DECENTRALIZED OUTPUT FEEDBACK CONTROL OF LARGE-SCALE INTERCONNECTED NONLINEAR SYSTEMS: THE LMI APPROACH [1]

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Abstract: The focus of the research is on the design of a decentralized output feedback controller for a class of large-scale systems using linear matrix inequalities (LMI). The class of large-scale systems is characterized by unmatched nonlinear interconnection functions that are uncertain but quadratically bounded in the overall system state. An elegant LMI solution to the problem is provided in (Siljak and Stipanovic 2001), but the method requires that the input matrix of each subsystem be invertible, i.e., each subsystem has as many independent control inputs as state variables. We provide an LMI solution that does not require invertibility of the input matrices of each subsystem. Simulation results on an example are given to validate the design.

Keywords: Decentralized control, large-scale systems, output feedback, LMI

1. INTRODUCTION

Large-scale interconnected systems can be found in such diverse fields as electrical power systems, space structures, manufacturing processes, transportation, and communication. An important motivation for the design of decentralized schemes is that the information exchange between subsystems of a large-scale system is not needed; thus, the individual subsystem controllers are simple and use only locally available information. Decentralized control of large-scale systems has received considerable interest in the systems and control literature. A large body of literature in decentralized control of large-scale systems can be found in (Siljak 1991). In (N. R. Sandell *et al.* 1978), a survey of early results in decentralized control of large scale systems was given. Decentralized control schemes that can achieve desired robust performance in the presence of uncertain interconnections can be found in (Ikeda 1989, Zhang *et al.* 1996, Gong 1995). A decentralized control scheme for robust stabilization of a class of nonlinear systems using the Linear Matrix Inequalities (LMI) framework was proposed in (Siljak and Stipanovic 2000).

In many practical situations, complete state measurements are not available at each individual subsystem for decentralized control; consequently, one has to consider decentralized feedback control based on measurements only or design decentralized observers to estimate the state of individual subsystems that can be used for estimated state feedback control. There has been a strong research effort in literature towards development of decentralized control schemes based on output feedback via construction of decentralized observers. Early work in this area can be found in (Viswanadham and Ramakrishna 1982, Ikeda 1989, Siljak 1991). Subsequent work in (Abdel-Jabbar *et al.* 1998, Aldeen and Marsh 1999, Jiang 2000, Narendra and Oleng 2002) has focused on the decentralized output feedback problem for a number of special classes

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of nonlinear systems. In (Siljak and Stipanovic 2001), the decentralized controller and observer design problems were formulated in the LMI framework for large-scale systems with nonlinear interconnections that are quadratically bounded. Autonomous linear decentralized observer-based output feedback controllers for all subsystems were obtained. The existence of a stabilizing controller and observer depended on the feasibility of solving an optimization problem in the LMI framework; further, for a solution to exist, this formulation also required, for each subsystem, the number of independent control inputs must be equal to the dimension of the state. A solution based on the concept of distance to controllability/observability was proposed in (Pagilla and Zhu 2004); the feasibility of the solution was dependent on satisfying the distance to controllability/observability of pairs of matrices being larger than a certain value.

In this paper, we provide a solution that does not require invertibility of subsystem input matrices as in (Siljak and Stipanovic 2001). The proposed LMI solution is obtained as a sequential two part optimization problem. The feasibility of both parts of the optimization problem is shown and discussed. The decentralized output feedback problem, the solution of (Siljak and Stipanovic 2001), and the proposed LMI solution are given in Section 2. Section 3 gives simulation results using the proposed method for an example. Conclusions are given in Section 4.

2. DECENTRALIZED OUTPUT FEEDBACK CONTROLLER DESIGN

The following class of large-scale interconnected nonlinear systems is considered:

$$\dot{x}_i(t) = A_i x_i(t) + B_i u_i(t) + h_i(t,x), \tag{1a}$$
$$y_i(t) = C_i x_i(t) \tag{1b}$$

