ROBUST GLOBAL EXPONENTIAL STABILIZATION OF AN UNDERACTUATED AIRSHIP

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Abstract: This paper considers the problem of controlling both the planar position and orientation of an underactuated airship with a reduced number of actuators. The airship is a nonholonomic system described by a set of nonlinear equations and the dynamics are subject to bounded uncertainties. A smooth and time-varying coordinate transformation is utilized to reduce the stabilization problem of the airship to that of a linear time-invariant system. A new robust feedback controller is presented for obtaining global exponential stabilization of the airship in the presence of the plant uncertainties. The proposed design method is simple and straightforward. Experiments are performed to validate the effectiveness of the proposed controller.

Key words: aerospace, autonomous vehicle, attitude and position control, nonlinear system, perturbed coefficients, uncertainty, global stability, exponentially stable, robust control

1. INTRODUCTION

Airship is a unique vehicle which enables safe and long-duration flight and station keeping. Recently, the interest to unmanned airships has been increasing. Many interesting projects are in progress such as Stratospheric Platform Project (Onda, 1999), in which the stratospheric airship is utilized as a mobile and low-cost platform for high-speed wireless communication instead of satellites. The position and attitude control is one of the most important component technologies.

This paper considers the problem of controlling both the planar position and orientation of an under-actuated airship, which is a nonholonomic system with fewer independent inputs than degree of freedom to be controlled. The problem is essentially similar to that of an underactuated ship system (Khoury and Gillett, 1999). The obstruction is the fact that there exists no continuous time-invariant state feedback controller to stabilize the system (Brockett, 1983). During the last few years, many interesting approaches have been proposed to stabilize the nonholonomic systems; for example, discontinuous feedback approach (Reyhanoglu, 1996, 1997), time-varying feedback approach (Pettersen and Nijimeijer, 2000; Do et al., 2002a,b; Jiang, 2002), averaging approach (Mazenc et al., 2002; Pettersen and Fossen, 2000). These studies mainly deal with the stabilization problem of underactuated ships.

Reyhanoglu (1996, 1997) applied a σ-process technique (Arnold, 1983; Astolfi, 1996) to the stabilization problem of an underactuated ship and derived a discontinuous coordinate transformation such that the stabilization problem can be reduced to the conventional pole-placement problem of a linear system. This approach makes the linear control theory applicable. Thus, it is useful not only for the design of stabilizing controllers but also for more sophisticated control problems. However, the method involves some troublesome problems that the coordinate transformation has some singular points, and asymptotic stability is not guaranteed for some initial states. Tian and Li (2002) generalized the coordinate transformation to a larger class of nonholonomic systems that include chained systems. These results did not consider uncertainties of the plant dynamics.

This paper considers a robust global stabilization problem for the case where the dynamics of the underactuated airship are subjected to bounded uncertainties. A smooth and time-varying coordinate transformation is utilized to reduce the problem of the airship to that of a linear system. The transformation is independent of the initial state of the system and is nonsingular. The following advantages are obtained. First, this paper deals with the case where the dynamics are known and presents a new controller to guarantee exponential stability of the system for any initial states. The design method based on the well-known linear control theory is simple and straightforward. Second, this paper considers the robust stabilization problem for the uncertain airship with unknown but bounded parameters. The new problem is reduced to the well-known quadric stabilization problem for an uncertain linear system (Petersen, 1987). The design method is simple, and it involves a certain algebraic Riccati equation. Finally, experimental results show the effectiveness of the proposed control system.

