tau \\ D &= D_c + C_c\hat{B}_T \\ &= D_c + C_c\left(\int_0^{T_p} e^{A_c\tau}B_c d\tau - \frac{1}{T_p}\int_0^{T_p}\int e^{A_c\tau}B_c d\tau d\tau\right) \\ F &= F_c + C_c\hat{E}_T \\ &= F_c + C_c\left(\int_0^{T_p} e^{A_c\tau}E_c d\tau - \frac{1}{T_p}\int_0^{T_p}\int e^{A_c\tau}E_c d\tau d\tau\right) \end{aligned}$$

Based on (2) one easily finds initial conditions for this system

$$\eta_l(0) = x_{0l} - \hat{B}_T u_l(0) - \hat{E}_T y_{l-1}(0), \quad l = 1,2,...$$

and

$$y_0 = y_0(t), \quad t \in [0,T] \tag{17}$$

Using trapezoidal approximation of integrals of state x, input u and output y in (1) one obtains the following matrices for TTT (Trapezoidal-Trapezoidal-Trapezoidal) model (2), similar to (14)

$$A = \left(I + A_c\frac{T_p}{2}\right)\left(I - A_c\frac{T_p}{2}\right)^{-1}$$

$$B = (I + A_c)B_c\frac{T_p}{2}, \quad E = (I + A_c)E_c\frac{T_p}{2}$$

$$C = C_c\left(I - A_c\frac{T_p}{2}\right)^{-1} \tag{18}$$

$$D = D_c + C_c\left(I - A_c\frac{T_p}{2}\right)^{-1}B_c\frac{T_p}{2}$$

$$F = F_c + C_c\left(I - A_c\frac{T_p}{2}\right)^{-1}E_c\frac{T_p}{2}$$

NUMERICAL EXAMPLES

Consider the first order repetitive process given in (Gałkowski, 2000) for $t\in[0,2]$ (Process 7)

$$\begin{aligned} \dot{x}_{l+1} &= -0.5x_{l+1} + u_{l+1} + 0.5y_l \\ y_{l+1} &= x_{l+1} + 0.9y_l \end{aligned} \tag{19}$$

Initial conditions and control input to the system were as follows

$$x_{l+1}(0)=0 \quad \text{for} \quad l=0,1,..., \quad y_0(t)=0 \quad \text{for} \quad t\in[0,2]$$

and

$$u_l(t) = \begin{cases} 0.5t & \text{for } l = 0 \\ 0 & \text{for } l > 0 \end{cases}, \quad t\in[0,2]$$

It is easy to note that the system is asymptotically stable in the time and repetition domains and its setting time $T_{95}\cong 6$ in time domain. Thus, in order to obtain well-defined model of the process along time t one should assume $T_p\le 1$. However, for $T_p=1$ we have $F=1.0804$ for DTT model (16). Thus, the discrete-time model is unstable and one should assume smaller sampling time. Note, that TTT model (18) is also unstable in this case, one obtains then $F=1.1$.

The system is also unstable for sampling time $T_p=0.5$. However, for $T_p=0.4$ we have the following matrices of the discrete-time model DTT (16) and TTT (18), respectively

$$A=0.8187,\ B=0.3308,\ E=0.1752,$$
$$C=1,\ D=0,\ F=0.9876$$

and

$$A=0.8182,\ B=0.3636,\ E=0.1818,$$
$$C=0.9091,\ D=0.1818,\ F=0.9909$$

In fig. 1 there are presented output of the real system ($T_p=0.001$) and DTT and TTT models with $T_p=0.4$. It is easy to see that both models are stable, but outputs calculated using the models are unacceptable, they are too big. However, output of the DTT model is significantly smaller.

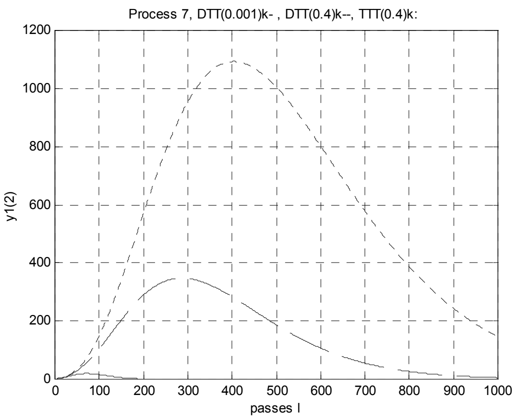

Fig. 1. Output in time $t=2$ versus repetitions l of the proposed DTT model (16) and TTT model (18) with sampling times $T_p=0.001$ and $T_p=0.4$ for system (19).