where $x_i \in \mathbb{R}^{n_i}$, $u_i \in \mathbb{R}^{m_i}$, $y_i \in \mathbb{R}^{l_i}$, $h_i \in \mathbb{R}^{n_i}$ are the state, input, output, and nonlinear interconnection function of the i-th subsystem, and $x = \begin{bmatrix} x_1^\top & x_2^\top & \dots & x_N^\top \end{bmatrix}^\top$ is the state of the overall system. The term $h_i(t,x)$ reflects the interconnection of the i-th subsystem with other subsystems and the uncertainty dynamics from the i-subsystem itself. It is assumed that the exact expression of $h_i(t,x)$ is unknown but is assumed to satisfy the quadratic constraints(Siljak and Stipanovic 2001):

$$h_i^T(t,x) h_i(t,x) \le \alpha_i^2 x^\top H_i^\top H_i x \tag{2}$$

where $\alpha_i > 0$ are interconnection bounds and H_i are bounding matrices. It is also assumed that (A_i, B_i) is a controllable pair and (C_i, A_i) is an observable pair for all $i \in I = \{1, 2, \dots, N\}$.

The objective is to design a totally decentralized observer-based linear controller that robustly regulates the state of the overall system without any information exchange between subsystems, this is, the local controller u_i is constrained to used only local output signal y_i. One specific practical application whose system model conforms to (1) with the quadratic interconnection bounds (2) is a multimachine power system consisting of N interconnected machines with steam valve control; the dynamic model is discussed in (Siljak *et al.* 2002).

The overall system (1) can be rewritten as

$$\dot{x}(t) = A_D x(t) + B_D u(t) + h(t,x), \tag{3a}$$
$$y(t) = C_D x(t) \tag{3b}$$

where $A_D = \mathrm{diag}(A_1, \dots, A_N)$, $B_D = \mathrm{diag}(B_1, \dots, B_N)$, $C_D = \mathrm{diag}(C_1, \dots, C_N)$, $u = \begin{bmatrix} u_1^\top & \dots & u_N^\top \end{bmatrix}^\top$, $y = \begin{bmatrix} y_1^\top & \dots & y_N^\top \end{bmatrix}^\top$, and $h = \begin{bmatrix} h_1^\top & \dots & h_N^\top \end{bmatrix}^\top$. The nonlinear interconnections $h(t,x)$ are bounded as follows:

$$h^\top(t,x) h(t,x) \le x^\top \Big(\sum_{i=1}^{N} \alpha_i^2 H_i^\top H_i\Big) x =: x^\top \Gamma^\top \Gamma x \tag{4}$$

The pair (A_D, B_D) is controllable and the pair (C_D, A_D) is observable, which is the direct result of each subsystem being controllable and observable.

Since the system (3) is linear with nonlinear interconnections, a common question to ask is under what conditions can we design a decentralized linear controller and a decentralized linear observer that will stabilize the system in the presence of bounded quadratic interconnections. Towards solving this problem, one can consider the following linear decentralized controller and observer:

$$u(t) = K_D \widehat{x}(t), \tag{5}$$
$$\dot{\widehat{x}}(t) = A_D \widehat{x}(t) + B_D u(t) + L_D (y(t) - C_D \widehat{x}(t)) \tag{6}$$

where $K_D = \mathrm{diag}(K_1, \dots, K_N)$, $L_D = \mathrm{diag}(L_1, \dots, L_N)$ are the controller and observer gain matrices, respectively. Rewriting (3) and (6) in the coordinates $x(t)$ and $\widetilde{x}(t)$, where $\widetilde{x}(t) \triangleq x(t) - \widehat{x}(t)$ is the estimation error, the closed-loop dynamics is

$$\dot{x}(t) = (A_D + B_D K_D) x(t) - B_D K_D \widetilde{x}(t) + h(t,x), \tag{7a}$$
$$\dot{\widetilde{x}}(t) = (A_D - L_D C_D) \widetilde{x}(t) + h(t,x). \tag{7b}$$

Let

$$A_c \triangleq A_D + B_D K_D, \quad A_o \triangleq A_D - L_D C_D. \tag{8}$$

Consider the following Lyapunov function candidate

$$V(x, \widetilde{x}) = x^\top \bar{P}_c x + \widetilde{x}^\top \bar{P}_o \widetilde{x}. \tag{9}$$

