STABILIZATION OF UNCERTAIN MARKOVIAN JUMP SINGULAR SYSTEMS WITH WIENER PROCESS

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Abstract: In this paper, firstly, one deals with the stability and the stabilizability problems for the class of Markovian jump continuous-time singular systems. Next, one will address the robustness problem. The proposed approaches derive sufficient conditions such that the regularity and the absence of impulses are assured as well as the stochastic stability and robust stochastic stability. Also, state feedback control laws are designed to guarantee that the resulting closed-loop system, with and without parameter uncertainties is regular, impulse free and stable too. All the obtained results are based on strict linear matrix inequality technique

Keywords: Singular system, Stochastic jump processes, Stability, Robust control.

1. INTRODUCTION

During the past decades, singular systems have received considerable interest, since this class is more suitable than the conventional ones in modelling practical systems in different areas such as electrical power systems, mechanical systems, robotics, chemical systems, see for instance, (Stykel, 2002; Dumont *et al.*, 2001; Lewis, 1986; Gilles, 1998). Unfortunately, a large number of industrial applications are subject to abrupt changes in their structures, which include for instance failures, repairs of machines in manufacturing systems, and modification of the operating point of a linearized model of a nonlinear systems. Consequently, Systems with this characterization can't be represented by deterministic models, but may be modelled by the stochastic hybrid systems which are becoming more and more popular in describing their dynamics behavior. For practical systems modelled by this class of systems, one refers the reader to (Boukas, 2005) and the references therein. Now, there exists a very rich list of references of articles dealing with control problems for singular systems and Markovian jump systems (Cao *et al.*, 2000; Boukas and Hang, 1999), however, because of the complexity of singular systems with Markovian jumps, there are rarely results for this kind of systems. Among these contributions, one quotes the ones obtained very recently by Boukas and Liu (2004), on stability and stabilization problems of continuous-time Markovian jump singular system with time delays. This motivates the authors to deal with the linear Markovian jump singular system with both parameter uncertainties and Wiener process. To the best of our knowledge, there are no results on robust stability and robust stabilizability for this kind of systems. Moreover, many of the existing techniques for control of singular systems assumed that the system under study is regular, however, this property may be destroyed by the state feedback inputs, also The linear matrix inequalities conditions proposed for the resolution of this type of problems, contain very frequently, equality constraints, which may cause numerical problems. In view of this, the aim of this paper is to derive sufficient conditions for the regularity, absence of impulses and stability of the Markovian jump linear singular system with Wiener disturbance, furthermore, a state feedback controller design method is addressed such that the resulting closed-loop system is regular impulse-free and stochastically asymptotically mean-square stable. The robustness problem will be also investigated. It should be pointed that the proposed conditions are strict LMI.

The rest of this paper is organized as follows. Section 2 states the problem to be studied. In Section 3, sufficient conditions are established to check the stochastic mean-square stability and design of the system under consideration. While the robust conditions stability and stabilizability are derived in Section 4. Finally, a numerical example is given in Section 5 to show the applicability of the proposed results.

Throughout this paper, the following notations will be used. The superscript "^{T}" denotes matrix transposition and for symmetric matrices $\mathbf{X}$ and $\mathbf{Y}$ the notation $\mathbf{X} > \mathbf{Y}$ (respectively $\mathbf{X} < \mathbf{Y}$) means that $(\mathbf{X} - \mathbf{Y})$ is positive-definite (resp. negative-definite). $\mathbb{I}$ denotes the identity matrix with the appropriate dimension. $\mathbb{E}[.]$ stands for the mathematical expectation operator. $\|.\|$ refers to the Euclidian norm of vectors.

