PID CONTROLLER SYNTHESIS FREE OF ANALYTICAL MODELS

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Abstract: In this paper, we show that the *complete* set of PID controllers that stabilizes a LTI plant and achieves prescribed performance levels, can be computed from knowledge of only the frequency response (Nyquist or Bode plot) of the plant and the number of RHP poles. The result is of practical importance in many situations where transfer function or state space models are not available and a design procedure that is free of analytical models is desired. The synthesis method also gives useful information on the frequency range over which accurate frequency response data on the plant is needed for PID design.

Keywords: PID controller, frequency domain data, model-free approach

1. INTRODUCTION

In classical control design, a single controller such as PID and phase lead/lag is designed from the frequency domain data (free of analytical model) or transfer function of the plant to be controlled. In modern and post modern control approah, an optimal controller of hight order is designed with respect to certain performance measure such as H_∞, H_2, and ℓ_1 based on analytical plant model. On the other hand, fuzzy neural control provides model free approaches to design a controller, but it lacks guarantee of stability and performance. In recent work (see (Keel et al., 2003; Tantaris et al., 2003; Datta et al., 2000) and references therein) the set of stabilizing first order and PID controllers have been determined for LTI systems in both the continuous-time and discrete-time cases. These sets define the freedom that is available to the control system designer to achieve other performance requirements and some progress has also been made in that direction(Ho and Lin, 2003).

The computation of the stabilizing sets described in the above references requires knowledge of the transfer function coefficients of the plant, or equivalently a state space model or realization. In practice such information is often unavailable. Instead it is more reasonable, in many situations, to assume that the frequency response of the plant can be measured experimentally. Indeed this is the basis of classical control theory methods, many popular tuning rules such as the Ziegler-Nichols procedure and the current practice in PID design for process control (see (Cominos and Munro, 2002; Åström and Hägglund, 1998; Chidambaram, 1998)).

In the present paper we show that the *complete* set of PID controllers stabilizing and achieving several meaningful performance specifications for a linear time-invariant (LTI) plant, can in fact be determined from knowledge of the plant frequency response and the number of unstable plant poles. This data can often be reliably determined experimentally and the number of unstable poles can be known from physical considerations. Thus the calculation of the stabilizing sets presented here rests solidly on raw plant-data rather than the parameters of a post-identification model derived from it. We feel that this fact alone has

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important consequences on robustness of stability and performance which deserve further study. Our derivation also gives explicit information on the frequency range over which accurate information of the plant frequency response is needed for PID control, and on the range of stabilizing gains.

2. MAIN RESULT

Consider a linear time-invariant plant, with underlying transfer function $P(s)$ with $n(m)$ poles (zeros). We assume that the *only* information available to the designer is:
1. Knowledge of the frequency response magnitude and phase, equivalently, $P(j\omega)$, $\omega \in [0, \infty)$.
2. Knowledge of the *number* of RHP poles p_r.
We also assume that the plant has no $j\omega$ poles or zeros so that the magnitude and its inverse and phase are well-defined for all $\omega \geq 0$. Write

$$P(j\omega) = |P(j\omega)|e^{j\phi(\omega)} = P_r(\omega) + jP_i(\omega) \quad (1)$$

where $|P(j\omega)|$ denote the *magnitude* and $\phi(\omega)$ the *phase* of the plant, at the frequency ω. Let the PID controller be of the form

$$C(s) = \frac{K_i + K_p s + K_d s^2}{s(1+sT)}, \quad T > 0 \quad (2)$$

where T is assumed to be fixed and small. Then the main result of the paper is stated in the following theorem.

Theorem 1.
A. The complete set of stabilizing PID gains for a given LTI plant can be found from the frequency response data $P(j\omega)$ and the knowledge of the number of RHP poles.
B. The set of stabilizing PID gains can be computed by the following procedure:

0. Determine the relative degree $n-m$ from the high frequency slope of the Bode magnitude plot. Determine z_r from the net phase change from the Bode phase plot as in Lemma 2 below.