The time derivative of $V(x,\widetilde{x})$ along the trajectories of (7) is given by

$$\dot{V}(x,\widetilde{x}) = \begin{bmatrix} x \\ \widetilde{x} \\ h \end{bmatrix}^\top \begin{bmatrix} A_c^\top \bar{P}_c + \bar{P}_c A_c & -\bar{P}_c B_D K_D & \bar{P}_c \\ -K_D^\top B_D^\top \bar{P}_c & A_o^\top \bar{P}_o + \bar{P}_o A_o & \bar{P}_o \\ \bar{P}_c & \bar{P}_o & 0 \end{bmatrix} \begin{bmatrix} x \\ \widetilde{x} \\ h \end{bmatrix}. \tag{10}$$

The interconnection condition (4) is equivalent to

$$\begin{bmatrix} x \\ \widetilde{x} \\ h \end{bmatrix}^\top \begin{bmatrix} -\Gamma^\top \Gamma & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & I \end{bmatrix} \begin{bmatrix} x \\ \widetilde{x} \\ h \end{bmatrix} \le 0. \tag{11}$$

The stabilization of the system (7) requires that

$$\dot{V}(x, \widetilde{x}) < 0 \tag{12}$$

for all $x, \widetilde{x} \neq 0$; together with the condition given by (11), one can obtain (Boyd *et al.* 1994) that if

$$\begin{bmatrix} A_c^\top \bar{P}_c + \bar{P}_c A_c & -\bar{P}_c B_D K_D & \bar{P}_c \\ -K_D^\top B_D^\top \bar{P}_c & A_o^\top \bar{P}_o + \bar{P}_o A_o \bar{P}_o & \\ \bar{P}_c & \bar{P}_o & 0 \end{bmatrix} - \tau \begin{bmatrix} -\Gamma^\top \Gamma & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & I \end{bmatrix} < 0, \tag{13a}$$

$$\bar{P}_c > 0, \ \bar{P}_o > 0, \ \tau > 0, \tag{13b}$$

then the inequality (12) is satisfied. Let

$$P_c \triangleq \frac{\bar{P}_c}{\tau}, \quad P_o \triangleq \frac{\bar{P}_o}{\tau}.$$

The condition given by (13) is equivalent to

$$\begin{bmatrix} A_c^\top P_c + P_c A_c + \Gamma^\top \Gamma & -P_c B_D K_D & P_c \\ -K_D^\top B_D^\top P_c & A_o^\top P_o + P_o A_o & P_o \\ P_c & P_o & -I \end{bmatrix} < 0, \tag{14a}$$

$$P_c > 0, \ P_o > 0. \tag{14b}$$

Considering (4) and (8), and applying the Schur complement to the inequality (14), results in

$$P_c > 0, P_o > 0, \tag{15a}$$

$$\begin{bmatrix} W_C & -P_c B_D K_D & P_c & \alpha_1 H_1^\top & \dots & \alpha_N H_N^\top \\ -(P_c B_D K_D)^\top & W_O & P_o & 0 & \dots & 0 \\ P_c & P_o & -I & 0 & \dots & 0 \\ \alpha_1 H_1 & 0 & 0 & -I & \dots & 0 \\ \vdots & 0 & \vdots & \vdots & \ddots & \vdots \\ \alpha_N H_N & 0 & 0 & 0 & \dots & -I \end{bmatrix} < 0 \tag{15b}$$

where $W_C \triangleq A_D^\top P_c + P_c A_D + (P_c B_D K_D)^\top + (P_c B_D K_D)$ and $W_O \triangleq A_D^\top P_o + P_o A_D - P_o L_D C_D - (P_o L_D C_D)^\top$. Rearranging entries and scaling corresponding columns and rows related to $H_i, i = 1, \dots, N$, on the left hand side matrix (15b), one obtains

$$P_c > 0, P_o > 0, \tag{16a}$$

$$\begin{bmatrix} W_C & H_1^\top & \dots & H_N^\top & -P_c B_D K_D & P_c \\ H_1 & -\gamma_1 I & \dots & 0 & 0 & 0 \\ \vdots & \vdots & \ddots & \vdots & 0 & 0 \\ H_N & 0 & \dots & -\gamma_N I & 0 & 0 \\ -(P_c B_D K_D)^\top & 0 & 0 & 0 & W_O & P_o \\ P_c & 0 & 0 & 0 & P_o & -I \end{bmatrix} < 0 \tag{16b}$$

where $\gamma_i = \dfrac{1}{\alpha_i^2} > 0$. Now the problem of stabilization of the large-scale system (1) by decentralized output feedback control is transferred to the problem of finding finding $\gamma_i > 0, i \in I$, such that inequalities in (16) are satisfied. Further, if the following optimization problem

$$\text{Minimize} \sum_{i=1}^{N} \gamma_i \text{ subject to Equation (16)} \tag{17}$$

is feasible, the selection of the control gain matrix K_D and observer gain matrix L_D not only stabilizes the overall system (7), but also simultaneously maximize the interconnection bounds α_i.