Next we have calculated models DST (12) and TST (14) with $T_p=0.2$ for the system receiving the following matrices

$$A = 0.9048,\ B = 0.1903,\ E = 0.0907,$$
$$C = 1,\ D = 0,\ F = 0.9468$$

and, respectively

$$A = 0.9048,\ B = 0.2000,\ E = 0.0952,$$
$$C = 0.9524,\ D = 0,\ F = 0.9476$$

In fig. 2 and 3 there are presented graphs similar to the ones given in (Gałkowski, 2000). We see that DTT and TTT models generate almost exact outputs of the system. They are much better then the output calculated by method proposed in (Gałkowski, 2000), compare fig. 3 with fig. 4 and 7 presented in (Gałkowski, 2000). Models DST and TST give very similar outputs but much worse then DTT or TTT models. In general, it is clear that the difference depends if nature of the input signal to the model state equation is properly recognized and taken into account. If the signal is different than step-wise then for larger sampling period we get worse results using step approximation than trapezoidal approximation. The same problem occurs when one uses trapezoidal approximation for step-wise signal. The difference should be made at the stage when the model is chosen. Of course, if the sampling time is small enough there is no difference. This can be seen in fig. 4 where continuous line in the middle refers to the output of the real system.

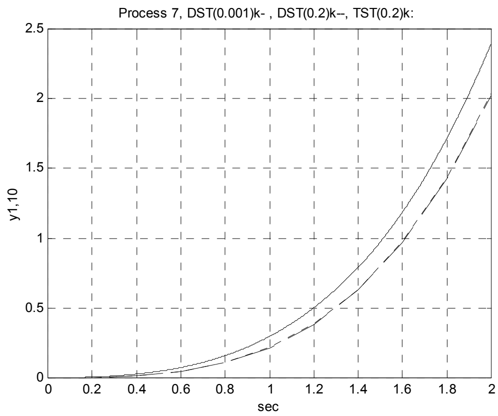

Fig. 2. Output in the 10th repetition of the proposed DST model (12) and TST model (14) with sampling times T_p=0.001 and T_p=0.2 for system (19).

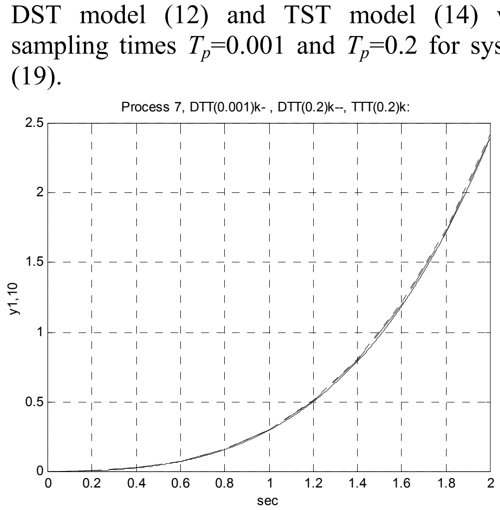

Fig. 3. Output in the 10th repetition of the proposed DTT model (16) and TTT model (18) with sampling times T_p=0.001 and T_p=0.2 for system (20).

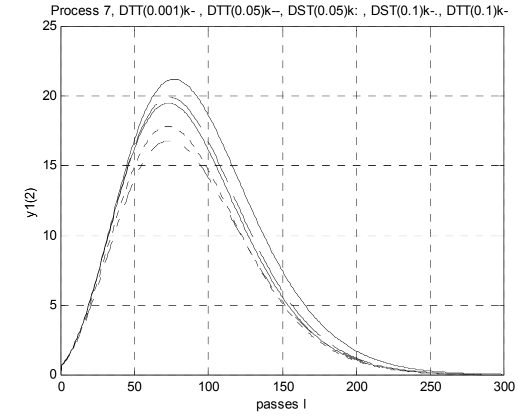

Fig. 4. Output in time t=2 versus repetitions l of the proposed DTT model (16) and TTT model (18) with sampling times T_p=0.001 and T_p=0.2 for system (20).