1. Fix $K_p = K_p^*$, solve

$$K_p^* = -\frac{P_r(\omega) + \omega T P_i(\omega)}{|P(j\omega)|^2} = -\frac{\cos\phi(\omega) + \omega T \sin\phi(\omega)}{|P(j\omega)|} \quad (3)$$

and let $\omega_1 < \omega_2 < \cdots < \omega_{l-1}$ denote the distinct frequencies which are solutions of (3).

2. Set $\omega_0 = 0$, $\omega_l = \infty$ and determine all strings of integers $i_t \in \{+1, 0, -1\}$ and $j \in \{-1, +1\}$ such that:
For $n-m$ even:

$$\left\{i_0 - 2i_1 + 2i_2 + \cdots + (-1)^{i-1}2i_{l-1} + (-1)^l i_l\right\} \cdot (-1)^{l-1} j = n - m + 2z_r + 2 \quad (4)$$

For $n-m$ odd:

$$\left\{i_0 - 2i_1 + 2i_2 + \cdots + (-1)^{i-1}2i_{l-1}\right\} \cdot (-1)^{l-1} j = n - m + 2z_r + 2. \quad (5)$$

3. For the fixed $K_p = K_p^*$ chosen in Step 1, solve for the stabilizing (K_i, K_d) from:

$$\left[K_i - K_d\omega_t^2 + \frac{\omega_t \sin\phi(\omega_t) - \omega_t^2 T \cos\phi(\omega_t)}{|P(j\omega)|}\right] i_t > 0 \quad (6)$$

for $t = 0, 1, \cdots$.

4. Repeat the previous three steps by updating K_p over prescribed ranges.

3. PROOF OF THE MAIN RESULT

The proof of Theorem 1 is based on the following Lemmas. As we see below the knowledge of p_r along with the Bode plot is sufficient to determine z_r as well as the relative degree $n-m$.

Lemma 2.
A. In the Bode magnitude plot of the LTI system $P(j\omega)$, the high frequency slope is $-(n-m)$20dB/decade where $n-m$ is the relative degree of the plant $P(s)$.
B. The net change of phase of $P(j\omega), \omega \in [0, \infty)$, denoted $\Delta_0^\infty(\phi)$ is:

$$\Delta_0^\infty(\phi) = -\left[(n-m) - 2(p_r - z_r)\right]\frac{\pi}{2}$$

where p_r and z_r are numbers of RHP poles and zeros of $P(s)$, respectively.

PROOF. The statement A is obvious from the property of Bode magnitude plot. Let p_l and z_l are number of LHP poles and zeros of $P(s)$. Then the net phase change of $P(j\omega)$ for $\omega \in [0, \infty)$ is

$$\Delta_0^\infty(\phi) = \left[(z_l - z_r) - (p_l - p_r)\right]\frac{\pi}{2}$$
$$= -\left[(n-m) - 2(p_r - z_r)\right]\frac{\pi}{2}.$$

The high frequency slope of the Bode magnitude plot determines $n-m$ and the net change in phase determines z_r if p_r is known or vice versa.

Consider an n^{th} degree polynomial

$$a(s) = a_n s^n + a_{n-1}s^{n-1} + \cdots + a_1 s + a_0$$

and write $a(j\omega) = a_R(\omega) + ja_I(\omega)$ where $a_R(\omega)$ and $a_I(\omega)$ are real polynomials. Let us first assume that $a(s)$ is a real polynomial and let $0 = \omega_0 < \omega_1 < \omega_2 < \cdots < \omega_{l-1}$ denote real positive, distinct, finite zeros $a_I(\omega) = 0$ of odd multiplicities. Then we define

$$i_0 = \text{sgn}\left[a_R^{p_0}(0)\right], \ i_k = \text{sgn}\left[a_R(\omega_k)\right], \ k = 1, \cdots, l$$

and $j = \text{sgn}[a_I(\infty)]$ where p_0 the multiplicity of $\omega_0 = 0$ as a zero of $a_I(\omega) = 0$. Let $\sigma(a)$ denote the signature of $a(s)$, that is the difference between the numbers of open LHP and open RHP zero of $a(s)$. We have the following signature formula for a real polynomial from (Datta et al., 2000).