In the optimization problem given by (17), variables are P_c, P_o, K_D, L_D and $\gamma_i, i \in I$. Since there are coupled term of matrix variables P_c and K_D, and P_o and L_D in the matrix inequality (16b), the optimization (17) is not on a convex set. One has to find a way to transform the inequality (16b) to a form which is affine in variables. To achieve this, one can introduce variables

$$M_D \triangleq P_c B_D K_D, \quad N_D \triangleq P_o L_D. \tag{18}$$

Then, the optimization problem (17) becomes

$$\text{Minimize} \sum_{i=1}^{N} \gamma_i \text{ subject to}$$

$$P_c > 0, P_o > 0, \tag{19a}$$

$$\begin{bmatrix} W_C & H_1^\top & \dots & H_N^\top & -M_D & P_c \\ H_1 & -\gamma_1 I & \dots & 0 & 0 & 0 \\ \vdots & \vdots & \ddots & \vdots & 0 & 0 \\ H_N & 0 & \dots & -\gamma_N I & 0 & 0 \\ -M_D^\top & 0 & 0 & 0 & W_O & P_o \\ P_c & 0 & 0 & 0 & P_o & -I \end{bmatrix} < 0 \tag{19b}$$

The solution to the optimization problem (19) gives rise to M_D and N_D. The controller and observer gain matrices were obtained from M_D and N_D in (Siljak and Stipanovic 2001) in the following manner. The observer gain matrix L_D can be computed using (18) as

$$L_D = P_o^{-1} N_D.$$

However, controller gain matrix K_D can be obtained only in the case when B_D is invertible, that is,

$$K_D = B_D^{-1} P_c^{-1} M_D.$$

Obviously, invertibility of B_D requires that $B_i, i \in I$ be invertible, which is too restrictive. When all the B_i are not invertible, it is not possible to obtain the control gain matrix K_D from the optimization problem (19). The following addresses the proposed LMI solution to the case when B_i are not invertible.

One can pre-multiply and post-multiply the left hand side of (16b) by $\text{diag}(P_c^{-1}, I)$ and define $Y = P_c^{-1}$ to obtain following conditions which are equivalent to (16):

$$Y > 0, P_o > 0, \tag{20a}$$

$$\begin{bmatrix} W_C' & Y H_1^\top & \dots & Y H_N^\top & -B_D K_D & I \\ H_1 Y & -\gamma_1 I & \dots & 0 & 0 & 0 \\ \vdots & \vdots & \ddots & \vdots & 0 & 0 \\ H_N Y & 0 & \dots & -\gamma_N I & 0 & 0 \\ -(B_D K_D)^\top & 0 & 0 & 0 & W_O & P_o \\ I & 0 & 0 & 0 & P_o & -I \end{bmatrix} < 0 \tag{20b}$$

where $W_C' \triangleq Y A_D^\top + A_D Y + (B_D K_D Y)^\top + (B_D K_D Y)$.

Let $\bar{M}_D \triangleq K_D Y$, $\begin{bmatrix} S_1 & S_2 \end{bmatrix} \triangleq \begin{bmatrix} -B_D K_D & I \\ 0 & 0 \\ \vdots & \vdots \\ 0 & 0 \end{bmatrix}$. Now, the problem is to find Y, P_o, K_D, L_D, and $\gamma_i, i \in I$, which can be found by the following two steps.