2. PROBLEM STATEMENT

Let us consider the class of uncertain Markovian jump continuous-time singular linear system defined on the probability space (Ω, F, P), with the following dynamics:

$$\begin{cases} \mathbf{E}d\mathbf{x_t} = \mathbf{A}(r_t,t)\mathbf{x_t}dt + \mathbf{B}(r_t,t)\mathbf{u_t}dt \\ +\mathbb{W}(r_t,t)\mathbf{x_t}dw(t), \mathbf{x}(0) = \mathbf{x_0} \end{cases} \tag{1}$$

where $\mathbf{x_t} \in \mathbb{R}^{\mathbf{n}}$ is the state, $\mathbf{u_t} \in \mathbb{R}^p$ is the control at time t, $w(t)$ is a standard Wiener process that is supposed to be independent of the Markov process $\{r_t, t \geq 0\}$, $\mathbb{W}(r_t)$ is the noise matrix that is supposed to be known for each $r_t \in \mathcal{S}$ and the matrix $\mathbf{E} \in \mathbb{R}^{n\times n}$ may be singular, with $rank(\mathbf{E}) = n_E \leq n$, $\mathbf{A}(r_t,t)$ is the state matrix, $\mathbf{B}(r_t,t)$ is the control matrix, supposed to have the following forms:

$$\begin{cases} \mathbf{A}(r_t,t) = \mathbf{A}(r_t) + \mathbf{D_A}(r_t)\mathbf{F_A}(r_t,t)\mathbf{E_A}(r_t) \\ \mathbf{B}(r_t,t) = \mathbf{B}(r_t) + \mathbf{D_B}(r_t)\mathbf{F_B}(r_t,t)\mathbf{E_B}(r_t) \end{cases} \tag{2}$$

with $\mathbf{A}(r_t), \mathbf{D_A}(r_t), \mathbf{E_A}(r_t), \mathbf{B}(r_t), \mathbf{D_B}(r_t)$ and $\mathbf{E_B}(r_t)$ are real known matrices with appropriate dimensions, and $\mathbf{F_A}(r_t,t)$ and $\mathbf{F_B}(r_t,t)$ are unknown matrices that satisfy the following:

$$\begin{cases} \mathbf{F_A^T}(r_t,t)\mathbf{F_A}(r_t,t) < \mathbb{I} \\ \mathbf{F_B^T}(r_t,t)\mathbf{F_B}(r_t,t) < \mathbb{I} \end{cases} \tag{3}$$

The time-varying parameter uncertainties are said to be admissible if both (2) and (3) hold.
The continuous-time Markov process $\{r_t, t \geq 0\}$ takes its values in a finite set $S = \{1, 2, \ldots, N\}$ with the transition probability given by:

$$P\left[r_{t+\Delta t} = j | r_t = i\right] = \begin{cases} \lambda_{ij}\Delta t + o(\Delta t) & i \neq j \\ 1 + \lambda_{ii}\Delta t + o(\Delta t) & i = j \end{cases}$$

where $\Delta t > 0, \lim_{\Delta t \to 0} \frac{o(\Delta t)}{\Delta t} = 0$, and λ_{ij} is the transition probability rate from the mode i to the mode j at time t, which satisfies $\lambda_{ij} \geq 0$, for all $i, j, i \neq j$, and $\lambda_{ii} = -\sum_{j=1, j\neq i}^{N} \lambda_{ij}$.

In the rest of the paper, we address the stochastic stability problem for the class of system described by (1), with $\mathbf{u_t} = 0$, and its robustness. Also, the problem of designing a state feedback controller such that the closed-loop system is regular, impulse free and stochastically mean-square stable, will be studied, as well as the robustness control problem.

Before establishing these results, one recalls the following definitions for the unforced stochastic singular system (1) (i.e $\mathbf{u}(t) = 0$ for all $t \geq 0$):

Definition 2.1. (Dai, 1989) For any given two matrices $\mathbf{E}, \mathbf{A} \in \mathbb{R}^{n\times n}$ and $r_t \in S$

a. The pair $(\mathbf{E}, \mathbf{A}(i))$ is said to be regular if $det(s\mathbf{E} - \mathbf{A}(i))$ is not identically zero, for each i .

b. The pair $(\mathbf{E}, \mathbf{A}(i))$ is said to be impulse free if $deg(det(s\mathbf{E} - \mathbf{A}(i))) = rank(\mathbf{E})$.