Lemma 3. (Real Signature Formula)
For deg$[a(s)]$ even,

$$\sigma(a) = \{i_0 - 2i_1 + \cdots + (-1)^{l-1}2i_{l-1} + (-1)^l i_l\} \cdot (-1)^{l-1} \cdot j.$$

For deg$[a(s)]$ odd,

$$\sigma(a) = \{i_0 - 2i_1 + 2i_2 + \cdots + (-1)^{l-1}2i_{l-1}\} \cdot (-1)^{l-1} \cdot j.$$

If $a(s)$ is a complex polynomial, let $-\infty < \omega_1 < \omega_2 < \cdots < \omega_{l-1}$ denote real, distinct, finite zeros of $a_I(\omega) = 0$ of odd multiplicities and let $\omega_0 = -\infty, \omega_l = +\infty$.

Lemma 4. (Complex Signature Formula)
If deg$[a(s)]$ is even and the leading coefficient a_n is purely real or deg$[a(s)]$ is odd and a_n is purely imaginary, the

$$\sigma(a) = \frac{1}{2}\left\{(-1)^{l-1}i_0 + 2\sum_{r=1}^{l-1}(-1)^{l-1-r}i_r - i_l\right\} \cdot j.$$

If deg$[a(s)]$ is even and a_n is not purely real or deg$[a(s)]$ is odd and a_n is not purely imaginary, then

$$\sigma(a) = \frac{1}{2}\left\{2\sum_{r=1}^{l-1}(-1)^{l-1-r}i_r\right\} \cdot j.$$

Now let us consider the plant and PID controller pair of the form:

$$P(s) = \frac{N(s)}{D(s)} = \frac{N_e(s^2) + sN_o(s^2)}{D_e(s^2) + sD_o(s^2)} \qquad (7)$$

$$C(s) = \frac{K_i + K_p s + K_d s^2}{s(1 + sT)}, \quad T > 0 \qquad (8)$$

where deg$[D(s)] = n$ and deg$[N(s)] = m$. The characteristic polynomial is

$$\delta(s) = s(1 + sT)D(s) + \left(K_i + K_p s + K_d s^2\right) N(s).$$

and multiplying both sides by $N(-s)$ we have

$$\Pi(s) = s(1 + sT)D(s)N(-s) + \left(K_i + K_p s + K_d s^2\right) N(s)N(-s) \qquad (9)$$

with deg$\Pi(s) = n + m + 2$. Let $z_l(z_r)$ denote the number of LHP (RHP) zeros of the plant. Then $\delta(s)$ is *Hurwitz* if and only if the signature of $\Pi(s)$ (number of LHP roots - number of RHP roots) is:

$$\sigma(\Pi) = n + 2 + z_r - z_l. \qquad (10)$$

Condition (10) can also be written as

$$\sigma(\Pi) = n + 2 + z_r - (m - z_r) = \underbrace{n - m}_{\text{relative degree of plant}} + 2z_r + 2. \qquad (11)$$

From (9),

$$\Pi(j\omega) = j\omega(1 + j\omega T)D(j\omega)N(-j\omega) + \left(K_i + j\omega K_p - \omega^2 K_d\right) N(j\omega)N(-j\omega). \qquad (12)$$

Writing

$$\Pi(j\omega) = R(\omega, K_i, K_d) + jI(\omega, K_p) \qquad (13)$$

where

$$R(\omega) = -\omega^2 T\left[D_e(-\omega^2)N_e(-\omega^2) + \omega^2 D_o(-\omega^2)N_o(-\omega^2)\right] - \omega^2\left[(D_o(-\omega^2)N_e(-\omega^2) - N_o(-\omega^2)D_e(-\omega^2)\right] + \left(K_i - K_d\omega^2\right)\left[(N_e^2(-\omega^2) + \omega^2 N_o^2(-\omega^2)\right] \qquad (14)$$

$$I(\omega) = \omega\Big(D_e(-\omega^2)N_e(-\omega^2) + \omega^2 D_o(-\omega^2)N_o(-\omega^2) - \omega^2 T\left[D_o(-\omega^2)N_e(-\omega^2) - N_o(-\omega^2)D_e(-\omega^2)\right] + K_p\left[N_e^2(-\omega^2) + \omega^2 N_o^2(-\omega^2)\right]\Big) \qquad (15)$$

We note that K_p appears only in $I(\omega)$ and K_i, K_d only in $R(\omega)$, and we denote this by writing $R(\omega, K_i, K_d)$, $I(\omega, K_p)$ as above.

