EFFECTS OF LOCAL ACTUATOR ACTION ON THE CONTROL OF LARGE FLEXIBLE STRUCTURES

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Abstract:
Active vibration damping of flexible structures requires the positioning of actuators and sensors along the structure. When the actuators become particularly small in comparison to the structure a large number of modes is necessary to accurately represent the response of the structure. Neglecting these higher modes may result in a spillover phenomenon which is capable of destabilizing the closed loop system. Therefore proper correction methods to account for the dynamics of higher modes in the structural model have to be found. One such possibility is the utilization of Frequency Response Modes (FRM), a special form of the particular solution of the equations of motion of a flexible structure. In this paper the FRM is defined and used for model correction of a simply supported beam. Finally, correction terms are used to quantize local actuator action.

Keywords: actuator placement, local actuator action, Frequency Response Mode, model correction

1. INTRODUCTION

Structural control is a well established control application. Damping vibrations of machinery parts, vehicle- or space structures is necessary for their functionality as well as for the reduction of noise radiation.
For these applications the controller is designed using a low order model of the flexible structure. Such a model is usually obtained by finite element (FE) methods and thus has to be reduced for the purpose of controller design.
Since usually only the first few modes of the flexible structure are to be controlled, the FE model is often reduced by direct truncation, i.e. neglecting the higher modes of the system leading to a linear modal model. This may lead to problems - and in the worst case - to instabilities of the closed loop system because neglected dynamics are excited. This phenomenon is called spillover and is well documented in the literature (Balas, 1978) and (Balas, 1982).

This work points out, that especially when the the actuator is small in comparison to the flexible structure, the model of the system has to be corrected to reduce spillover effects. For example, this is important for the active vibration damping of a railcar-body (Horwatitsch, 2004). The proposed tool for this correction is called the 'Frequency Response Mode - FRM' which is a particular solution of the flexible system under consideration. In difference to the literature, where many investigations concerning beams and plates, e.g. (Halim and Moheimani, 2001), can be found, the local actuator action has not been examined yet. This is necessary because these local actions require higher modes for the accurate composition of the system response. Utilizing the FRM a simple yet efficient method for model correction is proposed. The performance of the new method to suppress spillover is demonstrated, and the consequences for actuator and sensor placement are stated. Furthermore, the principles of an extended positioning criterion are outlined.

This work summarizes the theory of FRM in section 2 to compare the results with the control engineering notation in section 3. In section 4 these results are verified using a very simple example of a beam with pinned-pinned boundary conditions. Finally, the basic idea concerning a criterion for the optimal placement of actuators and sensors is proposed to give some outlook on future work.

2. MODEL REDUCTION AND FREQUENCY RESPONSE MODE (FRM)

2.1 Modal model

The static and dynamic properties of large flexible structures with N degrees of freedom are generally described by the following system of linear differential equations

$$\boldsymbol{M}\ddot{\boldsymbol{x}} + \boldsymbol{C}\dot{\boldsymbol{x}} + \boldsymbol{K}\boldsymbol{x} = \boldsymbol{f}(t), \tag{1}$$

where $\boldsymbol{M}$ is the $(N \times N)$ mass matrix, $\boldsymbol{C}$ the $(N \times N)$ damping matrix, $\boldsymbol{K}$ the $(N \times N)$ stiffness matrix, $\boldsymbol{x}$ the $(N \times 1)$ vector of generalized coordinates and $\boldsymbol{f}(t)$ the $(N \times 1)$ vector of the generalized excitation forces. These matrices are normally computed by finite element methods. With the help of an appropriate transformation matrix $\boldsymbol{\Phi}$ a transformation of (1) to diagonal form is accomplished. If one substitutes

$$\boldsymbol{x}(t) = \boldsymbol{\Phi}\boldsymbol{z}(t) = \sum_{j=1}^{N} \boldsymbol{\phi}_j z_j(t), \tag{2}$$

where $\boldsymbol{\Phi}$ contains the eigenvectors of (1) as column vectors, into (1) and multiplies (1) with $\boldsymbol{\Phi}^T$ from the left, one yields