Definition 2.2. System (1), with $\mathbf{u_t} = 0$, for all $t \geq 0$, is said to be:

(i) stochastically asymptotically stable in mean-square sense (SASMSS), if for any initial condition $(\mathbf{x_0}, r_0)$:

$$\lim_{t\to 0} \mathbb{E}\|\mathbf{x}(t)\|^2 = 0 \tag{4}$$

(ii) stochastically stabilizable in the above sense, if there exists a linear state feedback

$$\mathbf{u}(t) = \mathbf{K}(r_t)\mathbf{x_t} \tag{5}$$

with $\mathbf{K}(i)$ is a gain controller for each $i \in \mathcal{S}$, such that the closed-loop system is (SASMSS), for every initial condition $(\mathbf{x_0}, r_0)$.

the following Lemmas will be used for the proof of our results:

Lemma 2.1. (Peterson, 1987) Let $\mathbf{\Omega}$, $\mathbf{F}$ and $\mathbf{\Xi}$ be real matrices of appropriate dimensions with $\mathbf{F}^T\mathbf{F} \leq \mathbb{I}$. For any scalar $\varepsilon > 0$:

$$\mathbf{\Omega}\mathbf{F}\mathbf{\Xi} + \mathbf{\Xi}^T\mathbf{F}^T\mathbf{\Omega}^T \leq \varepsilon\mathbf{\Omega}\mathbf{\Omega}^T + \varepsilon^{-1}\mathbf{\Xi}^T\mathbf{\Xi} \quad (6)$$

Lemma 2.2. (Cao *et al.*, 2000) For any real matrices $\mathbf{\Lambda}$, $\mathbf{\Pi}$ and $\mathbf{Y}$ satisfying $\mathbf{Y} > 0$. The following holds :

$$\mathbf{\Lambda}\mathbf{\Pi} + \mathbf{\Lambda}^T\mathbf{\Pi}^T \leq \mathbf{\Lambda}\mathbf{Y}^{-1}\mathbf{\Lambda}^T + \mathbf{\Pi}^T\mathbf{Y}\mathbf{\Pi} \quad (7)$$

Lemma 2.3. (Boukas and Liu, 2004) Any matrices $\mathbf{U},\ \mathbf{V} \in \mathbb{R}^{n*n}$ with $\mathbf{V} > 0$, satisfy :

$$\mathbf{U}\mathbf{V}^{-1}\mathbf{U}^T \geq \mathbf{U} + \mathbf{U}^T -\ \mathbf{V} \quad (8)$$

In the next section, one starts by considering that all the uncertainties are equal to zero, then, the stochastic asymptotic stability condition for the nominal system under study, will be established, on another side, one will design a state feedback controller of the form (5) such that the resulting closed-loop system is regular, impulse free and stochastically asymptotically mean-square stable (RISS) as well.

3. STABILITY AND STABILIZATION

As the stability is the first requirement of any control design, one starts by establishing a sufficient condition under which the unforced nominal system is (RISS) simultaneously. The following theorem gives this result:

Lemma 3.1. If there exist a set of symmetric and positive-definite matrices $\mathbf{P} = (\mathbf{P}(1), \ldots, \mathbf{P}(N))$ and a set of matrices $\mathbf{H} = (\mathbf{H}(1), \ldots, \mathbf{H}(N))$ such that the following LMI holds for every $i \in \mathcal{S}$:

$$\mathbf{\Theta}(i) = \begin{bmatrix} \mathbb{J}(i) & \mathbb{W}^T(i)\mathbf{E}^T\mathbf{P}(i) \\ \mathbf{P}(i)\mathbf{E}\mathbb{W}(i) & -\mathbf{P}(i) \end{bmatrix} < 0 \quad (9)$$

where:
$\mathbb{J}(i) = \mathbf{E}^T\mathbf{P}(i)\mathbf{A}(i) + \mathbf{A}^T(i)\mathbf{P}(i)\mathbf{E}$
$+ \mathbf{H}^T(i)\mathbf{R}^T(i)\mathbf{A}(i) + \mathbf{A}^T(i)\mathbf{R}(i)\mathbf{H}(i)$
$+\sum_{j=1}^{N} \lambda_{ij}\mathbf{E}^T\mathbf{P}(j)\mathbf{E}$, $\mathbf{R}(i) \in \mathbb{R}^{n\times n_E}$ is any matrix such that $\mathbf{E}^T\mathbf{R}(i) = 0$, then the nominal system (1) with $\mathbf{u_t} = 0$, $t \geq 0$ is (RISS).

The proof of this Lemma is omitted due to the limited paper length.