Let $0 < \omega_1 < \omega_2 < \cdots < \omega_{l-1}$ denote the real, positive, distinct, finite zeros of $I(\omega) = 0$, of odd multiplicities, define $\omega_l = \infty$ and

$$i_0 = \text{sgn}\left[R^{p_0}(0)\right], \; i_k = \text{sgn}\left[R(\omega_k)\right], \; k = 1, \cdots, l$$

where p_0 is multiplicity of $\omega_0 = 0$ as a zero of $I(\omega) = 0$ and $j = \text{sgn}[I(\infty)]$. Then we have the following result from (Datta et al., 2000).

Lemma 5. (Datta et al., 2000) The set of PID stabilizing controllers can be found as follows: Fix $K = K_p^*$ and set

$$I(\omega, K_p^*) = 0. \qquad (16)$$

Let $i_t \in \{+1, 0, -1\}$ and $j \in \{+1, -1\}$. Determine strings of integers $\{i_0, i_1, \cdots\}$ satisfying:
For $n - m$ even:

$$\{i_0 - 2i_1 + 2i_2 + \cdots + (-1)^{i-1}2i_{l-1} + (-1)^l i_l\} \cdot (-1)^{l-1} j = n - m + 2z_r + 2$$

For $n - m$ odd:

$$\{i_0 - 2i_1 + 2i_2 + \cdots + (-1)^{i-1}2i_{l-1}\} \cdot (-1)^{l-1} j = n - m + 2z_r + 2.$$

For each string satisfying (4) or (5), the conditions for stability are:

$$\text{sgn}\left[R(\omega_t, K_i, K_d)\right] i_t > 0, \quad \text{for} \quad t = 0, 1, 2, \cdots \qquad (17)$$

PROOF. (*Theorem 1*) The proof of the theorem consists in showing that the stability conditions in Lemma 5 (eqs.(16)-(17)) are computable *only* from the plant frequency response.

From (7):

$$|P(j\omega)|^2 = \frac{|N(j\omega)|^2}{|D(j\omega)|^2} = \frac{N_e^2(-\omega^2) + \omega^2 N_o^2(-\omega^2)}{D_e^2(-\omega^2) + \omega^2 D_o^2(-\omega^2)}$$

$$P_r(\omega) = \frac{N_e(-\omega^2)D_e(-\omega^2) + \omega^2 D_o(-\omega^2)N_o(-\omega^2)}{D_e^2(-\omega^2) + \omega^2 D_o^2(-\omega^2)}$$

$$P_i(\omega) = \frac{\omega\Big(N_o(-\omega^2)D_e(-\omega^2) - D_o(-\omega^2)N_e(-\omega^2)\Big)}{D_e^2(-\omega^2) + \omega^2 D_o^2(-\omega^2)}.$$

With $D^2(\omega) = |D(j\omega)|^2$ and $N^2(\omega) = |N(j\omega)|^2$, we can rewrite (14) and (15) as:

$$I(\omega, K_p) = \omega\Big(P_r(\omega)D^2(\omega) + \omega T P_i(\omega)D^2(\omega) + K_p N^2(\omega)\Big)$$

$$R(\omega, K_i, K_d) = -\omega^2 T P_r(\omega)D^2(\omega) + \omega P_i(\omega)D^2(\omega) + \left(K_i - K_d\omega^2\right)N^2(\omega).$$

Since $D^2(\omega) > 0$ and $N^2(\omega) > 0$, it follows that the zeros of (3) are identical to those of (16), and (6) is identical to (17).

4. AN EXAMPLE

To illustrate the main result, we take a set of frequency data points for the *stable* plant used in (Datta et al., 2000). Let the data points be

$$\mathbf{P}(j\omega) := \{P(j\omega), \omega \in (0, 10) \text{ sampled every } 0.01\}.$$

The Nyquist and Bode plot are shown in Fig. 1 and Fig. 2, respectively.

