A SLIDING OBSERVER FOR CLOSED-CIRCUIT UNDERWATER BREATHING APPARATUS

Francesco Garofalo * **Luigi Iannelli** **
Sabato Manfredi * **Stefania Santini** *

* *Dipartimento di Informatica e Sistemistica*
Università degli Studi di Napoli Federico II
via Claudio, 21 - 80125 Napoli, Italy.
** *Dipartimento di Ingegneria*
Università degli Studi del Sannio di Benevento
Piazza Roma, 21 - 82100 Benevento, Italy.

Abstract: In this paper the estimation problem of the metabolic consumption rate in an underwater breathing apparatus is addressed. A sliding mode observer designed on a nonlinear model of the rebreather system is proposed. The design is based on Lyapunov techniques and a convergence analysis of the estimation error has been carried on. The effectiveness of the sliding observer has been validated through extensive simulations.

Keywords: Nonlinear control systems, Sliding mode, Observers, Lyapunov methods, Real-time systems.

1. INTRODUCTION AND PROBLEM MOTIVATION

Closed-circuit underwater breathing apparatus, commonly known as 'rebreather', is a device that combines the mobility of a free-swimming diver with the great autonomy necessary for a deep depth dive: essentially it permits completely autonomous diver operations at a very deep depth without an umbilical.

The term closed-circuit rebreather (CCR) refers to the recirculation of the breathable mixture. The macroscopic chemical effects on breathed gasses are (partial) oxygen subtraction for metabolic use with a consequent carbon dioxide increase. All other gasses different from oxygen are inert with respect to the respiration process and flow through the lungs without being chemically transformed. This means that exhaled gasses can be recycled (or better re-breathed), provided that oxygen content is restored and carbon dioxide is removed.

In order to recirculate gasses, all rebreather concepts include a mouthpiece, through which the diver breathes, connected with a collapsible bag that inflates when he exhales, and deflates when he inhales. This bag is usually called counterlung.

Complete collapse of the counterlung when external pressure increases is avoided by the action of a demand valve which is triggered when the counterlung is almost completely collapsed. This always guarantees the availability of a minimum breathable gas volume at any depth by the injection of a fresh gas mixture usually called 'diluent gas'. Similarly over-expansion of the counterlung when external pressure decreases is avoided by an overpressure relief valve.

To reestablish a breathable mixture, rebreathers must be equipped with a device (usually a chemical scrubber) for CO_2 removal and a supply valve for O_2 injection into the breathing loop. CCR has a feed-back electronic controller that, based on the measure of the oxygen level in the counterlung, injects pure oxygen by operating a solenoid supply valve, so as to regulate oxygen partial pressure to a given set-point value during the entire dive. The set point value should be selected by considering that

- the oxygen partial pressure inside the breathing loop should never fall below 0.16 [atm] to avoid hypoxia, *i.e.* the impossibility to fulfill the metabolic oxygen requirements. It can rapidly bring the diver to unconsciousness (Clark and Lambertsen, 1971).
- Breathing oxygen at high partial pressure (greater than 0.5 [atm]) can be toxic (Lambertsen et al., 1987). Central Nervous System (CNS) oxygen toxicity is related to both oxygen pressure level and duration of the exposure (Butler and Thalmann, 1984).

Controlling the oxygen partial pressure during all phases of a dive is a crucial task for the correct and safe use of a CCR. In practice it is convenient to keep oxygen partial pressure to the highest values compatible with CNS toxicity exposure limits in relation to the planned duration of the dive. This decreases the partial pressure of the inert gas in the mixture and hence decompression obligations (Buehlmann, 1995).

However O_2 partial pressure in the breathing loop is subject to variations due to disturbances such as

- the diver individual metabolism, which depends on the workload;
- the internal pressure of the counterlung which changes with the dive profile.

While variations of O_2 partial pressure due to the depth can be easily predicted, metabolic oxygen consumption rate can vary from person to person by a factor of 6 (or more) in normal conditions, and as much as 10-fold in extreme conditions, depending on the activity level.

So, in order to keep the oxygen partial pressure at a desired value, it is important to have knowledge of the actual metabolic consumption rate. Since the actual technology does not provide easy and reliable on-line measurements of the metabolism, here it is proposed the use of a state observer for the correct estimation of the metabolic consumption. In this way it is possible to compute that quantity in real time. This work opens to the possibility to have new CCR systems that exploit the state estimation for control purposes and diagnostics, as well.

