A NEW ANALYTICAL MODEL REPRESENTATION FOR VEHICLE DYNAMICS IN THE PLAN (X, Y)

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Abstract: An analytical investigation of a complete vehicle model dynamics, subjected to road disturbances is performed. In a reference plane (X, Y), the trajectory traced by the center of gravity of the vehicle will be simulated and shown, and how this trajectory varies if incidents (faults) happen such as variation in the inflation pressure, variation in the type of the contact area with the ground... Due to practical limitations, we cannot consider all the types of faults and their effects on the movement of the center of gravity such as vibration caused from the mechanical system. A complete model composed of a set of complex non linear differential equations requiring a limited set of easily obtained parameters will be shown to represent the motion of the center of gravity. In this work a vehicle without driver is considered, so his reactions on the steering angle, on the acceleration or braking will not be taken into account.

Keywords: Wheel pressure, Open loop system, Vehicle dynamical behavior.

1. INTRODUCTION

Demands for better ride comfort and controllability of road vehicles have motivated many industries to emerge the active technology in the construction of their vehicles; they have put the accent on accidents avoidance by using the safety active techniques. For developing accurate devices, it is necessary to estimate precisely the movement of the vehicle or to be more precise, the movement of the center of gravity. This estimation depends on the chosen model of the vehicle and on the type of the contact area with the ground i.e. (if the road is rigid, or plastic (that means it deforms plastically with the variation of the vertical charge i.e. the soil)).

The motivation of this paper comes from the thought that something has been lost during the evolution from older models to the newer ones, many researchers have investigated the tire and vehicle models, starting from a very simple model for tires in which the tires are represented by spring, or simple models for the vehicle which is a bicycle model or models of two degrees of freedom, arriving to complex and non linear models, for the wheels as in (James Lacombe (2000)), Jacob Svendenius(2003)),or complete models for the vehicle as in (Brigitte d'Andra-novel and Marco Pengov, (2002)) They have represented the movement of the center gravity by a set of non linear differential equations, without taking account to the variation of the slip angle; they have found models of rigid vehicles with rigid wheels moving on a rigid surface.

The objective of this paper is to construct a global model that can represent the vehicle and then to simulate its dynamical behavior without the presence of the driver, the importance of this simulation resides in the fact that if a problem or a fault has produced, for example, the inflation pressure in the tire, this simulation helps us to know an approximate behavior of the center of gravity, and how it leaves its initial or desired trajectory, and shows us where and when the driving will not be safe, in order and as a perspective of this work to inform the driver or to apply a command law that assures the safety driving in the presence of sudden faults.

This paper is organized as follows: In the section II we will deal with the modeling of the wheel and the forces generated on it during its contact with the ground, in this section we will see also the presence of the inflation pressure of the tire in the forces generated on the wheel, In section III we will describe the complete model of the vehicle and the equations of motion which describe the movement of the center of gravity , in section IV simulations and results, and finally conclusion and future work.

2. WHEEL MODELING AND THE INTERACTION WITH THE GROUND

This section deals with the modeling of the wheel, the forces acting on it due to its contact with the ground and the type of the ground under the wheel. Due to the fact that the deformation of the tire on a rigid surface is not the same as its deformation on a deformable surface (soil), and forces generated on the wheels are not the same if the type of the ground which is in contact with the wheel varies (asphalt, Concrete,...), we will consider in this paper that the ground under the wheel is rigid, but we will take into account the variation of the type of it's surface. The dynamical behavior of the wheel is characterized by the various forces applied to the surface of contact between the tire and the ground, these forces are:

- Vertical force,
- Longitudinal force,
- Lateral force.

The determination of these forces is not evident, several elements must be found before the calculation of these forces.

2.1 Slip angle

Due to the elasticity of the tire and under the effect of the transversal request, a couple of auto alignment is created, this couple modifies the initial direction of the wheel by an angle called slip angle.