Lemma 3.1 provides a sufficient condition for the Markovian jump singular system with Wiener disturbance to be (RISS). The proposed condition is a strict LMI, which is much more tractable and reliable in numerical computation than the non strict one as reported in a lot of works dealing with the problems of stability and stabilizability for singular systems, see for instance (Chun-Liang, 1999).

Now, one will synthesize a suitable state feedback controller such as the system in closed loop is (RISS). The following theorem summarizes this result:

Theorem 3.1. If there exist, a set of symmetric and positive definite matrices $\mathbf{P} = (\mathbf{P}(1), \ldots, \mathbf{P}(N))$ and a set of matrices $\mathbf{H} = (\mathbf{H}(1), \ldots, \mathbf{H}(N))$ and $\mathbf{X} = (\mathbf{X}(1), \ldots, \mathbf{X}(N))$ such that the following holds for each $i \in S$:

$$\begin{bmatrix} \mathbf{\Phi}(i) & \mathbb{W}^T(i)\mathbf{E}^T\mathbf{P}(i) & [\mathbf{P}(i)\mathbf{E} + \mathbf{R}(i)\mathbf{H}(i)]^T\mathbf{B}(i) \\ \mathbf{P}(i)\mathbf{E}\mathbb{W}(i) & -\mathbf{P}(i) & \mathbf{0} \\ \mathbf{B}^T(i)[\mathbf{P}(i)\mathbf{E} + \mathbf{R}(i)\mathbf{H}(i)] & \mathbf{0} & -\mathbf{P}(i) \end{bmatrix} < 0 \quad (10)$$

where:
$\mathbf{\Phi}(i) = [\mathbf{P}(i)\mathbf{E} + \mathbf{R}(i)\mathbf{H}(i)]^T\mathbf{A}(i) + \mathbf{X}^T(i) + \mathbf{X}(i)$
$+ \mathbf{A}^T(i)[\mathbf{P}(i)\mathbf{E} + \mathbf{R}(i)\mathbf{H}(i)] - \mathbf{P}^T(i)$
$+ \sum_{j=1}^{N} \lambda_{ij}\mathbf{E}^T\mathbf{P}(j)\mathbf{E}$, and $\mathbf{R}(i) \in \mathbb{R}^{n\times n_E}$ is any matrix such that $\mathbf{E}^T\mathbf{R}(i) = 0$, then the resulting closed-loop system under study (1) is (RISS), and the stabilizing controller gain is given by $\mathbf{K}(i) = \mathbf{P}^{-1}(i)\mathbf{X}(i)$, $i \in \mathcal{S}$.

Proof of the Theorem 3.1: For this purpose, plugging controller (5) in the dynamics (4) gives:

$$\mathbf{E}\mathbf{x_t}dt = \mathbf{A_c}(r_t)\mathbf{x_t}dt + \mathbb{W}(r_t)\mathbf{x_t}dw \quad (11)$$

with $\mathbf{A_c}(r_t) = \mathbf{A}(r_t) + \mathbf{B}(r_t)\mathbf{K}(r_t)$. Then, based on the results of Lemma 3.1, this closed-loop system is (RISS) if the following LMI holds for every $i \in \mathcal{S}$:

$$\begin{bmatrix} \mathbf{\Phi}(i) & \mathbb{W}^T(i)\mathbf{E}^T\mathbf{P}(i) \\ \mathbf{P}(i)\mathbf{E}\mathbb{W}(i) & -\mathbf{P}(i) \end{bmatrix} < 0 \quad (12)$$

Where:
$\mathbf{\Phi}(i) = \mathbf{E}^T\mathbf{P}(i)\mathbf{A_c}(i) + \mathbf{A_c}^T(i)\mathbf{P}(i)\mathbf{E}$
$+ \mathbf{H}^T(i)\mathbf{R}^T(i)\mathbf{A_c}(i) + \mathbf{A_c}^T(i)\mathbf{R}^T(i)\mathbf{H}(i)$
$+ \sum_{j=1}^{N} \lambda_{ij}\mathbf{E}^T\mathbf{P}(j)\mathbf{E}$.