$$\boldsymbol{\mu}\ddot{\boldsymbol{z}} + 2\boldsymbol{\mu}\boldsymbol{\zeta}\boldsymbol{\Omega}\dot{\boldsymbol{z}} + \boldsymbol{\mu}\boldsymbol{\Omega}^2\boldsymbol{z} = \boldsymbol{\Phi}^T\boldsymbol{f}(t) \tag{3}$$

with

$$\begin{aligned} \boldsymbol{\Phi}^T\boldsymbol{M}\boldsymbol{\Phi} &= diag(\mu_i) = \boldsymbol{\mu} \\ \boldsymbol{\Phi}^T\boldsymbol{K}\boldsymbol{\Phi} &= diag(\mu_i\omega_i^2) = \boldsymbol{\mu}\boldsymbol{\Omega}^2 \\ \boldsymbol{\Phi}^T\boldsymbol{C}\boldsymbol{\Phi} &= 2diag(\mu_i\zeta_i\omega_i) = 2\boldsymbol{\mu}\boldsymbol{\zeta}\boldsymbol{\Omega}. \end{aligned} \tag{4}$$

Here $\boldsymbol{\mu}$ is the matrix of modal masses, $\boldsymbol{\zeta}$ the matrix of modal damping and $\boldsymbol{\Omega}$ the matrix of eigenfrequencies of (1).
Representation (3) of (1) in modal coordinates is also called modal decomposition. By (3) the system is completely decoupled and represented by N single degree of freedom oscillators.
The number N is often very large and has to be reduced to $n < N$ modes for control purposes. When n eigenmodes are used to represent the system dynamics, additional functions become necessary for the accurate description of the elastic deformations. For that purpose the usage of particular solutions of (1) is advantageous as stated in (Dietz, 1999).

2.2 Particular solution of (1) and frequency response mode

For a harmonic excitation $\boldsymbol{f}(t)$

$$\boldsymbol{f}(t) = \boldsymbol{F}e^{j\Omega_0 t} \tag{5}$$

the particular solution $\boldsymbol{x}_p(t)$ can be written as

$$\boldsymbol{x}_p(t) = \boldsymbol{X}_p e^{j\Omega_0 t}. \tag{6}$$

If (5) and (6) are substituted into (1) and (2) is applied to (1),

$$\boldsymbol{X}_p = \sum_{i=1}^{N} \frac{\phi_i\phi_i^T}{\mu_i(\omega_i^2 - \Omega_0^2 + 2j\zeta_i\Omega_0\omega_i)}\boldsymbol{F} \tag{7}$$

follows. From (7) one observes that the particular solution is composed by the eigenvectors ϕ_i which is determined by simulation and used additionally to a set of n eigenvectors. Since this method is applicable to free floating bodies, as opposed to static modes, no problems finding appropriate boundary conditions arise. For each excitation or control force a particular solution $\boldsymbol{X}_p$ as response for unitary amplitude is computed.
Eliminating the effect of the first n eigenmodes from $\boldsymbol{X}_p$ according to (7), leads to

$$\boldsymbol{X}_p^{\perp} = \sum_{i=n+1}^{N} \frac{\boldsymbol{\phi}_i\boldsymbol{\phi}_i^T}{\mu_i(\omega_i^2 - \Omega_0^2 + 2j\Omega_0\zeta_i\omega_i)}\boldsymbol{F}. \tag{8}$$

The part of the particular solution $\boldsymbol{X}_p^{\perp}$ which is orthogonal to the first n eigenmodes of (1) is called *Frequency Response Mode (FRM)*. For k excitation and control forces the solution $\boldsymbol{x}(t)$ of (1) for arbitrary excitation frequency ω is approximated by

$$\boldsymbol{x}(t) \simeq \sum_{i=1}^{n} \boldsymbol{\phi}_i\boldsymbol{z}_i(t) + \sum_{j=1}^{k} \boldsymbol{X}_{p,j}^{\perp}\alpha_{p,j}(t), \tag{9}$$

where $\alpha_{p,j}(t)$ is an appropriate scaling function. The quality of the approximation (9) strongly depends on the frequency content of the excitation and the control forces.