2. PROBLEM FORMULATION

Consider the problem of controlling the Cartesian position and orientation of an airship with two independent propellers as shown in Fig.1. (x,y) denotes the earth-fixed position of the center of mass of the airship, θ denotes the orientation angle and u, v and r are the surge, sway, yaw velocities in the vehicle-fixed frame, respectively. For simplicity, the airship is symmetric and the origin of the vehicle-fixed frame is assumed to be located at the center of the mass. It is also assumed that the airship is neutrally buoyant without rolling and pitch motions and the actuators of the airship are two independent propellers without side thrusters. Then the dynamic equations of motion of the vehicle can be expressed in the vehicle-fixed frame as

$$\begin{aligned} m_1\dot{u} &= m_2 vr - d_{11}u + \tau_u \\ m_2\dot{v} &= -m_1 ur - d_{22}v \\ m_3\dot{r} &= (m_1 - m_2)uv - d_{33}r + \tau_r, \end{aligned} \qquad (1)$$

where $m_i > 0$ and $d_{ii} > 0, i = 1,2,3$ are the inertia constants and the damping ones, including hydrodynamic added mass effects. For simplicity, it is assumed that $m = m_1 = m_2$. τ_u *and* τ_r denote the external force and torque generated by the two propellers. The kinematical model that describes the geometrical relationship between the earth-fixed and the vehicle-fixed motion is given by

$$\begin{aligned} \dot{x} &= u\cos\theta - v\sin\theta \\ \dot{y} &= u\sin\theta + v\cos\theta \\ \dot{\theta} &= r\ . \end{aligned} \qquad (2)$$

Let

$$\begin{aligned} z_1 &= x\cos\theta + y\sin\theta \\ z_2 &= -x\sin\theta + y\cos\theta\ . \end{aligned} \qquad (3)$$

By putting together the kinematics and the dynamics, the following state equation of the airship is obtained.

$$\Sigma_1 : \frac{d}{dt}\begin{bmatrix} u \\ v \\ z_1 \\ z_2 \end{bmatrix} = \begin{bmatrix} -d_1 & r & 0 & 0 \\ -r & -d_2 & 0 & 0 \\ 1 & 0 & 0 & r \\ 0 & 1 & -r & 0 \end{bmatrix}\begin{bmatrix} u \\ v \\ z_1 \\ z_2 \end{bmatrix} + \begin{bmatrix} \frac{1}{m} \\ 0 \\ 0 \\ 0 \end{bmatrix}\tau_u$$

$$\Sigma_2 : \begin{cases} \frac{d}{dt} r = -d_3 r + \frac{1}{m_3}\tau_r \\ \frac{d}{dt}\theta = r \end{cases}, \qquad (4)$$

where

$$d_i = d_{ii}/m_i\ , \quad i = 1,2,3\,.$$

The objective of this paper is to find the control inputs τ_u, τ_r such that the systems of both Σ_1 and Σ_2 are exponentially globally stable at the origin.

3. PRELIMINARY

This section shows the key lemma to solve the stabilization problem.

Lemma 1 (Slotine and Li, 1991)

Consider the following time-varying system

$$\frac{d}{dt}\boldsymbol{x} = \left(A_1 + A_2(t)\right)\boldsymbol{x}\,, \qquad (5)$$

where A_1 is a constant and Hurwitz matrix and $A_2(t)$ is a matrix such that $A_2(t) \to 0$ $(t \to \infty)$ and

$$\int_0^\infty \| A_2(t) \| dt < \infty\ . \qquad (6)$$

Then the system is globally exponentially stable at the origin.

4. GLOBAL STABILIZATION

This section deals with the case where the damping constants of $d_i, i = 1,2,3$ are exactly known and considers a state feedback stabilization problem such that the subsystems of Σ_1 and Σ_2 are both exponentially globally stable at the origin. To begin with, let us focus on the subsystem Σ_2, called the linear subsystem. Consider the feedback controller described by

$$\begin{aligned} \tau_r &= m_3(-k_r\, r - k_\theta\,\theta + a\, e^{-k_3 t}) \\ k_r &= k_1 + k_2 - d_3 \\ k_\theta &= k_1\, k_2\ , \end{aligned} \qquad (7)$$

where $[k_1, k_2, k_3, a] \in \boldsymbol{R}^4$ are free parameters, s.t.,

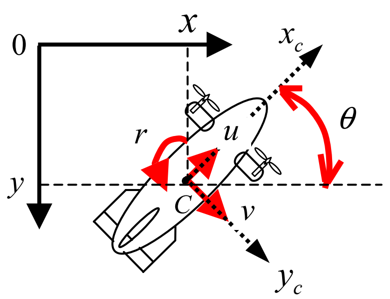

Fig.1 Model of an airship with two propellers.