Step 1. Maximize the interconnection bounds α_i by solving the following optimization problem:

$$\text{Minimize } \sum_{i=1}^{N} \gamma_i \text{ subject to}$$

$$Y > 0, F_{opt} = \begin{bmatrix} W_C' & YH_1^\top & \dots & YH_N^\top \\ H_1Y & -\gamma_1 I & \dots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ H_NY & 0 & \dots & -\gamma_N I \end{bmatrix} < 0. \quad (21)$$

Step 2. Find P_o and N_D by using K_D obtained from Step 1 and solving the following optimization problem

$$\text{Minimize } \sum_{i=1}^{N} \beta_i \text{ subject to}$$

$$P_o > 0, \Lambda > 0, \begin{bmatrix} \Lambda F_{opt} & S_1 & S_2 \\ S_1^\top & W_O & P_o \\ S_2^\top & P_o & -I \end{bmatrix} < 0. \quad (22)$$

where $\Lambda = \text{diag}(\beta_1 I_1, \beta_2 I_2, \dots, \beta_N I_N, \beta_1 I_1, \beta_2 I_2, \dots, \beta_N I_N)$, and I_i denotes the $n_i \times n_i$ identity matrix. The matrices F_{opt} and S_1 in Step 2 are obtained from Step 1.

The control gain K_D is obtained from Step 1 as

$$K_D = \bar{M}_D Y^{-1}, \quad (23)$$

and the observer gain L_D is obtained from Step 2 as

$$L_D = P_o^{-1} N_D. \quad (24)$$

Remark 1. Unlike the case when B_D is invertible, inequalities given by (21) and (22) cannot be solved simultaneously. The optimization problem (21) of step 1 must be solved followed by step 2.

Remark 2. Since Y, A_D, B_D and $\bar{M}_D$ are all block diagonal matrices, it is not difficult to show that $\Lambda F_{opt} = \Lambda^{1/2} F_{opt} \Lambda^{1/2} < 0$ when $F_{opt} < 0$. Also, if $\beta_i > 1, i \in I$, $\Lambda F_{opt} < F_{opt} < 0$. Further, notice that the solution K_D obtained from the optimization problem $F_{opt} < 0$ is unchanged if we solve the optimization problem $\Lambda F_{opt} < 0$ because of the chosen structure of Λ.

Remark 3. If $\Lambda = I$, the LMI (22) may not be feasible for the selection of F_{opt} and K_D resulting from the optimization problem (21). On the other hand, by choosing Λ as a matrix variable, the LMI (22) becomes feasible, which will be shown in the following.

The following lemmas illustrate the feasibility of the LMI problems (21) and (22).

Lemma 1. The optimization problem given by (21) is feasible if $(A_i, B_i), i = 1, \dots, N$, is a controllable pair.

Proof. To prove the LMI optimization problem (21) is feasible, one needs to show that there exists a solution that satisfies the inequality (21). In view of (21) and H_i being constant matrices, to show that there exist $Y > 0$, $\bar{M}_D$, $\gamma_i > 0, i = 1, \dots, N$, such that $F_{opt} < 0$, it is sufficient to show that

$$\text{there exists a } Y > 0,\ \bar{M}_D \text{ such that } W_C' < 0 \quad (25)$$

because of the existence of large enough γ_i to dominate the off-diagonal block elements H_i in (21). Notice that

$$\begin{aligned} W_C' &= YA_D^\top + A_D Y + (B_D \bar{M}_D)^\top + B_D \bar{M}_D \\ &= P_c^{-1} A_D^\top + A_D P_c^{-1} + (B_D K_D P_c^{-1})^\top + B_D K_D P_c^{-1} \\ &= P_c^{-1}\big((A_D + B_D K_D)^\top P_c + P_c(A_D + B_D K_D)\big) P_c^{-1}. \end{aligned}$$

Since (A_i, B_i) is a controllable pair (which implies (A_D, B_D) is a controllable pair), there exist a $P_c > 0$ and a K_D such that

$$(A_D + B_D K_D)^\top P_c + P_c(A_D + B_D K_D) < 0.$$

Therefore, the statement (25) is true. This completes the proof. ■

Lemma 2. If $(A_i, C_i), i = 1, \dots, N$ is an observable pair, the optimization problem (22) is feasible.