Note that at the moment the use of CCR is limited to military applications and scientific diving. It is common opinion that the diffusion of these devices will increase as soon as control and estimation problems will be properly and reliably solved.

2. THE REBREATHER SYSTEM

In this section a physics based analytical model is presented for predicting the oxygen level in rebreather counterlung for various dive profiles and diver activity levels. This open-loop model will be useful to design the state observer and to asses its performances.

The model considers the counterlung as an adiabatic collapsible recipient which contains the breathable mixture with the following assumptions: *i)* the breathable gas is composed by two species: inert gas (usually nitrogen and/or helium) and oxygen; *ii)* the breathing gas behaves as perfect mixture; *iii)* the recipient internal pressure instantaneously equals the external pressure. The respiration process is simply modelled as a (partial) oxygen subtraction from the collapsible bag at a time varying rate. No carbon dioxide presence is assumed in the breathable gas, *i.e.* perfect CO_2 adsorption is considered.

According to these assumptions, it is possible to derive the counterlung dynamic model as a balance of volume flow rates [liter/min] (see (Garofalo et al., 2003) and (Garofalo et al., 2004) for details):

$$\dot{p}_{\mathrm{pO_2}} = \left(\frac{p_\mathrm{a}}{V} - \frac{p_{\mathrm{pO_2}} p_\mathrm{a}}{p_\mathrm{e} V}\right)(s - m) + \frac{p_{\mathrm{pO_2}}}{p_\mathrm{e}}\dot{p}_\mathrm{e} + + \left(\frac{p_\mathrm{e}\beta}{V} - \frac{p_{\mathrm{pO_2}}}{V}\right) s_{\mathrm{dv}}, \tag{1a}$$

$$\dot{V} = \frac{p_\mathrm{a}}{p_\mathrm{e}}(s - m) - \frac{V}{p_\mathrm{e}}\dot{p}_\mathrm{e} + s_{\mathrm{dv}} - q, \tag{1b}$$

where $p_{\mathrm{pO_2}}$ is the oxygen partial pressure level [atm], V is the counterlung total volume, defined as the sum of the oxygen and inert partial volumes, p_e is the hydrostatic pressure [atm], s is the flow of the supply valve [liter/min], m is the oxygen metabolic volume rate consumption [liter/min], p_a is the pressure at the sea level [atm], s_{dv} is the demand valve flow [liter/min], q is the exhaust flow through the relief valve [liter/min] and β is the oxygen fraction of the gas mixture supplied by the demand valve.

Equations (1) show that the simplified model of the rebreather is nonlinear with respect to state variables and control input. They also show, as expected, that counterlung volume dynamics do affect oxygen partial pressure dynamics while the converse is not true.

Model (1) has been validated through extensive simulations on experimental data collected during different dives. It shows a good agreement with the real scenario and some validation results can be found in (Garofalo et al., 2003) and (Garofalo et al., 2004).

3. THE CLOSED LOOP-PLANT

The CCR has to be controlled so as

(1) to limit the excursions of the counterlung total volume in an interval $[\mathrm{V_m}, \mathrm{V_M}]$ (this both to guarantee at any time a minimal breathable volume for the diver and to avoid its damage by over-expansion);
(2) to keep oxygen partial pressure below a given value, selecting a trade-off between decompression obligations and oxygen toxicity risk mitigation.

The first control requirement is achieved by the 'demand valve-relief valve' pneumatic system which behaves as static nonlinear feedback of the actual volume with the characteristic given in Figure 1. Due to the nature of the pneumatic system, it is possible to assume an almost instantaneous volume variation given by the mechanical feedback, thus any related dynamics can be neglected. This system is capable of bounding the counterlung volume in spite of external pressure variation and oxygen subtraction/injection (see equation (1b)). The second requirement, i.e. the

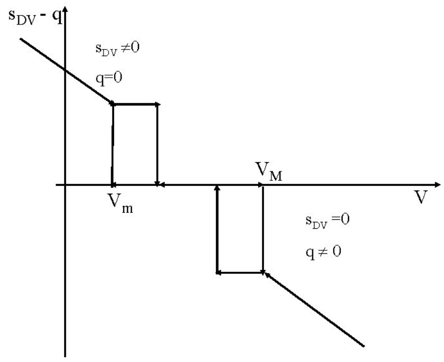

Fig. 1. Characteristic of the static nonlinear feedback of the actual volume V.

regulation of the $p_{\mathrm{pO_2}}$, is achieved by controlling the oxygen flow from the solenoidal valve, thus compensating for metabolic oxygen consumption and variations of external pressure (see equation (1a)). The solenoid valve is an on-off actuator and the maximum value of the supplied flow is a nonlinear function of the external pressure p_e (see (Garofalo et al., 2004) for details).