3. THE ASSOCIATED CONTROL PROBLEM

To solve the standard control problem the necessary transfer functions are defined next. Laplace transformation of (7) yields $\boldsymbol{X}(s) = \boldsymbol{G}(s)\boldsymbol{F}(s)$ with

$$\boldsymbol{G}(s) = \sum_{i=1}^{N} \frac{\phi_i \phi_i^T}{\mu_i(s^2 + 2\zeta_i\omega_i s + \omega_i^2)}. \quad (10)$$

$\boldsymbol{G}(s)$ is called transfer function matrix.

The element $\boldsymbol{G}_{ij}$ describes the influence on the deflection $x_i(t)$ of the i-th node of the excitation force $f_j(t)$ acting on the j-th node . If the bandwidth of the excitation force is smaller than the bandwidth of the reduced system (order n) $\boldsymbol{G}(s)$ is approximately described by

$$\boldsymbol{G} \simeq \boldsymbol{G}_{\text{corr}} = \sum_{i=1}^{n} \frac{\phi_i \phi_i^T}{\mu_i(s^2 + 2\zeta_i\omega_i s + \omega_i^2)} + \boldsymbol{K} \quad (11)$$

with the constant part

$$\boldsymbol{K} = \sum_{i=n+1}^{N} \frac{\phi_i \phi_i^T}{\mu_i \omega_i^2}. \quad (12)$$

One has to observe, that the correction of the reduced system given by (12) only affects the zeros of the transfer function G_{ij}.

4. EXAMPLE: SIMPLE BEAM WITH PINNED-PINNED BOUNDARY CONDITIONS

In this chapter a transfer function from the actuator voltage to the sensor voltage is corrected with the help of a feedthrough term K. Thus, one takes into account the contribution of the higher system modes to the measurement signal to avoid spillover effects which have been treated extensively in (Balas, 1978) and (Balas, 1982).

For this purpose a beam with a pair of moments respectively force excitation and piezoelectric patches is considered. The determination of a FRM is demonstrated first, which takes into account local actuator action.

4.1 Basic equations for simple beams

The derivation of the differential equations as well as other formulas can be found in (Timoshenko *et al.*, 1974). The equation of motion for a beam excited by a pair of moments (Figure 1) is

$$EJ\frac{\partial^4 w(x,t)}{\partial x^4} + \rho A \frac{\partial^2 w(x,t)}{\partial t^2} = \frac{\partial^2 M(x,t)}{\partial x^2} \quad (13)$$

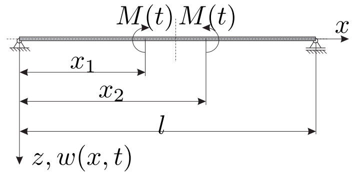

Fig. 1. beam with pair of moments

with E the modulus of elasticity, J the moment of inertia about the beam's neutral axis, ρ the density, A the cross sectional area of the beam and $w(x,t)$ the deflection of the beam. The separation of variables

$$w(x,t) = \sum_{i=1}^{\infty} W_i(x) z_i(t) \quad (14)$$

(in analogy to the transformation matrix $\boldsymbol{\Phi}$ in section 2) satisfies (13). Again, the $z_i(t)$ are the modal coordinates, and $W_i(x)$ are the (continuous) eigenfunctions. The eigenfunctions become

$$W_i(x) = \sqrt{\frac{2}{l}} \sin i\pi\frac{x}{l} \text{ (with } i = 1, 2, 3, \ldots). \quad (15)$$

The particular solution of (13) for harmonic excitation $M(t) = \hat{M}\sin\omega t$ is given by

$$w(x,t) = \sum_{i=1}^{\infty} \frac{W_i(x)\hat{M}\sin\omega t}{\rho A\omega_i^2\left(1 - \left(\frac{\omega}{\omega_i}\right)^2\right)} \left(W'_{i,1} - W'_{i,2}\right). \quad (16)$$

In (16) $W'_{i,j}$ denotes the value of the derivative of W_i at the position x_j.

4.2 FRM for the simple beam

The simulation results correspond to the data given in Table 1.