$$k_1 > k_2 > k_3 > 0, \quad a \neq 0. \tag{8}$$

Then the subsystem Σ_2 is expressed as

$$\ddot{\theta}(t) + (k_1 + k_2)\dot{\theta}(t) + k_1 k_2 \theta(t) = a e^{-k_3 t}. \tag{9}$$

By solving the differential equation, the responses of the yaw angle and the yaw velocity are obtained as

$$\theta(t) = \theta_1 e^{-k_1 t} + \theta_2 e^{-k_2 t} + \theta_3 e^{-k_3 t}$$
$$r(t) = -\theta_1 k_1 e^{-k_1 t} - \theta_2 k_2 e^{-k_2 t} - \theta_3 k_3 e^{-k_3 t}, \tag{10}$$

where

$$\theta_1 = \frac{k_2\theta(0) + r(0)}{k_2 - k_1} + \frac{a}{(k_2 - k_1)(k_3 - k_1)}$$
$$\theta_2 = \frac{k_1\theta(0) + r(0)}{k_1 - k_2} + \frac{a}{(k_1 - k_2)(k_3 - k_2)}$$
$$\theta_3 = \frac{a}{(k_3 - k_1)(k_3 - k_2)}. \tag{11}$$

Since $k_1 > k_2 > k_3 > 0$, the subsystem Σ_2 is exponentially globally stable at the origin.

Next, let us focus on the subsystem Σ_1 called the nonlinear subsystem. Note that, from Eqs.(4) and (10), the subsystem Σ_1 becomes a linear time-varying system consisting of the time varying element of $r(t)$. It may look natural to decompose the subsystem into a time-invariant part and time-varying part consisting of $r(t)$ only. Unfortunately, however, the time-invariant system is not stabilizable, and Lemma 1 does not apply to the decomposition to solve the stabilization problem. The key idea is, then, the reduction of the nonlinear subsystem by applying a smooth coordinate transformation as follows:

$$\bar{u} = u, \ \bar{v} = v e^{k_3 t}, \ \bar{z}_1 = z_1, \ \bar{z}_2 = z_2 e^{k_3 t}. \tag{12}$$

Define the new state vector as

$$\bar{\boldsymbol{x}} = [\bar{u}, \bar{v}, \bar{z}_1, \bar{z}_2]^T \in \boldsymbol{R}^4. \tag{13}$$

Then the subsystem Σ_1 is represented as

$$\dot{\bar{\boldsymbol{x}}} = \bar{A}(t)\bar{\boldsymbol{x}} + \bar{\boldsymbol{b}}\tau_u, \tag{14}$$

where

$$\bar{A}(t) = \begin{bmatrix} -d_1 & r(t)e^{-k_3 t} & 0 & 0 \\ -r(t)e^{k_3 t} & -d_2 + k_3 & 0 & 0 \\ 1 & 0 & 0 & r(t)e^{-k_3 t} \\ 0 & 1 & -r(t)e^{k_3 t} & k_3 \end{bmatrix}$$

$$\bar{\boldsymbol{b}} = [\tfrac{1}{m} \ 0 \ 0 \ 0]^T. \tag{15}$$

Note that the (2,1) and (4,3) elements have nonzero constant term of $\theta_3 k_3$ as follows:

$$r(t)e^{k_3 t} = -\theta_3 k_3 - \alpha(t)$$
$$\alpha(t) = \theta_1 k_1 e^{-(k_1 - k_3)t} + \theta_2 k_2 e^{-(k_2 - k_3)t}. \tag{16}$$