The closed-loop plant consists of a PI control law on the $p_{\mathrm{pO_2}}$ measurement, implemented by a pulse width modulation (PWM) technique that takes into account the on-off behavior of the actuator. It is well known that the integral action of the controller guarantees that, at the steady state behavior, constant disturbances are fully rejected so when the external pressure and the metabolic consumption rate are constant, the desired set-point is tracked. For the considered plant (see equations (1)) this implies that the control signal balances the metabolic consumption rate in an average sense (considering the PWM implementation). Hardware-in-the-loop results of the controller effectiveness are available in (Garofalo et al., 2004).

4. SLIDING OBSERVER

For a general nonlinear system, separation principle does not exist and the observation problem is strongly dependent on the control law and the inputs of the system. Thus, in the following the closed loop plant will be considered, since the aim of the paper is to design a state observer for a given control law.

The problem of estimating the counterlung volume and diver metabolism is addressed by designing a sliding state observer. Sliding observers estimate the system states directly according to the nonlinear nature of the plant (Slotine et al., 1987). Moreover their simple structure, robustness with respect to disturbances, parameter deviations and noise, (Sungwongwanich et al., 1990), (Tursini et al., 2000) makes them an attractive choice in our kind of problem. Obviously other kind of techniques, such as Extended Kalman Filter, can be also used, but they require a bigger computation effort and they lack of well defined tuning criteria. In this paper some results about the performances of a sliding observer will be presented, while a comparison with other techniques, like Kalman Filter, will be object of future work.

The observer is designed by neglecting the static nonlinear feed-back of the actual volume, as previously discussed in Section 3.

Letting $x = [p_{\mathrm{pO_2}}, \mathrm{V}]$ be the state vector and $u = s$ be the control input, the open-loop model equations (1) can be rewritten as

$$\dot{x}_1 = \left(\frac{p_\mathrm{a}}{x_2} - \frac{x_1 p_\mathrm{a}}{x_2 p_\mathrm{e}}\right)(u - m) + \frac{x_1}{p_\mathrm{e}}\dot{p}_\mathrm{e} \quad (2a)$$

$$\dot{x}_2 = \frac{p_\mathrm{a}}{p_\mathrm{e}}(u - m) - \frac{x_2}{p_\mathrm{e}}\dot{p}_\mathrm{e}. \quad (2b)$$

Note that p_e, $\dot{p}_\mathrm{e}$ and m act as disturbances for the plant. The variables $p_{\mathrm{pO_2}}$ and p_e are measured and $\dot{p}_\mathrm{e}$ can be computed on-line while m is unknown.

In order to estimate the disturbance m, it is convenient to enlarge the state vector and rewrite the equations (2) as

$$\dot{x}_1 = \frac{p_\mathrm{a}}{p_\mathrm{e}} \frac{(p_\mathrm{e} - x_1)}{x_2}(u - x_3) + \frac{x_1}{p_\mathrm{e}}\dot{p}_\mathrm{e}, \tag{3a}$$

$$\dot{x}_2 = \frac{p_\mathrm{a}}{p_\mathrm{e}}(u - x_3) - \frac{x_2}{p_\mathrm{e}}\dot{p}_\mathrm{e}, \tag{3b}$$

$$\dot{x}_3 = 0, \tag{3c}$$

by assuming a slowly time-varying metabolism.