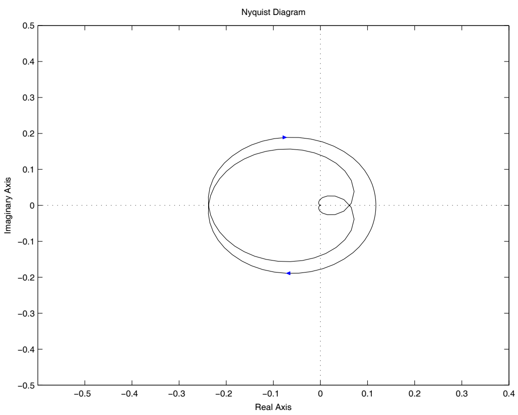

Fig. 1. Nyquist plot of $P(j\omega)$

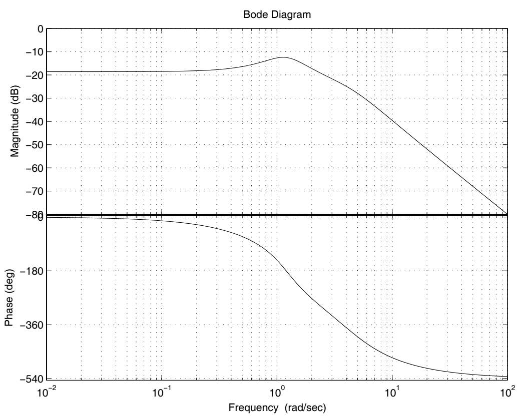

Fig. 2. Bode plots of $P(j\omega)$

The high frequency slope of the Bode magnitude plot is $-40db/decade$ and thus $n - m = 2$. The total change of phase is -540 degrees and so

$$-6\frac{\pi}{2} = -\Big((n-m) - 2(p_r - z_r)\Big)\frac{\pi}{2} \qquad (18)$$

and since the plant is stable, $p_r = 0$, giving (from Lemma 3) $z_r = 2$ The required signature for stability can now be determined and is

$$\sigma(\Pi) = (n - m) + 2z_r + 2 = (2) + 2(2) + 2 = 8.$$

Since $n - m$ is even, we have

$$i_0 - 2i_1 + 2i_2 - 2i_3 + 2i_4 - \cdots + (-1)^l i_l = 8,$$

and it is clear that at least four terms are required to satisfy the above. In other words $l \geq 4$. From Fig. 3 it is easy to see that (3) has at most three positive frequencies as solutions and therefore we have $i_0 - 2i_1 + 2i_2 - 2i_3 + i_4 = 8$. It is easy to see that $i_4 = \text{sgn}\,[R(\infty, K_i, K_d)] = 1$ independent of K_i and K_d. This means that K_p must be chosen so that $I(\omega, K_p^*) = 0$ has exactly three positive real zeros. This gives the feasible range of K_p values as shown in Fig. 3 that depicts the function:

$$-\frac{\cos\phi(\omega) + \omega T \sin\phi(\omega)}{|P(j\omega)|}.$$

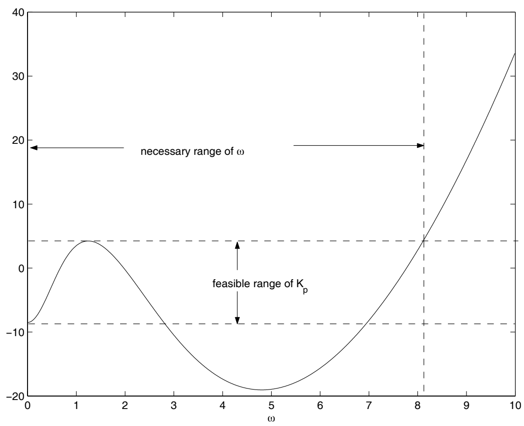

Fig. 3. Finding the feasible range of K_p

In Fig. 3, we also observe that the *frequency range over which plant data must accurately known for PID control* is $[0, 8.2]$.

We now fix $K_p = 1$ and compute the set of ω's that satisfies

$$-\frac{\cos\phi(\omega) + \omega T \sin\phi(\omega)}{|P(j\omega)|} = 1.$$

To find the set of ω's satisfying the above, we plot the above functions as shown in Figure 4.