The subsystem Σ_1 is decomposed as follows:

$$\dot{\bar{\boldsymbol{x}}} = (\bar{A}_1 + \bar{A}_2(t))\bar{\boldsymbol{x}} + \bar{\boldsymbol{b}}\tau_u, \tag{17}$$

where

$$\bar{A}_1 = \begin{bmatrix} -d_1 & 0 & 0 & 0 \\ \theta_3 k_3 & -(d_2 - k_3) & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & \theta_3 k_3 & k_3 \end{bmatrix}$$

$$\bar{A}_2(t) = \begin{bmatrix} 0 & r(t)e^{-k_3 t} & 0 & 0 \\ \alpha(t) & 0 & 0 & 0 \\ 0 & 0 & 0 & r(t)e^{-k_3 t} \\ 0 & 0 & \alpha(t) & 0 \end{bmatrix}. \tag{18}$$

It is easily checked that the time-varying element of $\bar{A}_2(t)$ satisfies both $\bar{A}_2(t) \to 0 \ (t \to \infty)$ and Eq.(6). The following lemma guarantees the controllability of the reduced subsystem (17) removing the time-varying matrix $\bar{A}_2(t)$.

Lemma 2

Consider the following time-invariant system

$$\dot{\bar{\boldsymbol{x}}} = \bar{A}_1\bar{\boldsymbol{x}} + \bar{\boldsymbol{b}}\tau_u. \tag{19}$$

This system is controllable if and only if

$$k_3 \neq d_2. \tag{20}$$

Proof: The proof is straightforward, and is omitted.

The following theorem presents a simple and explicit design method of the stabilizing controller.

Theorem 1

Design

$$\bar{k} = [\bar{k}_1, \bar{k}_2, \bar{k}_3, \bar{k}_4] \in R^4 \tag{21}$$

such that the matrix of $(\bar{A}_1 - \bar{b}\bar{k})$ is Hurwitz. Consider the state feedback controller of Eq.(7) and

$$\tau_u = -(\bar{k}_1 u + \bar{k}_2 v e^{k_3 t} + \bar{k}_3 z_1 + \bar{k}_4 z_2 e^{k_3 t}). \tag{22}$$

$[k_1, k_2, k_3, a] \in \boldsymbol{R}^4$ in Eq.(7) satisfies Eqs.(8) and (20), Then the system of Eq.(4) is exponentially stable at the origin for any initial states.

Proof: From Eq.(22), the subsystem Σ_1 is represented by

$$\dot{\bar{\boldsymbol{x}}} = ((\bar{A}_1 - \bar{\boldsymbol{b}}\bar{\boldsymbol{k}}) + \bar{A}_2(t))\bar{\boldsymbol{x}}. \tag{23}$$

Since the matrix of $(\bar{A}_1 - \bar{b}\bar{k})$ is Hurwitz, Lemma 1 guarantees that the system (23) is globally exponentially stable at the origin.

Remark 1: Lemma 2 makes clear that there exists a feedback gain $\bar{\boldsymbol{k}} \in \boldsymbol{R}^4$ such that $(\bar{A}_1 - \bar{b}\bar{k})$ is Hurwitz. Moreover the design method is simple and straightforward by applying the conventional linear control theory.

Remark 2: Compared with the previous results (Reyhanoglu, 1996, 1997; Tian and Li, 2002), the proposed method has two merits. First, the coordinate transformation is smooth and is non-singular. Second, the linear time-invariant part $(\bar{A}_1, \bar{b})$ of the reduced system is independent of the initial state. Accordingly, the proposed method can guarantee that exponential stability of the system holds for any initial states.

Remark 3: Theorem 1 shows that by using the state feedback controller (7) and the coordinate transformation (12), the stabilization problem of the nonlinear system (4) is reduced to that of the linear time-invariant system (19). This approach makes the linear control theory applicable. Thus, it is useful for more sophisticated control problems. In the next section, we will apply the reduction to a robust stabilization problem.

5. ROBUST GLOBAL STABILIZATION

This section deals with the case where the damping constants are subjected to bounded uncertainties as follows:

$$\underline{d}_i \le d_i \le \overline{d}_i, \quad i = 1,2,3, \tag{24}$$

where $\underline{d}_i, \overline{d}_i > 0,\ i = 1,2,3$ are known in advance. The aim of this section is to solve a robust stabilization problem such that the systems of Σ_1 and Σ_2 are both exponentially globally stable at the origin in the presence of the uncertainties.