Starting from equations (3) the sliding mode asymptotic observer uses p_{pO_2} and p_e measurements for estimating the state and can be written as (Slotine et al., 1987)

$$\dot{\hat{x}}_1 = \frac{p_\mathrm{a}}{p_\mathrm{e}} \frac{(p_\mathrm{e} - \hat{x}_1)}{\hat{x}_2}(u - \hat{x}_3) + \frac{\hat{x}_1}{p_\mathrm{e}}\dot{p}_\mathrm{e} + \\ + k_1 \operatorname{sgn}(x_1 - \hat{x}_1), \tag{4a}$$

$$\dot{\hat{x}}_2 = \frac{p_\mathrm{a}}{p_\mathrm{e}}(u - \hat{x}_3) - \frac{\hat{x}_2}{p_\mathrm{e}}\dot{p}_\mathrm{e} + \\ + k_2 \operatorname{sgn}(x_1 - \hat{x}_1), \tag{4b}$$

$$\dot{\hat{x}}_3 = k_3 \operatorname{sgn}(x_1 - \hat{x}_1). \tag{4c}$$

From equations (3) and (4) the error dynamics are

$$\dot{e}_1 = (\dot{x}_1 - \dot{\hat{x}}_1) = -\frac{p_\mathrm{a}}{p_\mathrm{e} x_2 \hat{x}_2} e_2 (p_\mathrm{e} - \hat{x}_1)(u - x_3) + \\ -\frac{p_\mathrm{a}}{p_\mathrm{e} x_2} e_1 (u - x_3) - \frac{p_\mathrm{a}}{p_\mathrm{e} \hat{x}_2} e_3 (p_\mathrm{e} - x_1) + e_1 \frac{\dot{p}_\mathrm{e}}{p_\mathrm{e}} + \\ - k_1 \operatorname{sgn}(e_1) = \Delta(x, \hat{x}, u) - k_1 \operatorname{sgn}(e_1), \tag{5a}$$

$$\dot{e}_2 = (\dot{x}_2 - \dot{\hat{x}}_2) = -\frac{p_\mathrm{a}}{p_\mathrm{e}} e_3 - \frac{\dot{p}_\mathrm{e}}{p_\mathrm{e}} e_2 + \\ - k_2 \operatorname{sgn}(e_1), \tag{5b}$$

$$\dot{e}_3 = (\dot{x}_3 - \dot{\hat{x}}_3) = -k_3 \operatorname{sgn}(e_1), \tag{5c}$$

with

$$\Delta(x, \hat{x}, u) = -\frac{p_\mathrm{a}}{p_\mathrm{e}} \left[\frac{e_3}{\hat{x}_2}(p_\mathrm{e} - x_1) \right] + \frac{\dot{p}_\mathrm{e}}{p_\mathrm{e}} e_1 + \\ -\frac{p_\mathrm{a}}{p_\mathrm{e}} \left[\frac{e_1}{x_2}(u - x_3) + \frac{e_2}{x_2 \hat{x}_2}(p_\mathrm{e} - \hat{x}_1)(u - x_3) \right] \tag{6}$$

The sliding manifold is $e_1 = 0$, while the reaching condition can be written as

$$\eta + |\Delta(x, \hat{x}, u)| \leq k_1 \tag{7}$$

with an arbitrary constant $\eta > 0$. Along the sliding manifold it holds $\dot{e}_1 = 0$. In this situation, according to the Filippov conditions (Filippov, 1988), from equation (5a) $\Delta(x, \hat{x}, u)$ can be written as a convex combination of the values $+k_1$ and $-k_1$

$$\Delta(x, \hat{x}, u) = k_1 \gamma - (1 - \gamma) k_1 \Rightarrow \gamma = \frac{\Delta(x, \hat{x}, u)}{2k_1} + \frac{1}{2}.$$

With this γ on the sliding manifold it follows

$$\operatorname{sgn}(e_1) = \gamma - (1 - \gamma) = \frac{\Delta(x, \hat{x}, u)}{k_1}. \tag{8}$$

Taking into account equation (8), during the sliding motion the following error dynamics occur

$$\dot{e}_2 = -\frac{p_\mathrm{a}}{p_\mathrm{e}} e_3 - \frac{\dot{p}_\mathrm{e}}{p_\mathrm{e}} e_2 - \frac{k_2}{k_1} \Delta(x, \hat{x}, u)|_{e_1 = 0}, \tag{9a}$$