Table 1. material and geometry data

parameter	numerical value	unit
E	$2 \cdot 10^{11}$	N/m^2
b	0.02	m
h	0.005	m
A $(= bh)$	$1 \cdot 10^{-4}$	m^2
J $(= \frac{bh^3}{12})$	$2.0833 \cdot 10^{-10}$	m^4
ρ	7850	$\frac{kg}{m^3}$
$\hat{M}$	10	Nm
x_1	0.48	m
x_2	0.52	m

As excitation frequency Ω_0 for the calculation of the FRM the first eigenfrequency divided by two is used ($\Omega_0 = 35.95 s^{-1}$). This approach is proposed in the manual of the multi body simulation program SIMPACK, when the higher modes only contribute 'statically' to the particular solution.

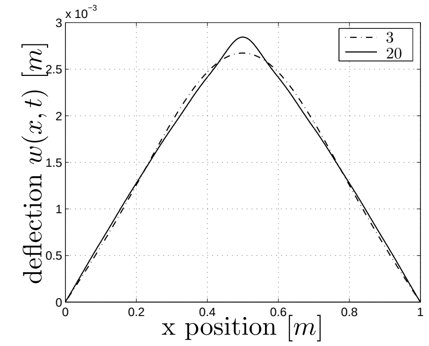

Fig. 2. particular solution; $\Omega_0 = 35.95s^{-1}$

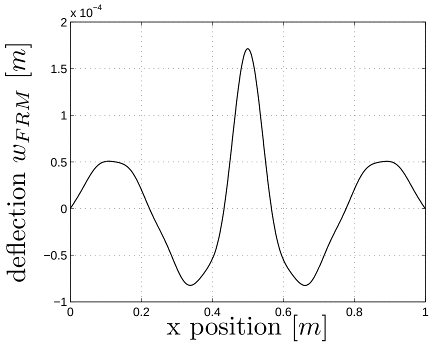

Fig. 3. FRM

In Figure 2 the particular solution is printed for the 'full' (20 modes) and for the 'reduced' system (only 3 modes).

One can observe the contribution of the higher modes, which comes from the local impact of the moment pair. In the extreme case of vanishing actuator length the exact solution is of triangular shape and the number of necessary eigenfunctions for a given accuracy becomes very large.
The difference between full and reduced solution according to (16) is the FRM which is depicted in Figure 3.

Figure 4 shows the comparison between full ('20'), reduced ('3') and corrected system ('FRM') for $\omega = \frac{\omega_2+\omega_3}{2} = 467.38s^{-1}$.

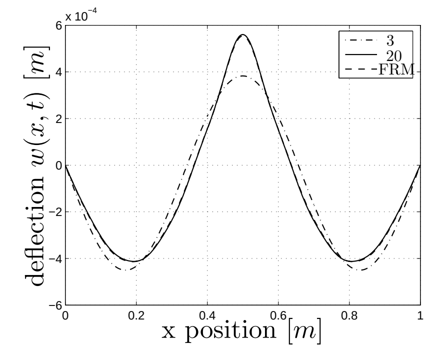

Fig. 4. particular solutions; $\omega = \frac{\omega_2+\omega_3}{2}$

There is hardly any difference between the full ('20') and the corrected ('FRM') solution observable. Note that excitation frequencies larger than ω_3 would result in deviations.

4.3 Actuator-sensor pair on the simple beam

In this section the transfer function from the actuator voltage to the sensor output will be stated which follows (Halim and Moheimani, 2001).
The pair of moments from Figure 1 e.g. is generated by a piezoelectric actuator depicted in Figure 5.

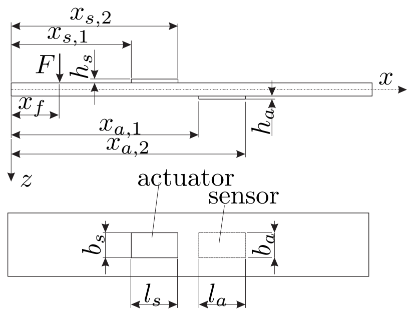

Fig. 5. sensor and actuator placement

The solution of (13) extended by modal damping for a single pizoelectric actuator in modal coordinates is

$$\ddot{z}_i + 2\zeta_i\omega_i\dot{z}_i + \omega_i^2 z_i = \frac{\bar{K}}{\rho A}\Psi_{a,i}V_a(t) \qquad (17)$$

where

$$\Psi_{a,i} = [W_i^{'}(x_{a2}) - W_i^{'}(x_{a1})] \qquad (18)$$

and $\bar{K}$ depends on the properties of the piezoelectric material. In (17) $V_a(t)$ is the actuator voltage. All other variables are defined in Table 2.