First, let us focus on the linear subsystem Σ_2. Consider the feedback controller described by

$$\begin{aligned} \tau_r &= m_3(-k_r\, r - k_\theta\, \theta + a\, e^{-k_3 t}) \\ k_r &= k_1 + k_2 - \underline{d}_3 \\ k_\theta &= k_1\, k_2, \end{aligned} \tag{25}$$

where $[k_1, k_2, k_3, a] \in \boldsymbol{R}^4$ are free parameters that satisfy Eq.(8). The subsystem Σ_2 is expressed as

$$\begin{aligned} \ddot{\theta}(t) + (k_1 + k_2 + \Delta_d)\,\dot{\theta}(t) + k_1\, k_2\, \theta(t) = a\, e^{-k_3 t} \\ \Delta_d = d_3 - \underline{d}_3. \end{aligned} \tag{26}$$

From Eq.(24), it follows that

$$0 \le \Delta_d \le \overline{d}_3 - \underline{d}_3. \tag{27}$$

The following lemma is obtained.

Lemma 3

The responses of the yaw angle and the velocity are obtained by

$$\begin{aligned} \theta(t) &= \theta_1\, e^{-\lambda_1 t} + \theta_2\, e^{-\lambda_2 t} + \theta_3\, e^{-k_3 t} \\ r(t) &= -\theta_1\, \lambda_1\, e^{-\lambda_1 t} - \theta_2\, \lambda_2\, e^{-\lambda_2 t} - \theta_3\, k_3\, e^{-k_3 t}, \end{aligned} \tag{28}$$

where

$$\lambda_1 = \frac{(\Delta_d + k_1 + k_2) + \sqrt{\Delta_d^2 + 2\Delta_d(k_1 + k_2) + (k_1 - k_2)^2}}{2}$$

$$\lambda_2 = \frac{(\Delta_d + k_1 + k_2) - \sqrt{\Delta_d^2 + 2\Delta_d(k_1 + k_2) + (k_1 - k_2)^2}}{2}$$

$$\theta_1 = \frac{\lambda_2 \theta(0) + r(0)}{\lambda_2 - \lambda_1} + \frac{a}{(\lambda_2 - \lambda_1)(k_3 - \lambda_1)}$$

$$\theta_2 = \frac{\lambda_1\, \theta(0) + r(0)}{\lambda_1 - \lambda_2} + \frac{a}{(\lambda_1 - \lambda_2)(k_3 - \lambda_2)}$$

$$\theta_3 = \frac{a}{(\lambda_1 - k_3)(\lambda_2 - k_3)}. \tag{29}$$

Since for any d_3 satisfying Eq.(24),

$$\lambda_1 > \lambda_2 > 0, \quad k_3 > 0, \tag{30}$$

the subsystem Σ_2 is globally exponentially stable at the origin. Choose $k_3 \in \boldsymbol{R}$ such that

$$0 < k_3 < \frac{(\overline{d}_3 + k_r) - \sqrt{(\overline{d}_3 + k_r)^2 - 4k_1 k_2}}{2}. \tag{31}$$

Then for any d_3 satisfying Eq.(24), it follows that

$$\lambda_1 > \lambda_2 > k_3 > 0. \tag{32}$$

Proof: The proof is straightforward and is omitted.