$$\dot{e}_3 = -\frac{k_3}{k_1} \Delta(x, \hat{x}, u)|_{e_1 = 0}, \tag{9b}$$

or, extensively,

$$\dot{e}_2 = -\frac{p_\mathrm{a}}{p_\mathrm{e}} e_3 - \frac{\dot{p}_\mathrm{e}}{p_\mathrm{e}} e_2 + \frac{k_2}{k_1} \frac{p_\mathrm{a}}{p_\mathrm{e}} \left(\frac{p_\mathrm{e} - \hat{x}_1}{\hat{x}_2} \right) \times \\ \times \left[\frac{e_2}{x_2}(u - x_3) + e_3 \right], \tag{10a}$$

$$\dot{e}_3 = \frac{k_3}{k_1} \frac{p_\mathrm{a}}{p_\mathrm{e}} \left(\frac{p_\mathrm{e} - \hat{x}_1}{\hat{x}_2} \right) \times \\ \times \left[\frac{e_2}{x_2}(u - x_3) + e_3 \right]. \tag{10b}$$

In order to give a constant convergence rate to the error dynamics (see equations (10)), the observer gains k_2 and k_3 can be chosen in the following suitable way

$$k_2 = \frac{k_1 p_\mathrm{a} \hat{x}_2}{p_\mathrm{e} - \hat{x}_1} \alpha_2(\hat{x}, u), \tag{11a}$$

$$k_3 = -\frac{k_1 p_\mathrm{a} \hat{x}_2}{p_\mathrm{e} - \hat{x}_1} \alpha_3(\hat{x}, u). \tag{11b}$$

These gains are functions of the estimated state and the known and measurable signals like the control input and the external pressure. Furthermore the α functions will be designed as in Section 4.1 for coping with the dependence of the error dynamics from the input signal u.

Inserting equations (11) in (10), finally it comes out

$$\dot{e}_2 = -\frac{p_\mathrm{a}}{p_\mathrm{e}} e_3 - \frac{\dot{p}_\mathrm{e}}{p_\mathrm{e}} e_2 + \frac{\alpha_2(\hat{x}, u)}{p_\mathrm{e}} \times \\ \times \left[\frac{e_2}{x_2}(u - x_3) + e_3 \right], \tag{12a}$$

$$\dot{e}_3 = -\frac{\alpha_3(\hat{x}, u)}{p_\mathrm{e}} \left[\frac{e_2}{x_2}(u - x_3) + e_3 \right]. \tag{12b}$$

4.1 Convergence of the observation error

The design of the sliding observer requires two sequential steps. The first one is the fulfill of the reaching condition (7) by properly selecting the gain k_1. By some physical insights ($\Delta(x, \hat{x}, u)$ can be bounded) it is possible to choose a gain k_1 such that the reaching condition (7) is always fulfilled during normal operating conditions. Once the sliding manifold is reached, then the reduced dynamics are given by the above equations (12). The second step is to guarantee the asymptotic stability of the reduced error system.

A Lyapunov analysis will be carried out in this section in order to derive the conditions on the observer gains k_2, k_3 that satisfy the stability requirements. The stability investigation will neglect environmental disturbance. In fact, letting $\dot{p}_e = 0$ in (12), the reduced error system is

$$\dot{e}_2 = -\frac{p_a}{p_e}e_3 + \frac{\alpha_2(\hat{x},u)}{p_e}\left[\frac{e_2}{x_2}(u-x_3)+e_3\right] \tag{13a}$$

$$\dot{e}_3 = -\frac{\alpha_3(\hat{x},u)}{p_e}\left[\frac{e_2}{x_2}(u-x_3)+e_3\right]. \tag{13b}$$

For this addressed problem the following Lyapunov function has been chosen

$$V(e_2,e_3) = \frac{1}{2}c_2e_2^2 + \frac{1}{2}c_3e_3^2. \tag{14}$$

Taking into account equations (13) and (14), after some algebraic computations the time derivative of the Lyapunov function is

$$\begin{aligned}\dot{V}(e_2,e_3) = &-e_2^2\left[-c_2\alpha_2(\hat{x},u)\frac{(u-x_3)}{p_e x_2}\right] + \\ &-e_3^2\left[c_3\frac{\alpha_3(\hat{x},u)}{p_e}\right] + \\ &\underbrace{-e_2e_3\left[c_2\frac{p_a}{p_e} - c_2\frac{\alpha_2(\hat{x},u)}{p_e} + c_3\alpha_3(\hat{x},u)\frac{(u-x_3)}{p_e x_2}\right]}_{\Phi}.\end{aligned} \tag{15}$$