Table 2. data of the piezo patches

parameter	variable	num. value
act.start.pos.	$x_{a,1}$	0.40 m
act.end.pos.	$x_{a,2}$	0.44 m
sens.start.pos.	$x_{s,1}$	0.40 m
sens.end.pos.	$x_{s,2}$	0.44 m
act.height	h_a	0.001 m
sens.height	h_s	0.0001 m
act.length	l_a	0.040 m
sens.length	l_s	0.040 m
act.width	b_a	0.020 m
sens.width	b_s	0.020 m
constant	$\bar{K}$	$-4.794 \cdot 10^{-4}$ C
constant	Γ	$-1.096 \cdot 10^{-4}$ Nm/As

If (17) is Laplace transformed and inserted into the Laplace transformed version of (14) one obtains

$$G(s,x) = \frac{w(s,x)}{V_a(s)} = \frac{\bar{K}}{\rho A}\sum_{i=1}^{\infty}\frac{W_i(x)\Psi_{a,i}}{s^2 + 2\zeta_i\omega_i s + \omega_i^2} \qquad (19)$$

the transfer function from the actuator voltage $V_a(t)$ to the beam deflection $w(x,t)$.

If a piezoelectric patch is used as sensor, the sensor output voltage $V_s(t)$ becomes with (14)

$$V_{s,l}(t) = \Gamma \sum_{i=1}^{\infty} \Psi_{s,i}\ z_i(t) \tag{20}$$

where $\Psi_{s,i} = [W_i'(x_{s2}) - W_i'(x_{s1})]$ and Γ again depends on the properties of the piezoelectric material.

Laplace transforming (20) and using (19) yields the transfer function from the actuator voltage $V_a(t)$ to the sensor voltage $V_s(t)$

$$G_{Vs}(s) = \Gamma \frac{\bar{K}}{\rho A} \sum_{i=1}^{\infty} \frac{\Psi_{s,i}\Psi_{a,i}}{s^2 + 2\zeta_i\omega_i s + \omega_i^2}. \tag{21}$$

With the help of (21) the necessity of model correction using a feedthrough term K shall be demonstrated next.

4.4 *Local actuator action*

Using an actuator, which in comparison to the structure to be controlled is relatively small, makes the correction factor K defined by (11) more important. This will be demonstrated in the following assuming that

$$l_a \ll l. \tag{22}$$

We define

$$F_i = \frac{\Psi_{a,i}(l_a)}{\Psi_{a,i}(l_{a,ref})} \tag{23}$$

with $\Psi_{a,i}$ as in (18) and beginning and the end positions of the actuator at

$$x_{a1} = x_m - \frac{l_a}{2}, \quad x_{a2} = x_m + \frac{l_a}{2}$$

where x_m is the actuator middle position. In (23) $l_{a,ref}$ stands some reference value of the actuator length. Utilizing the eigenfunctions of the simply supported beam (15) one obtains

$$F_i = \frac{\sin\left(\frac{i\pi}{l}\frac{l_a}{2}\right)}{\sin\left(\frac{i\pi}{l}\frac{l_{a,ref}}{2}\right)} \tag{24}$$

which is completely independent of the actuator middle position x_m. The derivative of (23) with respect to l_a is

$$\frac{dF_i}{dl_a} = \frac{1}{\Psi_{a,i}(l_{a,ref})}\frac{d\Psi_{a,i}(l_a)}{dl_a} \tag{25}$$

which, using (15), yields

$$\frac{dF_i}{dl_a} = \frac{i\pi}{2l}\frac{\cos\left(\frac{i\pi l_a}{2l}\right)}{\sin\left(\frac{i\pi l_{a,ref}}{2l}\right)}. \tag{26}$$

Using (22), (26) can be linearized for both small l_a and $l_{a,ref}$ yielding

$$\frac{dF_i}{dl_a} = \frac{1}{l_{a,ref}}. \tag{27}$$

From (27) one concludes, that for small actuator length in comparison to the wave length of the highest mode for increasing l_a the contribution of each mode to the total response remains constant. This can also be seen from the plot of F_i in Figure 6 for small values of $\frac{l_a}{l}$.