From this we found the solutions $\{\omega_1, \omega_2, \omega_3\} = \{0,\ 0.742,\ 1.865,\ 7.854\}$. This leads to the requirement $i_0 - 2i_1 + 2i_2 - 2i_3 = 7$ giving the feasible string

$$\mathcal{F} = \{i_0, i_1, i_2, i_3\} = \{1,\ -1,\ 1,\ -1\}.$$

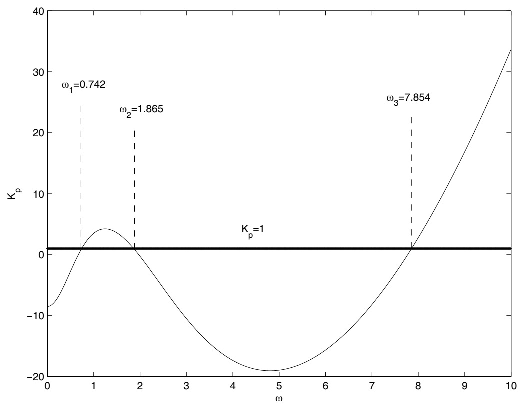

Fig. 4. Finding the set of ωs satisfying the given K_p

Thus, we have the following set of linear inequalities for stability:

$$\begin{aligned} 0.0138K_i &> 0 \\ -0.1390 + 0.0364K_i - 0.0201K_d &< 0 \\ 0.2791 + 0.0229K_i - 0.0797K_d &> 0 \\ -0.1349 + 0.0003K_i - 0.0182K_d &< 0 \end{aligned}$$

The complete set of stabilizing PID gains for $K_p = 1$ is given in Fig. 5. For a small value of T, the region obtained here was verified to be identical to that obtained in (Datta et al., 2000).

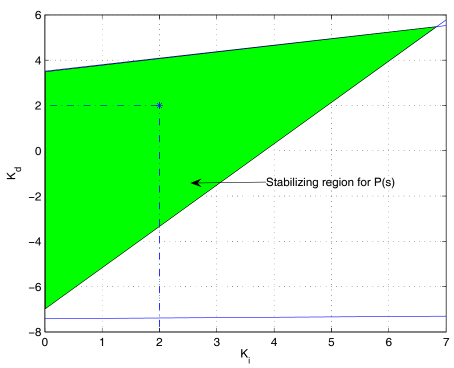

Fig. 5. The complete set of Stabilizing PID gains when $K_p = 1$

By sweeping over K_p we have the entire stabilizing PID gains as shown in Fig. 6.
Note that the range of K_p over which the search needs to be carried out is also obvious from Fig. 3 as discussed and it is $K_p \in [-8.5, 4.2]$.

5. PERFORMANCE SPECIFICATIONS AND COMPLEX STABILIZATION

Many performance attainment problems can be cast as simultaneous stabilization of the plant $P(s)$ and families of real and complex plants. For example:
1. The problem of achieving a gain margin is

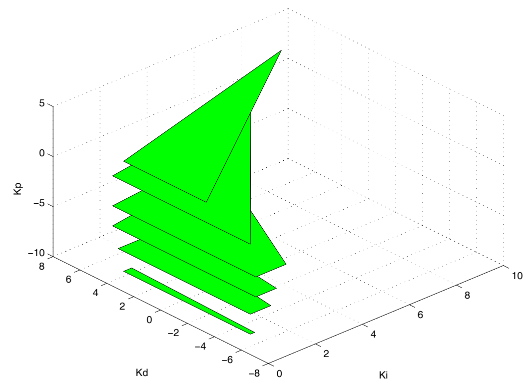

Fig. 6. Entire set of stabilizing PID gains

equivalent to samultaneously stabilizing the plant $P(s)$ and the family of *real* plants

$$\mathcal{P}^c(s) = \{KP(s) : K \in [K_{\min}, K_{\max}]\}.$$

2. The problem of achieving prescribed phase margin θ_m is equivalent to simultaneously stabilizing the plant $P(s)$ and the family of *complex* plants

$$\mathcal{P}^c(s) = \left\{e^{-j\theta}P(s) : \theta \in [0, \theta_m]\right\}.$$