Now, replace $\mathbf{A_c}(i)$ by its expression then $\mathbf{\Phi}(i)$ becomes:
$\mathbf{\Phi}(i) = [\mathbf{P}(i)\mathbf{E}+\mathbf{R}(i)\mathbf{H}(i)]^T\mathbf{A}(i)+\mathbf{A}^T(i)[\mathbf{P}(i)\mathbf{E}+\mathbf{R}(i)\mathbf{H}(i)] + [\mathbf{P}(i)\mathbf{E} + \mathbf{R}(i)\mathbf{H}(i)]^T\mathbf{B}^T(i)\mathbf{K}^T(i)$
$\times\mathbf{K}(i)\mathbf{B}(i)[\mathbf{P}(i)\mathbf{E}+\mathbf{R}(i)\mathbf{H}(i)]+\sum_{j=1}^{N} \lambda_{ij}\mathbf{E}^T\mathbf{P}(j)$.

remark 3.1. It must be remarked that the introduction of any matrices $\mathbf{H}(i)$ and $\mathbf{R}(i)$ such that

$\mathbf{E}^T\mathbf{R}(i) = 0$, will remove the equality $\mathbf{E}^\top\mathbf{P}(i) = \mathbf{P}^\top(i)\mathbf{E}$ usually used in a lot of papers dealing with the problem of the control for singular system, which may cause some numerical problems, by using the equality $\left[\mathbf{E}^T\mathbf{P}(i)+\mathbf{H}^T(i)\mathbf{R}^T(i)\right]\mathbf{E} = \mathbf{E}^T\left[\mathbf{P}(i)\mathbf{E}+\mathbf{R}(i)\mathbf{H}(i)\right]\}$ that is automatically verified. It should be noted that, as matrix E is independent of the mode, without loss of generality, we can also choose the matrix $\mathbf{R}(i)$ independent of the mode. The same remark can be made for the matrix $\mathbf{H}(i)$.

On the other hand, by applying Lemma 2.2 to the term $[\mathbf{P}(i)\mathbf{E}+\mathbf{R}(i)\mathbf{H}(i)]^T\mathbf{B}(i)\mathbf{K}(i)+\mathbf{K}^T(i)\mathbf{B}^T(i)$ $\times[\mathbf{P}(i)\mathbf{E}+\mathbf{R}(i)\mathbf{H}(i)]$, letting $\mathbf{X}(i) = \mathbf{P}(i)\mathbf{K}(i)$ and according to Lemma 2.3, one obtains:

$$\begin{aligned}
&[\mathbf{P}(i)\mathbf{E} + \mathbf{R}(i)\mathbf{H}(i)]^T\mathbf{B}(i)\mathbf{K}(i)\\
&+\mathbf{K}^T(i)\mathbf{B}^T(i)[\mathbf{P}(i)\mathbf{E} + \mathbf{R}(i)\mathbf{H}(i)]\\
&\geq \mathbf{X}^T(i) + \mathbf{X}(i) - \mathbf{P}^T(i)\\
&+[\mathbf{P}(i)\mathbf{E} + \mathbf{R}(i)\mathbf{H}(i)]^T\mathbf{B}(i)\mathbf{P}^{-1}(i)\\
&\times\mathbf{B}^T(i)[\mathbf{P}(i)\mathbf{E} + \mathbf{R}(i)\mathbf{H}(i)]
\end{aligned}$$

Hence, substituting (13) into (12), and applying Schur complement formula, yields (10), this ends the proof.

4. ROBUST STABILITY AND ROBUST STABILIZATION

The objective of this section is to derive sufficient conditions for robust stochastic mean-square stability for the class of system under study. Our attention is also to develop a state feedback controller of the form (5) that will robustly stochastically stabilizes the closed-loop system. One starts by giving the following theorem that states the condition under which the unforced system (1) is robustly stochastically stable (RSS) :

Theorem 4.1. If there exist, a set of symmetric and positive-definite matrices $\mathbf{P} = (\mathbf{P}(1), \ldots, \mathbf{P}(N))$, a set of matrices $\mathbf{H} = (\mathbf{H}(1), \ldots, \mathbf{H}(N))$ and a set of positive scalars $\varepsilon_A = (\varepsilon_A(1), \ldots, \varepsilon_A(N))$ such that the following LMI holds for every $\mathbf{i} \in \mathcal{S}$ and all admissible uncertainties:

$$\begin{bmatrix}
\mathbf{\Gamma}(i) & \mathbb{W}^T(i)\mathbf{E}^T\mathbf{P}(i) & (\mathbf{E}^T\mathbf{P}(i) + \mathbf{H}^T(i)\mathbf{R}^T(i))\mathbf{D_A}(i)\\
\mathbf{P}(i)\mathbf{E}\mathbb{W}(i) & -\mathbf{P}(i) & \mathbf{0}\\
(\mathbf{P}(i)\mathbf{E} + \mathbf{R}(i)\mathbf{H}(i))\mathbf{D_A}(i)^T & \mathbf{0} & -\varepsilon_A(i)\mathbb{I}
\end{bmatrix} < 0 \quad (13)$$

with:
$\mathbf{\Gamma}(i) = \mathbf{E}^T\mathbf{P}(i)\mathbf{A}(i) + \mathbf{A}^T(i)\mathbf{P}(i)\mathbf{E}$
$+ \mathbf{H}^T(i)\mathbf{R}^T(i)\mathbf{A}(i) + \mathbf{A}^T(i)\mathbf{R}(i)\mathbf{H}(i)$
$+ \sum_{j=1}^N \lambda_{ij}\mathbf{E}^T\mathbf{P}(j)\mathbf{E} + \varepsilon_A(i)\mathbf{E_A}(i)^T\mathbf{E_A}(i)$,
and $\mathbf{R}(i) \in \mathbb{R}^{n\times n_E}$ is any matrix such that $\mathbf{E}^T\mathbf{R}(i) = 0$,
then, the unforced system (1) is regular impulse-free and robustly stochastically mean-square stable (RIRSS).

Proof: Based on the result of the theorem 3.1, and using (2), the system (1) is stochastically stable if the following holds:

$$\begin{aligned}
&\mathbf{E}^T\mathbf{P}(i)\mathbf{A}(i) + \mathbf{A}^T(i)\mathbf{P}(i)\mathbf{E}\\
&+\mathbf{H}^T(i)\mathbf{R}^T(i)\mathbf{A}(i) + \mathbf{A}^T(i)\mathbf{R}(i)\mathbf{H}(i)\\
&+(\mathbf{E}^T\mathbf{P}(i) + \mathbf{H}^T(i)\mathbf{R}^T(i))\mathbf{D_A}(i)\mathbf{F_A}(i,t)\mathbf{E_A}(i)\\
&+\mathbf{E_A^T(i)}\mathbf{F_A}^T(i,t)\mathbf{D_A}^T(i)(\mathbf{P}(i)\mathbf{E} + \mathbf{R}(i)\mathbf{H}(i))\\
&+\mathbb{W}^T(i)\mathbf{E}^T\mathbf{P}(i)\mathbf{E}\mathbb{W}(i) + \sum_{j=1}^N \lambda_{ij}\mathbf{E}^T\mathbf{P}(j)\mathbf{E} \quad (14)
\end{aligned}$$

Then by applying Lemma 2.1 to (14), and after using Schur complement, one obtains (13).

Now, one can design a suitable state feedback controller that robustly stochastically stabilizes (1). To this end, let us set (5) in the system dynamics, this yields to the following:

$$\mathbf{E}\mathbf{x_t}dt = \mathbf{A_c}(r_t,t)\mathbf{x_t}dt + \mathbb{W}(r_t)\mathbf{x_t}dw(t) \quad (15)$$

with $\mathbf{A_c}(r_t,t) = \mathbf{A}(r_t,t) + \mathbf{B}(r_t,t)\mathbf{K}(r_t)$.
Under the condition of the theorem (4.1), the closed-loop system is (RSS) if the following LMI holds for every $i \in \mathcal{S}$:

$$\begin{bmatrix}
\mathbf{\Gamma}(i) & \mathbb{W}^T(i)\mathbf{E}^T\mathbf{P}(i) & (\mathbf{E}^T\mathbf{P}(i) + \mathbf{H}^T(i)\mathbf{R}^T(i))\mathbf{D_A}(i)\\
\mathbf{P}(i)\mathbf{E}\mathbb{W}(i) & -\mathbf{P}(i) & \mathbf{0}\\
(\mathbf{P}(i)\mathbf{E} + \mathbf{R}(i)\mathbf{H}(i))\mathbf{D_A}(i)^T & \mathbf{0} & -\varepsilon_\mathbf{A}(\mathbf{i})\mathbb{I}
\end{bmatrix} < 0 \quad (16)$$