In order to show that $\dot{V}(e_2,e_3)$ is negative definite, firstly it will be neglected the term Φ in (15) (this assumption will be justified below)

$$\begin{aligned}\dot{V}(e_2,e_3) \cong &-e_2^2\left[-c_2\alpha_2(\hat{x},u)\frac{(u-x_3)}{p_e x_2}\right] + \\ &-e_3^2\left[c_3\frac{\alpha_3(\hat{x},u)}{p_e}\right].\end{aligned} \tag{16}$$

It has to be remarked that in closed-loop plant, for the actuator characteristics, the input signal $u(t)$ can assume only two values $u = 0$ or $u = u_{\max} > x_3(t) > 0$. In this way, by choosing a proper switching law for the α_2 as

$$\alpha_2(\hat{x},u) = \begin{cases} \bar{\alpha}_2 > 0 & u = 0, \\ -\bar{\alpha}_2 < 0 & u = u_{\max}, \end{cases} \tag{17}$$

the term multiplying e_2^2 in equation (16) is always positive. Now, setting $\alpha_3(\hat{x},u) = \bar{\alpha}_3 > 0$, the right hand side of equation (16) is always negative. By these choices of the α functions, the observer stability is showed when the hypothesis of the negligibility of the Φ term holds.

Some engineering considerations on this particular application can be made in order to validate our assumption. The PI controller output described in Section 3 is modulated by using a PWM technique. As previously highlighted in steady state behavior (at a fixed depth) the effectiveness of the control action guarantees $u = x_3$ in a average sense. Taking into account this feature, equation (13b) shows that $e_3(t)$ converges to zero independently on the volume estimation error $e_2(t)$. Consequently the term Φ in equation (15) is negligible after a transient depending on the observer gain α_3 (see equation (13b)).

Note that the stability analysis has been investigated in the absence of environmental disturbances. The robustness of the observer with respect to depth variations has been checked through extensive simulations and some examples will be showed in Section 5.

5. SIMULATION RESULTS

The effectiveness of the proposed sliding observer has been validated through simulations. The nonlinear model of the rebreather system has been identified from experimental data and a typical dive profile has been simulated. During the mission the set-point is varied and external disturbances such as the metabolic consumption and depth variation are simulated.

The observer showed good performances in estimating the metabolic consumption rate also during critical phases such as depth variations during which the time derivative of the external pressure acts as a disturbance. In Figure 2 the comparison between the actual oxygen partial pressure and the estimated one is reported: the sliding condition is assured.

In Figure 3 and 4 the performances of the observer are highlighted: it is able to estimate quite well both volume and metabolism. The spikes in the estimation error of the metabolism are due essentially to the variation of the external pressure: the convergence analysis of Section 4 has been carried on neglecting the influence of $\dot{p}_e$. Anyway simulations show some robustness to the disturbances.

6. CONCLUSIONS

In this paper it has been proposed a sliding observer for estimating volume and metabolic consumption rate in a rebreather system. A Lyapunov convergence analysis has been carried on allowing to derive some criteria for designing the observer. The effectiveness of the proposed approach has been validated through simulations. The implementation of such technique is under investigation and some hardware-in-the-loop simulations are going on by using standard simulation software (MATLAB$^{\text{TM}}$/Real Time Workshop) and low cost micro-controllers. A future work will include such results showing the real performances of the observer under realistic environmental conditions.

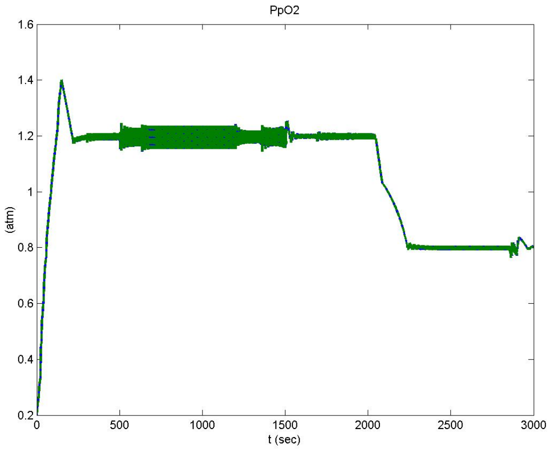

Fig. 2. Profile of the actual (*solid*) p_{PO_2} and the estimated (*dash*) one.