Proof. We first prove that

$$\text{there exists a } P_o > 0 \text{ and } N_D \text{ such that } \begin{bmatrix} W_O & P_o \\ P_o & -I \end{bmatrix} < 0. \quad (26)$$

Applying the Schur complement to the above matrix inequality yields the following equivalent inequality

$$W_O + P_o P_o < 0. \quad (27)$$

Recall that $N_D = P_o L_D$ and $W_O = A_D^\top P_o + P_o A_D - P_o L_D C_D - (P_o L_D C_D)^\top$. Equation (27) can be rewritten as

$$P_o\big((A_D - L_D C_D) Y_o + Y_o (A_D - L_D C_D)^\top + I\big) P_o < 0 \quad (28)$$

where $Y_o = P_o^{-1}$. Since (A_i, C_i) is an observable pair (which implies (A_D, C_D) is an observable pair), there exists a $Y_o > 0$ and an L_D such that

$$(A_D - L_D C_D) Y_o + Y_o (A_D - L_D C_D)^\top + I < 0.$$

Hence, the statement (26) is true.

Since $F_{opt} < 0$ and the statement (26) is true, all the principal minors of the matrix on the left hand side of (22) are negative. Since S_1 and S_2 are constant matrices after solving the optimization given in Step 1, to guarantee that (22) holds, it is sufficient to let the principal minor ΛF_{opt} dominate the off-diagonal block elements S_1 and S_2; this can be achieved by a large $\Lambda > 0$. This completes the proof. ■

Remark 4. The final uncertainty gains are $\beta_i \gamma_i, i \in I$, where γ_i is obtained from the optimization problem (21) and β_i is obtained from (22).

The LMI optimization problems given by (21) and (22) do not pose any restrictions on the size of the matrix variables Y, $\bar{M}_D$, P_o and N_D. Consequently, the results of these two optimization problems may yield very large controller and observer gain matrices K_D and L_D, respectively. In view of (23) and (24), one can restrict K_D and L_D by posing constraints on the matrices Y, $\bar{M}_D$, P_o and N_D, and a further constraint on γ_i (Siljak and Stipanovic 2000) as

$$\gamma_i - \frac{1}{\bar{\alpha}_i^2} < 0, \bar{\alpha}_i > 0; \quad Y_i^{-1} < \kappa_{Y_i} I, \kappa_{Y_i} > 0;$$
$$\bar{M}_{D_i} \bar{M}_{D_i}^\top < \kappa_{\bar{M}_{D_i}} I, \kappa_{\bar{M}_{D_i}} > 0; \quad (29)$$
$$\beta_i - \bar{\beta}_i > 0, \bar{\beta}_i > 0; \quad P_{o_i}^{-1} < \kappa_{P_{o_i}} I, \kappa_{P_{o_i}} > 0;$$
$$N_{D_i}^\top N_{D_i} < \kappa_{N_{D_i}} I, \kappa_{N_{D_i}} > 0 \quad (30)$$

where $\bar{M}_{D_i}$ and N_{D_i} are the i-th diagonal block of $\bar{M}_D$ and N_D, respectively. Equations (29) and (30) are respectively equivalent to

$$\gamma_i - \frac{1}{\bar{\alpha}_i^2} < 0, \quad \begin{bmatrix} -Y_i & -I \\ -I & -\kappa_{Y_i} I \end{bmatrix} < 0,$$
$$\begin{bmatrix} -\kappa_{\bar{M}_{D_i}} I & \bar{M}_{D_i} \\ \bar{M}_{D_i}^\top & -I \end{bmatrix} < 0, \quad \kappa_{Y_i}, \kappa_{\bar{M}_{D_i}} > 0, \quad (31)$$
$$\beta_i - \bar{\beta}_i > 0, \quad \begin{bmatrix} -P_{o_i} & -I \\ -I & -\kappa_{P_{o_i}} I \end{bmatrix} < 0,$$
$$\begin{bmatrix} -\kappa_{N_{D_i}} & N_{D_i}^\top \\ N_{D_i} & -I \end{bmatrix} < 0, \quad \kappa_{N_{D_i}}, \kappa_{P_{o_i}} > 0. \quad (32)$$

Combining (21) and (31), (22) and (32), and changing the optimization objectives to the minimization of $\sum_{i=1}^N (\gamma_i + \kappa_{Y_i} + \kappa_{\bar{M}_{D_i}})$ and $\sum_{i=1}^N (\beta_i + \kappa_{P_{o_i}} + \kappa_{N_{D_i}})$, respectively, results in the following two LMI optimization problems:

Step 1′. Maximize the interconnection bounds α_i by solving the following optimization problem:

$$\text{Minimize } \sum_{i=1}^N (\gamma_i + \kappa_{Y_i} + \kappa_{\bar{M}_{D_i}}) \text{ subject to}$$
$$\text{Equations (21) and (31).} \quad (33)$$

Step 2′. Find P_o and N_D by using K_D obtained from Step 1′ and solving the following optimization problem

$$\text{Minimize } \sum_{i=1}^N (\beta_i + \kappa_{P_{o_i}} + \kappa_{N_{D_i}}) \text{ subject to}$$
$$\text{Equations (22) and (32).} \quad (34)$$

Similar to Lemmas 1 and 2, it can be shown that the optimization problems (33) and (34) are feasible when all the subsystems are controllable and observable, provided that $\bar{\alpha}_i$ is chosen sufficient small. This is because one can choose large $\bar{\beta}$, $\kappa_{\bar{M}_{D_i}}$, κ_{Y_i}, $\kappa_{N_{D_i}}$ and $\kappa_{P_{o_i}}$, and small $\bar{\alpha}_i$ to satisfy (31) and (32).

The results of the LMI solution to the decentralized output feedback control problem for the large scale system (1) are summarized in the following theorem.

Theorem 1. Consider the large scale system (1) with the observer given by (6) and the controller given by (5). If

$$\alpha_i \le \min\left(\frac{1}{\sqrt{\gamma_i}}, \frac{1}{\sqrt{\beta_i \gamma_i}}\right) \quad (35)$$

where γ_i and β_i are solutions to the optimization problems (33) and (34), then the selection of controller and observer gain matrices given by (23) and (24) results in a stable closed-loop system.

3. NUMERICAL EXAMPLE AND SIMULATION

Consider the following large-scale system composed of two subsystems:

$$\dot{x}_1 = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix} x_1 + \begin{bmatrix} 0 \\ 1 \end{bmatrix} u_1 + h_1(x), y_1 = \begin{bmatrix} 1 & 0 \end{bmatrix} x_1; \quad (36a)$$
$$\dot{x}_2 = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix} x_2 + \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} u_2 + h_2(x), y_2 = \begin{bmatrix} 1 & 0 & 0 \end{bmatrix} x_2 \quad (36b)$$

where $x_1 = \begin{bmatrix} x_{11} & x_{12} \end{bmatrix}^\top$, $x_2 = \begin{bmatrix} x_{21} & x_{22} & x_{23} \end{bmatrix}^\top$, $x = \begin{bmatrix} x_1^\top & x_2^\top \end{bmatrix}^\top$, $h_1(x) = \alpha_1 \cos(x_{22}) H_1 x$, $h_2(x) = \alpha_1 \cos(x_{11}) H_2 x$, $\alpha_1 = \alpha_2 = 0.1$, $H_1 = \frac{1}{\sqrt{10}} \begin{bmatrix} 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 \end{bmatrix}$, and $H_2 = \frac{1}{\sqrt{15}} \begin{bmatrix} 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 \end{bmatrix}$.

Choosing $\bar{\alpha}_i = 0.001, \beta_i = 0.0001, i = 1, 2$, and solving the optimization problems (33) and (34) results in

$$M_D = \begin{bmatrix} 2.7773 & -3.0854 & 0 & 0 & 0 \\ 0 & 0 & -0.20276 & -1.6091 & -3.1217 \end{bmatrix},$$
$$Y = \begin{bmatrix} 25.4611 & -14.0572 & 0 & 0 & 0 \\ -14.0572 & 9.36744 & 0 & 0 & 0 \\ 0 & 0 & 47.4557 & -35.8544 & 6.51817 \\ 0 & 0 & -35.8544 & 41.4759 & -17.3584 \\ 0 & 0 & 6.51817 & -17.3584 & 14.7546 \end{bmatrix},$$
$$N_D = \begin{bmatrix} 0.65833 & 0 \\ 0.51052 & 0 \\ 0 & 1.193 \\ 0 & 0.98386 \\ 0 & -0.39479 \end{bmatrix},$$
$$P_o = \begin{bmatrix} 0.98889 & -0.39895 & 0 & 0 & 0 \\ -0.39895 & 0.59679 & 0 & 0 & 0 \\ 0 & 0 & 1.4986 & -0.29952 & -0.12104 \\ 0 & 0 & -0.29952 & 0.50936 & -0.43087 \\ 0 & 0 & -0.12104 & -0.43087 & 0.70333 \end{bmatrix},$$
$$\gamma_1 = 13.8634, \quad \gamma_2 = 2.7773, \quad \beta_1 = 4.9354, \quad \beta_2 = 11.3989.$$