with:
$\mathbf{\Gamma}(i) = \mathbf{E}^T\mathbf{P}(i)\mathbf{A}(i) + \mathbf{A}^T(i)\mathbf{P}(i)\mathbf{E}$
$+ \mathbf{H}^T(i)\mathbf{R}^T(i)\mathbf{A}(i) + \mathbf{A}^T(i)\mathbf{R}(i)\mathbf{H}(i)$
$+ \varepsilon_A(i)\mathbf{E_A}(i)^T\mathbf{E_A}(i) + \sum_{j=1}^N \lambda_{ij}\mathbf{E}^T\mathbf{P}(j)\mathbf{E}$
$+ (\mathbf{E}^T\mathbf{P}(i) + \mathbf{H}^T(i)\mathbf{R}^T(i))\mathbf{B}(i)\mathbf{K}(i)$
$+ \mathbf{K}^T(i)\mathbf{B}^T(i)(\mathbf{P}(i)\mathbf{E} + \mathbf{R}(i)\mathbf{H}(i))$
$+(\mathbf{E}^T\mathbf{P}(i)+\mathbf{H}^T(i)\mathbf{R}^T(i))\mathbf{D}_B(i)\mathbf{F}_B(i,t)\mathbf{E}_B(i)\mathbf{K}(i)$
$+ \mathbf{K}^T(i)\mathbf{E_B}^T(i)\mathbf{F_B^T}(i,t)\mathbf{D_B}^T(\mathbf{P}(i)\mathbf{E} + \mathbf{R}(i)\mathbf{H}(i))$

then by applying lemma 2.1 one obtains the following:

$$\begin{aligned}
&[\mathbf{P}(i)\mathbf{E} + \mathbf{R}(i)\mathbf{H}(i)]^T\mathbf{D_B}(i)\mathbf{F_B}(i,t)E_B(i)\mathbf{K}(i)\\
&+\mathbf{K}^T(i)\mathbf{E_B}^T(i)\mathbf{F_B^T}(i,t)\mathbf{D_B}^T[\mathbf{P}(i)\mathbf{E} + \mathbf{R}(i)\mathbf{H}(i)]\\
&\leq \varepsilon_B(i)\mathbf{K}^T(i)\mathbf{E_B}^T(i)\mathbf{E_B}(i)\mathbf{K}(i)\\
&+\varepsilon_B^{-1}(i)[\mathbf{P}(i)\mathbf{E} + \mathbf{R}(i)\mathbf{H}(i)]^T\mathbf{D_B}(i)\mathbf{D_B}^T(i)\\
&\times[\mathbf{E}\mathbf{P}(i) + \mathbf{R}(i)\mathbf{H}(i)]
\end{aligned}$$

this together with (13), and taking into consideration that $\mathbf{K}^T(i)\mathbf{E_B}^T(i)\mathbf{E_B}(i)\mathbf{K}(i) > 0$, (16) will be satisfied with:

$$
\begin{aligned}
\mathbf{\Gamma}(i) &= \mathbf{A}^T(i)[\mathbf{P}(i)\mathbf{E} + \mathbf{R}(i)\mathbf{H}(i)] + \mathbf{X}(i) + \mathbf{X}^T(i) \\
&+[\mathbf{P}(i)\mathbf{E} + \mathbf{R}(i)\mathbf{H}(i)]^T\mathbf{A}(i) - \mathbf{P}^T(i) \\
&+\varepsilon_B^{-1}(i)[\mathbf{P}(i)\mathbf{E} + \mathbf{R}(i)\mathbf{H}(i)]^T\mathbf{D_B}(i)\mathbf{D_B}^T(i) \\
&\times[\mathbf{P}(i)\mathbf{E} + \mathbf{R}(i)\mathbf{H}(i)] + \varepsilon_A(i)\mathbf{E_A}^T(i)\mathbf{E_A}(i) \\
&+[\mathbf{P}(i)\mathbf{E} + \mathbf{R}(i)\mathbf{H}(i)]^T\mathbf{B}(i)\mathbf{P}^{-1}(i)\mathbf{B}^T(i) \\
&\times[\mathbf{P}(i)\mathbf{E} + \mathbf{R}(i)\mathbf{H}(i)] + \sum_{j=1}^{N}\lambda_{ij}\mathbf{E}^T\mathbf{P}(j)\mathbf{E} \quad (17)
\end{aligned}
$$