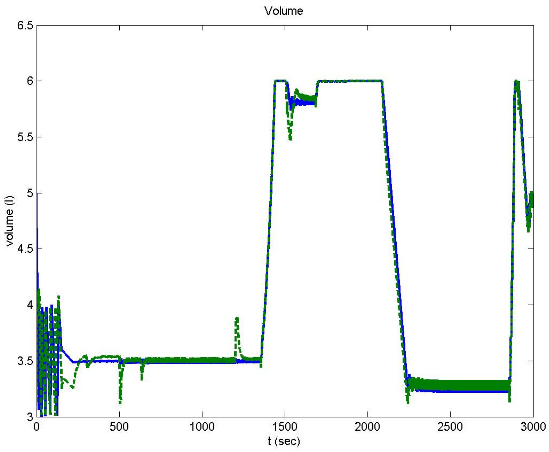

Fig. 3. Profile of the actual (*solid*) volume V and the estimated (*dash*) one.

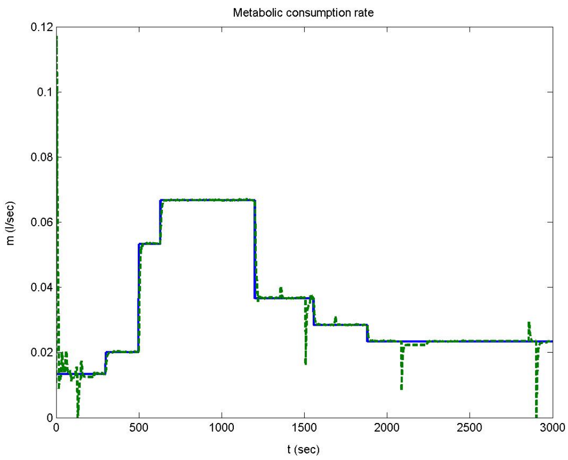

Fig. 4. Profile of the actual (*solid*) metabolic rate m and the estimated (*dash*) one.

7. ACKNOWLEDGMENT

The authors would like to thank Luca Avitabile for his competent contribution to simulations activities.

REFERENCES

AA. Buehlmann. *Tauchmedizin. Barotrauma. Gasembolie. Dekompression, Dekompressionskrankheit,Dekompressionscomputer.* Springer-Verlang, Berlin, 4th Edn., 1995.

F. K. Butler and E. D. Thalmann. Cns oxygen toxicity in closed-circuit scuba divers. *In: Underwater Physiology VIII. Eds. A.J. Bachrach and M.M. Matzen. UMS, Inc., Bethesda*, 1984.

J.M. Clark and C.J. Lambertsen. Pulmonary oxygen toxicity: a review. *Pharmacol, pp. 37–133*, 1971.

A. F. Filippov. *Differential Equations with Discontinous Rigthand Sides.* Kluwer Accademic Publishers, 1988.

F. Garofalo, S. Manfredi, and S. Santini. Modelling and control of oxygen partial pressure in an underwater breathing apparatus with gas recycle. *Proceedings of IEEE European Control Conference, England, September*, 2003.

F. Garofalo, S. Manfredi, and S. Santini. Electronic control of an underwater breathing apparatus. *Proceedings of IFAC Conference on Control Applications in Marine Systems Ancona, Italy, July 7-9*, 2004.

C.J. Lambertsen, J.M. Clark, and R. Gelfand. Definition of tolerance to continuous hyperoxia in man. an abstract report of predictive studies. *Underwater and Hyperbaric Physiology IX (A.A. Bove, A.J. Bacherach, and L.J. Greenbaum, eds), Undersea and Hyperbaric Medical Society, Bethesda. pp. 717-735.*, 1987.

J.J.A. Slotine, J.K. Hedrick, and E.A. Misawa. On sliding observers for nonlinear systems. *ASME J. Dynamic Systems, Measurements and Control, vol. 109*, 1987.

S. Sungwongwanich, S. Doki, T. Yonemoto, T. Furu-hashi, and S. Okuma. Design of sliding observers for robust estimation of rotor flux of induction motors. *Proceedings of Int. Power Electronics Conf.*, 1990.

M. Tursini, R. Petrella, and F. Parasiliti. Adaptive sliding mode observer for speed-sensorless control of induction motors. *IEEE Trans. on Industry Applications, vol. 36, no 5*, 2000.