3. The problem of achieving an H_∞ norm specification on the sensitivity function $S(s)$, that is,

$$\|W(s)S(s)\|_\infty < \gamma$$

is equivalent to simultaneously stabilizing the plant $P(s)$ and the family of *complex* plants

$$\mathcal{P}^c(s) = \left\{\left[\frac{1}{1 + \frac{1}{\gamma}e^{j\theta}W(s)}\right] P(s) : \theta \in [0, 2\pi]\right\}.$$

4. The problem of achieving an H_∞ norm specification on the complementary sensitivity function $T(s)$, that is,

$$\|W(s)T(s)\|_\infty < \gamma$$

is equivalent to simultaneuously stabilizing the plant $P(s)$ and the family of *complex* plants

$$\mathcal{P}^c(s) = \left\{P(s)\left[1 + \frac{1}{\gamma}e^{j\theta}W(s)\right] : \theta \in [0, 2\pi]\right\}.$$

Based on the above, we consider the problem of stabilizing a complex LTI plant with transfer function $P^c(s)$ using PID control. As was in the real case, we assume that the only information available to the designer is:
1. Knowledge of the frequency response magnitude and phase, equivalently, $P^c(j\omega)$, $\omega \in (-\infty, +\infty)$.
2. Knowledge of the number of RHP poles, p_r.

Theorem 6.
A. The complete set of stabilizing PID gains for a given complex LTI plant can be found from the frequency response data $P^c(j\omega)$ and the knowledge of the number of RHP poles

B. The set of stabilizing PID gains can be computed by the following procedure:
0. Determine the relative degree $n-m$ from the high frequency slope of the Bode magnitude plot. Determine z_r from the Bode phase plot
1. Fix $K_p = K_p^*$ and solve

$$K_p^* = -\frac{\cos\phi(\omega) + \omega T \sin\phi(\omega)}{|P^c(j\omega)|} \tag{19}$$

and let $\omega_1 < \omega_2 < \cdots < 0 < \cdots < \omega_{l-1}$ denote the distinct frequencies which are solutions of (19).
2. Set $\omega_0 = -\infty$, $\omega_l = +\infty$ and determine all strings of integers $i_t \in \{+1, 0, -1\}$ and $j \in \{-1, +1\}$ such that
For $n-m$ even:

$$\frac{1}{2}\left[(-1)^{l-1} i_0 + 2\sum_{r=1}^{l-1}(-1)^{l-1-r} i_r - i_l\right]\cdot j$$

For $n-m$ odd:

$$\frac{1}{2}\left[2\sum_{r=1}^{l-1}(-1)^{l-1-r} i_r\right]\cdot j$$

3. For the fixed $K_p = K_p^*$ chosen in Step 1, solve for the stabilizing (K_i, K_d) from:

$$\left[K_i - K_d\omega_t^2 + \frac{\omega_t \sin\phi(\omega_t) - \omega_t^2 T \cos\phi(\omega_t)}{|P^c(j\omega_t)|}\right] i_t > 0$$

for $t = 0, 1, \cdots$.
4. Repeat the previous three steps by updating K_p over prescribed ranges.

The detailed proof of this result is omitted.

6. CONCLUDING REMARKS

We have shown that the complete set of PID stabilizing controllers achieving stability and various meaningful performance specifications can be found from the frequency response of the plant and knowledge of the number of RHP plant poles. It is remarkable a) that this calculation can be done by a nested linear programming procedure and b) that only knowledge of the frequency response and number of RHP poles is sufficient. It is also clear that one can easily find the feasible ranges of K_p as shown in the example. For prescribed ranges of K_p to be considered, a designer can get the frequency range over which the accurate plant data is needed. This information may be used to improve the plant data if desired. It is not clear whether these advantages can be extended to other types of controllers and this is an area worth investigating since determining such stabilizing and performance sets is an important step toward lower order, robust and high performance controller design (see (Crowe and Johnson, 2002; Grimble, 2002; Haddad et al., 1993)). It is also worth investigating how this procedure can be modified to accommodate incomplete or finite frequency data. An important area of research is MIMO PID control (see (Zhuang and Atherton, 1994)) and the extension of the results given here to the multi-variable case.

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