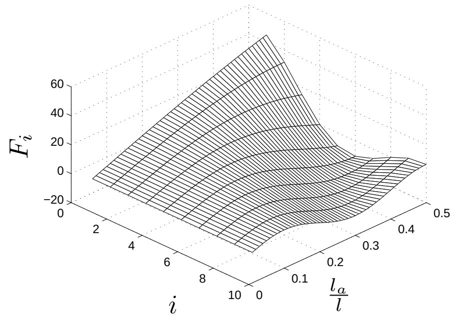

Fig. 6. F_i as function of mode number i and actuator length l_a (l=1m, $l_{a,ref}$=0.01m)

When the order of the length of the actuator becomes equal to or greater than the order of the wave length of a certain mode the contribution of this mode to the overall beam deflection decreases in comparison to the first mode.

Therefore, Figure 6 suggests that a model correction utilizing (11) is more important when the actuator becomes small compared to the size of the structure to be controlled. This fact is illustrated in Table 3. There

$$\frac{K_{1,3}}{K_{4,10}}$$

is the proportion of the static contribution from the first three modes to the last six ones, which clearly states that a correction according to (11) is more important when the actuator length decreases.

Table 3. comparison for different actuator lengths

l_a/l	**0.1**	**0.5**
F_1/F_9	~ 1.4	~ 9.0
$K_{1,3}/K_{3,10}$	~ 0.67	~ 6.04

5. CONSEQUENCES FOR ACTUATOR AND SENSOR PLACEMENT

For an effective control system the optimal placement of actuators and sensors along the structure to be controlled is crucial:

Observability and controllability:
From an engineering point of view, optimality with respect to controllability and observability could be defined as the highest possible efficiency of the actuators and sensors, respectively.
For piezoelectric elements positions with maximum curvature of the mode shapes are optimal with respect to observability and controllability. An approach which is based on the controllability (W_c) and the observability gramians (W_o) is given in (Leleu *et al.*, 2001).

Collocated vs. non-collocated control:
Placing actuators and sensors at the same location in form of pairs has several advantages according to (Preumont, 1999): a minimum phase system is obtained, and some control concepts (e.g. Direct Velocity Feedback) lead to an unconditionally stable closed loop behaviour if the actuator dynamics are neglected.
As mentioned in the beginning, a separation of the actuator and sensor location is advantageous when the actuator is very small in comparison with the flexible structure.

In this case the measured signal amplitude due to the local actuator action can be reduced and thus the sensor signal is mainly comprised by the eigenmodes to be controlled (compare the heights of the peaks in Figure 3). This fact becomes even more important in the presence of measurement noise where a high signal to noise ratio is desirable.

Criterion for actuator and sensor placement:
The demands concerning a suitable criterion for actuator/sensor placement have to combine the following points:

(1) Preferably high values for the controllability of the modes to be controlled. This leads to small control signals. At the same time preferably low values for the controllability of the residual modes.
(2) Preferably high values for the observability of the modes to be controlled. At the same time preferably low values for the observability of the residual modes to obtain a high noise rejection.

6. SUMMARY AND OUTLOOK

If in comparison to the structure to be controlled the actuators are relatively small, the local action of the actuator requires many modes for the representation of the system response. With decreasing actuator length the amount of the higher modes in the measurement signal of the sensor increases. To detect local actuator actions and avoid spillover an additional correction becomes unavoidable for control purposes. This correction can be generated by the collective acquisition of the influence of the higher modes in the form of the FRM. In section 3 the analogy between FRM and model correction in form of feedthrough terms has been shown.
Then the accuracy of the solution of the corrected model depends on the bandwidth of the excitation. The basics of a criterion proposed in section 5 provides the possibility of finding an optimal set of actuator/sensor locations for arbitrary flexible structures and real-world demands.
Future work comprises the development of the numerical optimization procedure and the application of the proposed methods to a practical problem. The structure to be controlled is the body of a railcar vehicle.

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