So the slip angle can be defined as the angle that exists between the velocity vector of the wheel and the plane of the wheel, it is calculated in the following way (Shinichiro Horiuchi et al (1999)):

$$\alpha 1 = \tan^{-1}\left(\frac{V_Y + L_1 \times V_\Psi}{V_X - t_f \times V_\Psi}\right) - \delta_f \qquad (1)$$

$$\alpha 2 = \tan^{-1}\left(\frac{V_Y + L_1 \times V_\Psi}{V_X + t_f \times V_\Psi}\right) - \delta_f \qquad (2)$$

$$\alpha 3 = \tan^{-1}\left(\frac{V_Y - L_2 \times V_\Psi}{V_X - t_r \times V_\Psi}\right) - \delta_r \qquad (3)$$

$$\alpha 4 = \tan^{-1}\left(\frac{V_Y - L_2 \times V_\Psi}{V_X + t_r \times V_\Psi}\right) - \delta_r \qquad (4)$$

α_i is the slip angle of the wheel i.
V_Ψ is the yaw velocity [rad/s].
δ_f is the front steering angle
δ_r is the rear steering angle
V_X is the longitudinal velocity of the center of gravity
V_Y is the longitudinal velocity of the center of gravity
L_1 is the distance between the center of gravity and the front axis center of gravity
L_2 is the distance between the center of gravity and the rear axis center of gravity
t_f is the half width of the front axes
t_r is the half width of the rear axes

2.2 Sliding

Sliding is composed of two components:

1. Longitudinal sliding
2. Lateral sliding

The longitudinal sliding is given by the following equation

$$\lambda_x = \frac{(v_{xi} - r_i \Omega_i)}{vi} \qquad (5)$$

where:
v_{xi} is the longitudinal velocity of the tire i
v_{yi} is the longitudinal velocity of the tire i
Ω_i and vi are respectively the angular and the resultant speed of the wheel i . vi is defined by:

$$vi = \sqrt{v_{xi}^2 + v_{yi}^2} \qquad (6)$$

The side slip is due to side elasticity of the tire and it is represented by the following formula:

$$\lambda_y = \frac{r_i \Omega_i \times \sin(\alpha_i)}{vi} = (1-\lambda_x)\sin(\alpha_i) \quad (7)$$

α_i and r_i are respectively the slip angle and the radius of the wheel i.
Moreover we can determine the total sliding from the following formula:

$$\lambda = \sqrt{\lambda_x^2 + \lambda_y^2} \quad \text{with} \quad 0 < \lambda < 1 \quad (8)$$

2.3 Modelling of the friction coefficient

The friction is the tire's characteristic of adherence, it is given by the longitudinal coefficient of adherence η_x which is nonlinear:

$$\eta_x(\lambda, V) = \left(C_1(1-\exp(-C_2\lambda)) - C_3\lambda\right)\exp(-C_4\lambda V) \quad (9)$$

This function depends essentially on the characteristics on the tire (standard, quality, wear, pressure of inflation, temperature), but especially on the type of coating of the road which is characterized by the coefficients C_1, C_2, C_3 and C_4. For various types of coating, the characteristics of the coating of the road are given in (Idar Petersen 2003)

2.4 *Rolling resistance*

The rolling resistance noted by F_{rr} is given by the following expression (Hassan Shraim et al (2004)):

$$F_{rri} = -K_{rr} F_{zi} \quad (10)$$

Where F_{zi} the vertical is load and
$0.01 < K_{rr} < 0.05$ is the rolling constant.

2.5 Surface of contact

The location and the dimension of the surface of contact S, where the tire forces are assumed to act, is of substantial interest. Finding this surface and calculating its dimension is necessary in order to know the values and the directions of these forces. In order to find the parameters of this surface we have to study the variation of the inflation pressure inside the wheel which causes the variations in the radius of the wheel and modifies the dimensions of the surface of contact.