Next, let us consider the nonlinear subsystem Σ_1. The smooth coordinate transformation (12) is applied. Then the subsystem Σ_1 is represented as

$$\dot{\boldsymbol{x}} = (\overline{A}_1 + \overline{A}_2(t))\boldsymbol{x} + \boldsymbol{b}\,\tau_u. \tag{33}$$

$\overline{A}_1$ and $\overline{A}_2(t)$ are given by Eq.(18) replaced by

$$\alpha(t) = \theta_1 \lambda_1 e^{-(\lambda_1 - k_3)t} + \theta_2 \lambda_2 e^{-(\lambda_2 - k_3)t}. \tag{34}$$

Since the time-varying $\overline{A}_2(t)$ satisfies both Eq.(6) and $\overline{A}_2(t) \to 0\ (t \to \infty)$, from Lemma 1, the robust stabilization problem is reduced to that of the linear time-invariant system given by

$$\dot{\boldsymbol{x}} = \overline{A}_1\, \boldsymbol{x} + \boldsymbol{b}\,\tau_u. \tag{35}$$

Note that the matrix $\overline{A}_1 \in \boldsymbol{R}^{4\times 4}$ involves a structured uncertainty on the coefficients of d_1, d_2 and θ_3. The bounds of d_1 and d_2 are given by Eq.(24). Since from Eq.(29), $\theta_3 \in \boldsymbol{R}$ is written as

$$\theta_3 = \frac{a}{k_3^2 - (\Delta_d + \underline{d}_3 + k_r)k_3 + k_1 k_2}, \tag{36}$$

it follows that

$$\underline{\theta}_3 \le \theta_3 \le \overline{\theta}_3 \quad (0 \le \Delta_d \le \overline{d}_3 - \underline{d}_3), \tag{37}$$

where

$$\underline{\theta}_3 = \theta_3\big|_{\Delta_d = 0} = \frac{a}{(k_3 - k_1)(k_3 - k_2)}$$

$$\overline{\theta}_3 = \theta_3\big|_{\Delta_d = \overline{d}_3 - \underline{d}_3} = \frac{a}{k_3^2 - (\overline{d}_3 + k_r)k_3 + k_1 k_2}.$$

These uncertainties can be represented as

$$\begin{aligned} d_i &= d_{i0} + \Delta_i, \quad |\Delta_i| \le \delta_i, \quad i = 1,2 \\ \theta_3 &= \theta_{30} + \Delta_3, \quad |\Delta_3| \le \delta_3, \end{aligned} \tag{38}$$

where

$$\begin{aligned} d_{i0} &= \frac{\overline{d}_i + \underline{d}_i}{2}, \quad \delta_i = \frac{\overline{d}_i - \underline{d}_i}{2}, \quad i = 1,2 \\ \theta_{30} &= \frac{\overline{\theta}_3 + \underline{\theta}_3}{2}, \quad \delta_3 = \frac{\overline{\theta}_3 - \underline{\theta}_3}{2}. \end{aligned} \tag{39}$$

Then $\overline{A}_1 \in \boldsymbol{R}^{4\times 4}$ can be separated into a known part and uncertain one as follows:

$$\bar{A}_1 = \bar{A}_{10} + D\Delta E \ , \quad (40)$$

where

$$\bar{A}_{10} = \begin{bmatrix} -d_{10} & 0 & 0 & 0 \\ \theta_{30}k_3 & -(d_{20}-k_3) & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & \theta_{30}k_3 & k_3 \end{bmatrix}$$

$$\Delta = diag\{\frac{\Delta_1}{\delta_1}, \frac{\Delta_2}{\delta_2}, \frac{\Delta_3}{\delta_3}, \frac{\Delta_3}{\delta_3}\}$$

$$D = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}, \ E = \begin{bmatrix} -\delta_1 & 0 & 0 & 0 \\ 0 & -\delta_2 & 0 & 0 \\ k_3\delta_3 & 0 & 0 & 0 \\ 0 & 0 & k_3\delta_3 & 0 \end{bmatrix}.$$

Note that the bound of the uncertainty Δ is given by

$$\|\Delta\| \le 1, \quad (41)$$

where $\| \ \|$ denotes the Euclidean norm. The reduced system (35) is expressed as

$$\dot{\bar{x}} = (\bar{A}_{10} + D\Delta E)\bar{x} + \bar{b}\,\tau_u \,. \quad (42)$$

The following lemma guarantees the controllability of the reduced subsystem (42) in the absence of the uncertainty Δ.