Gain matrices K_D and L_D are found to be

$$K_D = \begin{bmatrix} -0.42433 & -0.96614 & 0 & 0 & 0 \\ 0 & 0 & -0.8614 & -1.404 & -1.4828 \end{bmatrix},$$
$$L_D = \begin{bmatrix} 1.3841 & 0 \\ 1.7807 & 0 \\ 0 & 2.4812 \\ 0 & 6.8017 \\ 0 & 4.0325 \end{bmatrix}$$

by (23) and (24), respectively. It is easy to check that the condition given by (35) is satisfied. Hence, according to Theorem 1, the closed-loop system is quadratically stable.

The simulation results are shown in Figures 1 and 2. In Figure 1, the state x_{11} and its estimate $\widehat{x}_{11}$, the state x_{12} and its estimate $\widehat{x}_{12}$, and the control u_1 are shown in the first, second and third plot, respectively. Figure 2 shows the states x_2, their estimates $\widehat{x}_2$, and the control u_2. It can be observed from both the figures that the state of the overall system, x, and their estimates, $\widehat{x}$, converge to zero.

4. SUMMARY

In this paper, an LMI solution to the decentralized output feedback control problem for a class of large-scale interconnected nonlinear systems is given. The interconnecting nonlinearity of each subsystem was assumed to be bounded by a quadratic form of states of the overall system. Local output signals from each subsystem were used to generate the local control inputs and exact knowledge of the nonlinear interconnections is not required for the proposed solution. Simulation results on a numerical example verify the proposed design. The contribution of this research over prior work is that the requirement that input matrix of each subsystem be invertible is relaxed.

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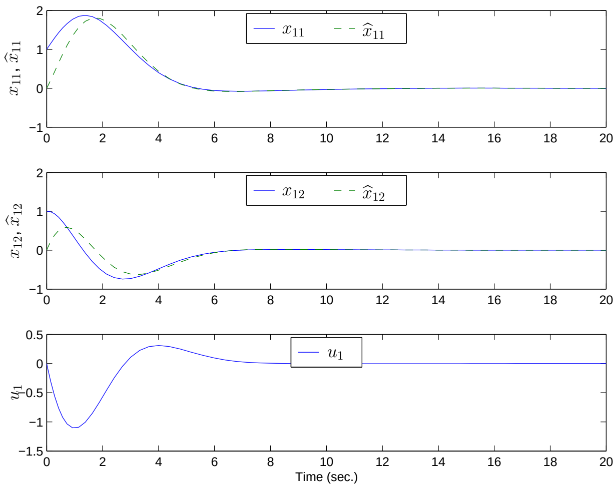

Fig. 1. Simulation result for the first subsystem (36a)

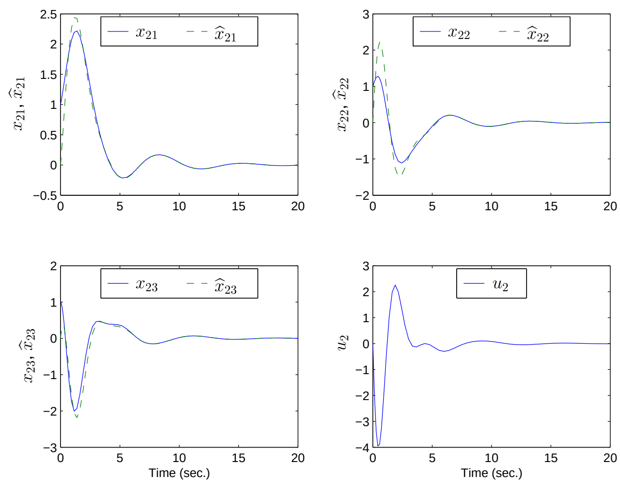

Fig. 2. Simulation result for the second subsystem (36b)