2.6 Inflation Pressure

The inflation pressure of the tire varies according to the variation of the quantity of air inside the air chamber, and it depends also on the variation of the temperature. The inflation pressure is an important factor that influences the behavior of the vehicle's dynamics. Indeed, any variation in the inflation pressure causes the variation in the dimensions of the surface of contact between the wheel and the ground, which involves a change in the dynamical radius of the wheel. This variation causes the change of rolling angle and modifies the vertical charge distribution on the four wheels.
The deflection of the tire depends on:

- The tire's rigidity
- The tire's structure
- The inflation pressure.

The distribution of the pressure is given according to the longitudinal co-ordinate X and it is assumed that the distribution of the vertical pressure is a parabolic curve given by the following formula:

$$q_z = \frac{3 \cdot F_z}{4 \cdot a}\left(1-(x/a)^2\right) \quad (11)$$

Where a can be determined easily if we determine the dynamical radius of the tire

$$\mathrm{r}_{1\mathrm{i}} = \mathrm{r} - \delta_i \quad (12)$$

Where $\mathrm{r}_{1\mathrm{i}}$ and δ_i are respectively the radius with deformation (dynamical radius) and the deformation of the wheel (i).

We will suppose that the area of contact is an ellipse in which its major axis is 2*a and its minor one is the width of the tire,In order to estimate the value of X_s figure (2), which is the transition point from the zone of sliding to the zone of adhesion, we can use the following formula:

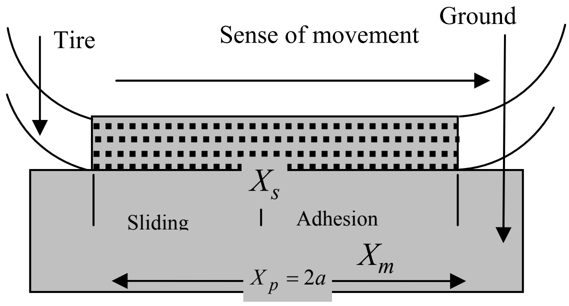

Fig. 1: surface of contact between the tire and the ground.

$$C_P \lambda (a - X_s) = \eta_x q_z(X_s) \quad (13)$$

Where C_p is the rigidity of the tire by a unit length of the width.

$$X_s = X_m - a \quad (14)$$

2.7 The wheel velocity

Each wheel i has its longitudinal speed v_{xi} and its lateral speed v_{yi} which are different from the other wheels and are given by the following formulas:
In our study and for simplification purpose only , we suppose that $t_f = t_r = l$
For the first wheel, longitudinal velocity is:

$$v_{x1} = (V_x - lV_\psi)\cos(\delta_f) + (V_y + L_1 V_\psi)\sin(\delta_f) \qquad (15)$$

and the lateral velocity

$$v_{y1} = -(V_x - lV_\psi)\sin(\delta_f) + (V_y + L_1 V_\psi)\cos(\delta_f) \qquad (16)$$

For the second wheel 2 we replace l by $-l$ and for the third wheel 3 we replace L_1 by $-L_2$ and δ_f by δ_r and for the fourth wheel 4 we replace l by $-l$, L_1 *by* $-L_2$ and δ_f by δ_r .

2.8 The vertical force

The vertical forces that we use in our model are function of the longitudinal, lateral acceleration and the height of the center of gravity, for example for the left front wheel, the vertical force can be represented as:
For the wheel front left:

$$F_{z1} = \left(\frac{M}{2L}\right) \times \left(-L_2 \ddot{Y}\frac{h}{l} - \ddot{X}h + gL_2\right) \qquad (17)$$

Where M is the total mass, $L = L_1 + L_2$

$\ddot{X}$, $\ddot{Y}$ is the longitudinal acceleration and the lateral acceleration, h is the height of the center of gravity