Lemma 4

Consider the following time-invariant system

$$\dot{\bar{x}} = \bar{A}_{10}\bar{x} + \bar{b}\,\tau_u \,. \quad (43)$$

This system is controllable if and only if

$$k_3 \ne d_{20} \,. \quad (44)$$

Then the perturbed system (42) is quadratically stabilizable via linear control if and only if given any positive definite matrix $Q \in \boldsymbol{R}^{4\times 4}$ and any $r > 0$, there exists a constant $\varepsilon > 0$ such that the Riccati equation

$$\bar{A}_{10}{}^T P + P\bar{A}_{10} - \frac{1}{\varepsilon r} P\bar{b}\bar{b}^T P + PDD^T P + E^T E + \varepsilon Q = 0 \quad (45)$$

has a positive definite solution $P \in \boldsymbol{R}^{4\times 4}$.

Proof: See Petersen (1987).

The following theorem presents a simple and explicit design method of the robust controller.

Theorem 2

Consider state feedback controller (25) and choose $k_3 \in \boldsymbol{R}$ satisfying both Eqs.(8), (31) and (44). Given any positive definite matrix $Q \in \boldsymbol{R}^{4\times 4}$ and any $r > 0$, if there exists a constant $\varepsilon > 0$ such that the Riccati equation (45) has a positive definite solution $P \in \boldsymbol{R}^{4\times 4}$, then there exists a controller such that the systems of Σ_1 and Σ_2 are both exponentially stable at the origin for both any uncertainties satisfying Eq.(24) and any initial states. Furthermore, a suitable robust control low is obtained by

$$\tau_u = -(\bar{k}_1 u + \bar{k}_2\, v e^{k_3 t} + \bar{k}_3 z_1 + \bar{k}_4 z_2\, e^{k_3 t}), \quad (46)$$

where the feedback gain

$$\bar{\boldsymbol{k}} = [\bar{k}_1, \bar{k}_2, \bar{k}_3, \bar{k}_4] \in R^4 \quad (47)$$

is given by

$$\bar{\boldsymbol{k}} = \frac{\gamma}{2r\varepsilon}\bar{\boldsymbol{b}}^T P, \quad (\gamma \ge 1)\,. \quad (48)$$

Proof: From Eqs.(33) and (40), the closed loop system Σ_1 with the state feedback controller (46) is represented by

$$\dot{\bar{x}} = (\bar{A}_{10} - \bar{b}\bar{k} + D\Delta E + \bar{A}_2(t))\bar{x}\,. \quad (49)$$

The feedback gain (48) stabilizes quadratically the following system removing the time-varying matrix $\bar{A}_2(t)$

$$\dot{\bar{x}} = (\bar{A}_{10} - \bar{b}\bar{k} + D\Delta E)\bar{x} \quad (50)$$

for any uncertainties satisfying Eq.(24) and any initial states. In other words, the uncertain matrix of $(\bar{A}_{10} - \bar{b}\bar{k} + D\Delta E)$ is Hurwitz. Accordingly, Lemma 1 guarantees that the system (49) is exponentially stable at the origin for any uncertainties of Eq.(24) and any initial states.

6. EXPERIMENTS

Indoor experiments were performed by using a radio-controlled blimp of 1 [m] in length and 0.5 [m] in diameter (Fig.2). The actuators are given by two independent propellers. Test flights were performed to obtain the following physical parameters:

$$\begin{aligned} &[m, m_3] = [0.072[kg], 0.018[kgm^2]] \\ &0.027 \le d_1 \le 0.105, \quad 0.029 \le d_2 \le 0.057, \\ &0.14 \le d_3 \le 0.33. \end{aligned} \quad (51)$$

The coefficients of the feedback controller were chosen as follows:

$$[k_1, k_2, k_3, a] = [0.32, 0.29, 0.13, 0.05], \quad (52)$$

$$\bar{\boldsymbol{k}} = [0.61,\ 0.11,\ 0.20,\ 0.11] \in \boldsymbol{R}^4\,. \quad (53)$$