Now consider the following first order repetitive process (Process 4) given in (Gramacki, 2000)

$$\begin{aligned} \dot{x}_{l+1} &= -16.36 x_{l+1} + 9.09 y_l \\ y_{l+1} &= x_{l+1} + 0.8 y_l \end{aligned} \qquad (20)$$

Initial conditions for the system were as follows

$$x_{l+1}(0)=0 \quad \text{for} \quad l=0,1,\ldots \quad \text{and} \quad y_0(t)=1 \quad \text{for} \quad t\in[0,2]$$

One easily finds that the system is asymptotically stable in the time and repetition domains and its setting time $T_{95}\cong 0.1834$. As previously mentioned, in order to obtain well-defined model of the process along time t, one should assume $T_p \le 0.03$. However, it is interesting to note that for greater T_p=0.1 we have DST model (12) stable but TST model (14) unstable. In the latter case we obtain F=1.0804.

For T_p=0.03 we have the following matrices of the discrete-time model DST (12) and TST (14)

$$A = 0.6121,\ E = 0.1771,\ C = 1,\ F = 0.8990$$

and, respectively

$$A = 0.6059,\ E = 0.2190,\ C = 0.8030,\ F = 0.9095$$

In fig. 5 we see that for larger sampling time that is a great difference between the same approximation of the input but different approximation of the state vector, trapezoidal and exact solution, TST and DST models. Continuous line refers to the real output of the system (T_p=0.001). From fig. 6 one finds that for small sampling time one gets the same output of the system independent on the approximation of the input signals to state equation. However, we note that using step approximation of the system output instead of trapezoidal one we have to increase calculation effort more than 10 times, the sampling time for DSS model should be 10 times smaller than for DST model for the system in order to obtain similar simulation results.

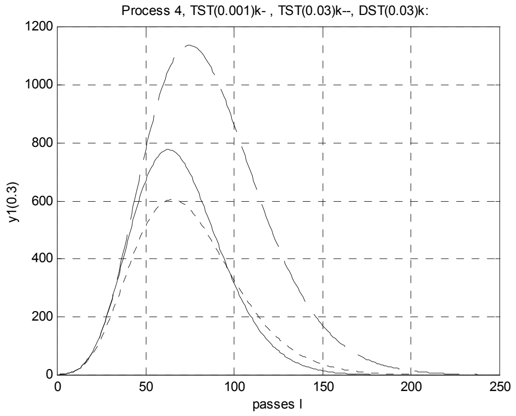

Fig. 5. Output in time t=0.3 versus repetitions l of the proposed DST model (12) and TST model (14) with sampling times T_p=0.001 and T_p=0.03 for repetitive process (20).

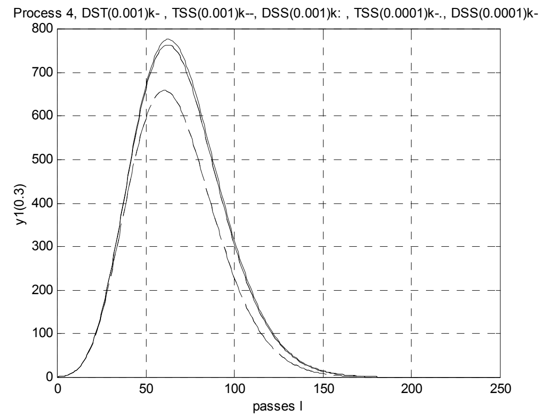

Fig. 6. Output in time t=0.3 versus repetitions l of the proposed DTT model (16) and TTT model (18) with sampling times T_p=0.001 and T_p=0.0001 for repetitive process (20).

CONCLUDING REMARKS

New method for calculation of discrete-time model for continuous-time repetitive processes is presented. Proposed model could be useful for computer simulation of repetitive processes. It has some advantages comparing with other models. This is illustrated by numerical examples of calculation of simple repetitive processes. Note, that one has to take into account the knowledge about the input signals to the repetitive system state equation, i.e. if it is a step-wise signal or not.

The proposed model usually enables ones to calculate discrete-time model with sampling time larger than using other methods. However, it should be noted, that since there is a change of matrix F in discrete-time model it can affect stability of the process along passes. In this case one should choose smaller sampling time in order to preserve stability of the repetition process. A simple suggestion for the choice of sampling time is given.

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