2.9 Longitudinal force

The total longitudinal force is equal to the sum of all the longitudinal forces acting on the wheel and which are expressed in the following way:
-The longitudinal force given by the static area (adhesion area between the wheel and the ground)
-The longitudinal force given by the sliding area (between the wheel and the ground)
- The longitudinal force given by the rolling resistance F_{rr}
The longitudinal force is given by:

$$F_{xi} = \frac{C_1}{2}\left(1 - \frac{Vxi}{\Omega i \cdot \left(r - \left(K1 + \frac{K2}{p_i}\right)\frac{Fzi}{1000}\right)}\right) X_{mi}^2 + F_{xsli} + F_{rri} \qquad (18)$$

Where

$$F_{xsl\,i} = \frac{\left(\frac{\Omega_i \cdot \left(r - \left(K1 + \frac{K2}{p_i}\right)\frac{Fzi}{1000}\right)}{Vxi} - 1\right)\mu \times F_{zi}(Xpi - Xmi)}{\left(Tan^2(\alpha_i) + \left(\frac{\Omega_i \cdot \left(r - \left(K1 + \frac{K2}{p_i}\right)\frac{F_{zi}}{1000}\right)}{Vxi} - 1\right)^2\right)^{0.5} Xpi} \qquad (19)$$

With C_1 is the longitudinal stiffness of the tire, r is the radius without any deflection, p_i is the inflation pressure of the wheel (i), K1 and K2 are two constants chosen depending on the maximum and the minimum deflection of the tire .

2.10 Lateral force

The total lateral force is given by the lateral effort integrated on the surface of contact. It corresponds to the sum of all the lateral forces acting on the wheel
1- The side force given by the static area (area of adhesion)
2- The side force given by the sliding area.

$$F_{Y_i} = \frac{Tan(\alpha_i)\mu F_{zi}(X_{p_i} - X_{mi})}{\left(Tan^2(\alpha_i) + \left(\frac{1}{v_{xi}}\Omega i\left(r - \left(K1 + \frac{K2}{p_i}\right)\frac{F_{zi}}{1000}\right) - 1\right)^2\right)^{0.5} X_{p_i}} + \frac{A}{3}X_{mi}^3 \qquad (20)$$

Where α is the slip angle, and A is a nonlinear function depending on (Hassan Shraim et al (2004)):

$$A = F(X_p, X_m, \alpha) \qquad (21)$$

$$F_{ysli} = \frac{Tan(\alpha_i)\mu F_{zi}(X_{p_i} - X_{mi})}{\left(Tan^2(\alpha_i) + \left(\frac{\Omega i}{v_{xi}}\left[r - \left(K1 + \frac{K2}{p_i}\right)\frac{F_{zi}}{1000}\right] - 1\right)^2\right)^{0.5} X_{pi}} \qquad (22)$$

$$\beta = \left(\frac{K}{EI}\right)^{\frac{1}{4}} \qquad (23)$$

E is the coefficient of elasticity, I is the moment of inertia.

3. COMPLETE MODEL FORMULATIONS

From the principles of dynamics, the vehicle's plane (X, Y) and yaw rate motions are expressed by the following equations of motion:

$$M\dot{V}_{COG} = \sum F \; external\ forces \tag{24}$$

The equilibrium of the moments at the center of gravity:

$$\sum \mathrm{M}_Z = I_Z \, \dot{V}_\Psi \tag{25}$$

The equation to link the longitudinal acceleration to the longitudinal velocity as a result of the introduced vehicle related coordinate system gives:

$$\ddot{X} = \dot{V}_X - V_\Psi V_Y \tag{26}$$

The equation to link the longitudinal acceleration to the longitudinal velocity as a result of the introduced vehicle related coordinate system gives:

$$\ddot{Y} = \dot{V}_Y + V_\Psi V_X \tag{27}$$

Therefore the dynamic equations of the longitudinal speed, lateral speed and yaw rate are given by:

$$\begin{aligned} M\dot{V}_X = {} & MV_\Psi V_Y + \cos\delta_f (F_{X1} + F_{X2}) + \cos\delta_r \times \\ & (F_{X3} + F_{X4}) - \sin\delta_f (F_{Y1} + F_{Y2}) - \\ & \sin\delta_r (F_{Y3} + F_{Y4}) \qquad (28) \end{aligned}$$