In the experiment, the initial states were set as follows:

$$\begin{aligned} &[x(0),\ y(0), \theta(0),\ u(0),\ v(0),\ r(0)] = [-0.6[m], \\ &0.6[m],\ 90[\deg], 0[m/s], 0[m/s], 0[\deg/s]]. \end{aligned} \quad (54)$$

Figure 3 shows the trajectory of the center of mass of the blimp, and Figure 4 gives a sequence of photographs of the blimp motion. Figures 5 shows

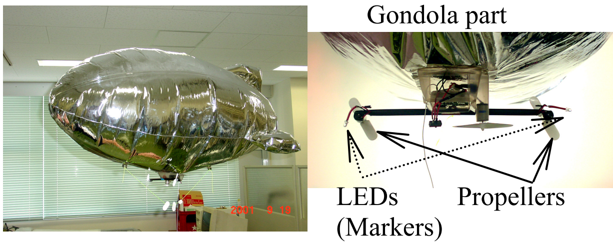

Fig.2 Blimp and propellers in experiment.

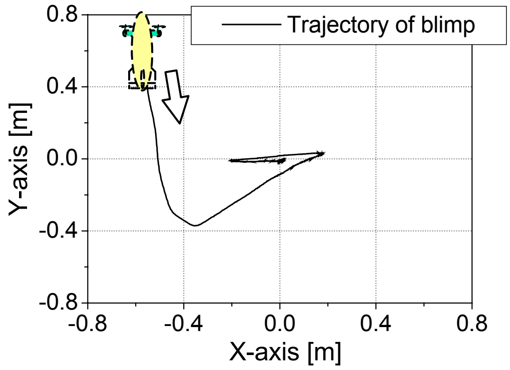

Fig.3 Trajectory of blimp (experiment).

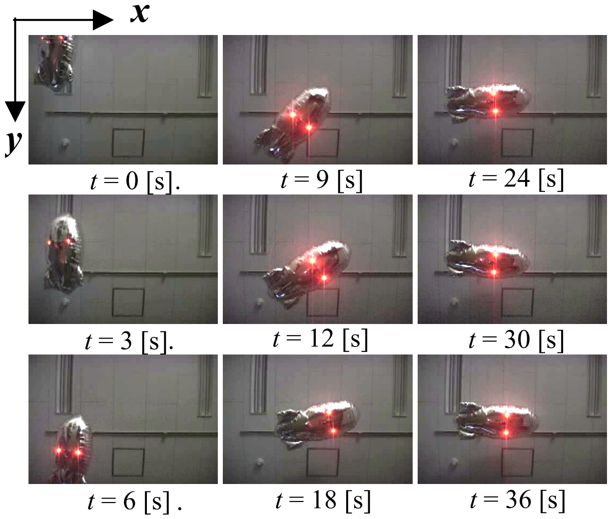

Fig.4 Sequence photographs of blimp motion (experiment).

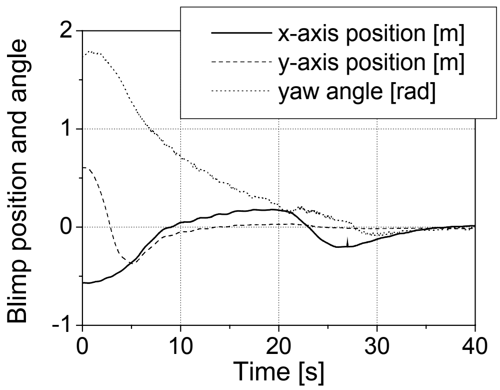

Fig.5 Time responses (experiment).

time responses of the states. These figures demonstrate that the proposed control system provides a smooth and effective switching motion, and both the position and the orientation converge to the origin and zero quickly, respectively.

7. CONCLUSIONS

This paper has presented a new robust feedback controller for global exponential stabilization of both the position and orientation of an underactuated airship in the presence of bounded uncertainties on the damping coefficients. Experiments using a blimp have demonstrated the effectiveness.

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