$$\begin{aligned} M\dot{V}_Y = {} & -MV_\Psi V_Y + \sin\delta_f (F_{X1} + F_{X2}) + \sin\delta_r \times \\ & (F_{X3} + F_{X4}) + \cos\delta_f (F_{Y1} + F_{Y2}) + \cos\delta_r \times \\ & (F_{Y3} + F_{Y4}) \quad (29) \end{aligned}$$

$$\begin{aligned} I_Z \dot{V}_\Psi = {} & l(\cos\delta_f (F_{X2} - F_{X1}) + \sin\delta_f (F_{Y1} - F_{Y2})) \\ & + L_1 (\sin\delta_f (F_{X1} + F_{X2}) + \cos\delta_f (F_{Y1} + F_{Y2})) \\ & - L_2 (\sin\delta_r (F_{X3} + F_{X4}) - \cos\delta_r (F_{Y3} + F_{Y4})) \\ & + l(\cos\delta_r (F_{X4} - F_{X3}) + \sin\delta_r (F_{Y3} - F_{Y4})) \end{aligned} \tag{30}$$

$$I_r \dot{\Omega}_i = -r_i F_{Xi} + C_{mi} - C_{fi} \qquad i = 1,2,3,4. \tag{31}$$

4. SIMULATION AND RESULTS

The set of simulations was developed with software MATLAB. A GUI "graphical use interface" was built in order to obtain a general

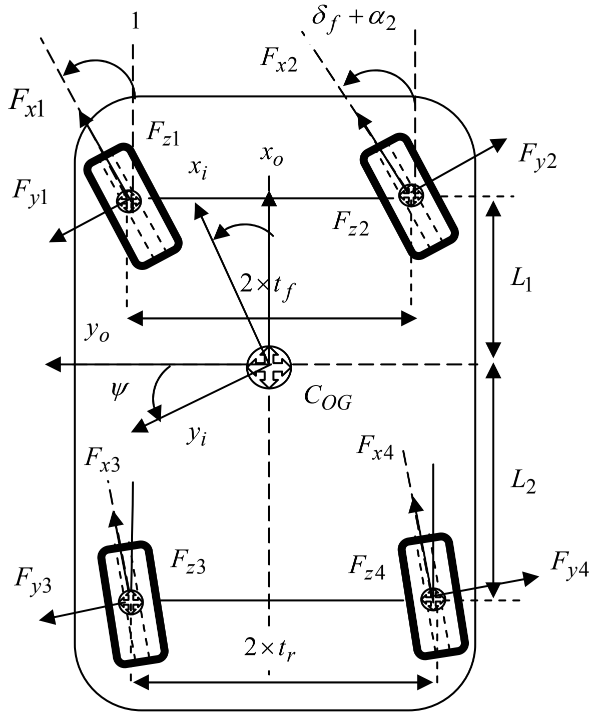

Fig 2:2 D-vehicle scheme

and applicable model to all types of the vehicles. In this section we will make a comparison between our work and the work that it is found in literature, in order to show the importance of the pneumatic aspects on the behavior of the vehicle. Actually, we have to consider the reaction of the driver, which turns the steering angle or brakes in the case of faults existence, but in this work we will show the trajectory traced by the center of gravity without driver, to show at which instant we have to inform the driver by a signal that he left the safety range, or to apply a command law that assists the driver in the catastrophic cases.

In figure 3 , we see two traces, the first one(pointed) is the trace of the position (X,Y)of the center of gravity by giving as input a certain couple and with front steering angle equal to zero, when we don't have any fault, in the second trace, the solid line one, we give the same input but after a certain time, an inflation pressure problem is produced on the forward left tire, it is seen that the center of gravity is strongly affected by the inflation pressure problem and the vehicle loses its stability and leaves its trajectory. The same simulation is made in figure4 but if the problem affects the forward right wheel, we will see how the center the gravity changes its trajectory in the direction of the left, figure 5 and figure 6 show the trajectory traced by the center of gravity if problems in the third and the fourth tire occurs respectively. in figure 7 the escape in pressure in the third wheel is greater than the case discussed before, so we will see that it turns faster than before with a greater steering angle. Another simulation has been made if two problems at the same instant happens on the two front wheel, we will see that no variation in the direction as in figure8.Reasonable results are shown.

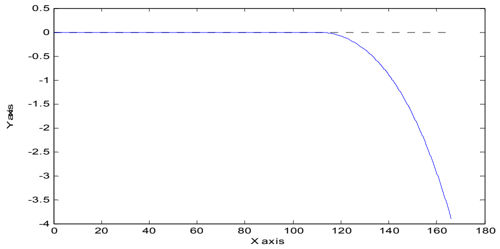

Fig 3 : inflation in the first tire.

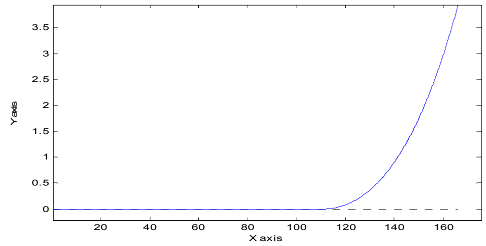

Fig 4: inflation in the second tire.

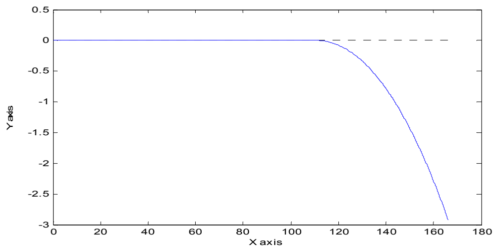

Fig 5: inflation in the third tire.

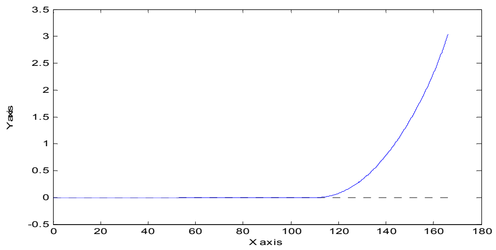

Fig 6: inflation in the fourth tire.

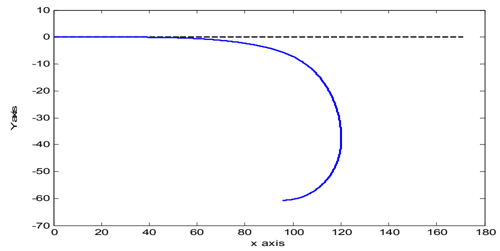

Fig 7: inflation in the third tire (different escape).

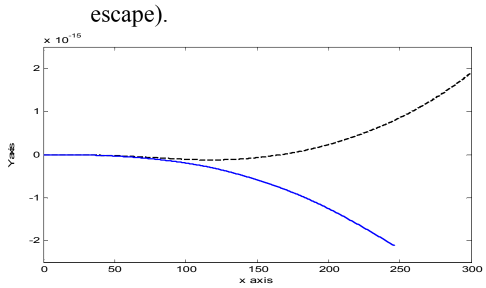

Fig 8: inflation in the first and the second tire at the same time (different escape).

5. CONCLUSION AND FUTURE WORK

In this study we have investigated a complete model of the vehicle, it treats the dynamic of the tire; which comprises firstly the modeling and the analysis of the lateral and the longitudinal forces in terms of the radius of the tire that varies by the variation of the inflation pressure, after that we have arrived to the equations of motion representing the motion of the center of gravity, and how the variation of the radius affects this motion. Simulations are made in the absence of the driver's reactions, and results show in the reference plane (X,Y) the motion of the vehicle with and without faults, in order and as a perspective to this work to study this motion by considering a ground with a plastic surface, and after that to inform the driver that he leaves the safety range or to apply a command law that acts as a driver assistant in the dangerous cases.

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