Hence, by using Schur complement formula to (17), (16) becomes:

$$
\left[\begin{array}{ccccc}
\mathbf{\Upsilon}(i) & \mathbb{W}^T(i)\mathbf{E}^T\mathbf{P}(i) & [\mathbf{P}(i)\mathbf{E} + \mathbf{R}(i)\mathbf{H}(i)]^T\mathbf{B}(i) & [\mathbf{P}(i)\mathbf{E} + \mathbf{R}(i)\mathbf{H}(i)]^T\mathbf{D}_A(i) & [\mathbf{P}(i)\mathbf{E} + \mathbf{R}(i)\mathbf{H}(i)]^T\mathbf{D}_B(i) \\
\mathbf{P}(i)\mathbf{E}\mathbb{W}(i) & -\mathbf{P}(i) & \mathbf{0} & \mathbf{0} & \mathbf{0} \\
\mathbf{B}^T(i)[\mathbf{P}(i)\mathbf{E} + \mathbf{R}(i)\mathbf{H}(i)] & & -\mathbf{P}(i) & \mathbf{0} & \mathbf{0} \\
\mathbf{D_A}^T(i)[\mathbf{P}(i)\mathbf{E} + \mathbf{R}(i)\mathbf{H}(i)] & \mathbf{0} & \mathbf{0} & -\varepsilon_A(i)\mathbb{I} & \mathbf{0} \\
\mathbf{D_B}^T(i)[\mathbf{P}(i)\mathbf{E} + \mathbf{R}(i)\mathbf{H}(i)](i) & \mathbf{0} & \mathbf{0} & \mathbf{0} & -\varepsilon_B(i)\mathbb{I}
\end{array}\right] < 0 \, (18)
$$

where:
$\mathbf{\Upsilon}(i) = [\mathbf{P}(i)\mathbf{E}+\mathbf{R}(i)\mathbf{H}(i)]^T\mathbf{A}(i)+\mathbf{A}^T(i)[\mathbf{P}(i)\mathbf{E}+\mathbf{R}(i)\mathbf{H}(i)]+\mathbf{X}(i)+\mathbf{X}^T(i)-\mathbf{P}^T(i)+\varepsilon_A(i)\mathbf{E_A}^T(i)\mathbf{E_A}(i) + \sum_{j=1}^{N}\lambda_{ij}\mathbf{E}^T\mathbf{P}(j)\mathbf{E}$

The following theorem summaries this result:

Theorem 4.2. If there exist, a set of symmetric and positive-definite matrices $\mathbf{P} = (\mathbf{P}(1), \ldots, \mathbf{P}(N))$, a set of matrices $\mathbf{H} = (\mathbf{H}(1), \ldots, \mathbf{H}(N)) > 0$, $\mathbf{X} = (\mathbf{X}(1), \ldots, \mathbf{X}(N)) > 0$ and a set of positive scalars $\varepsilon_A = (\varepsilon_A(1), \ldots, \varepsilon_A(N))$, and $\varepsilon_B = (\varepsilon_B(1), \ldots, \varepsilon_B(N))$, such that (18) holds for each $i \in S$, and all admissible uncertainties, then the closed-loop system is (RIRSS). In this case, the robustly stabilizing controller gain is given by $\mathbf{K}(i) = \mathbf{P}^{-1}(i)\mathbf{X}(i)$.

In the following section, one will demonstrate the validity of the proposed results by considering the following numerical example.

5. EXAMPLES

let us suppose that the generator matrix $\mathbf{\Lambda}$ and the matrix $\mathbf{E}$ are given by:

$$
\mathbf{\Lambda} = \begin{bmatrix} -2.00 & 2.00 \\ 1.00 & -1.00 \end{bmatrix}, \mathbf{E} = \begin{bmatrix} 2.00 & 1.00 & 0.00 \\ 2.00 & 1.00 & 0.00 \\ 0.00 & 0.00 & 1.00 \end{bmatrix}.
$$

Let the dynamics and the uncertainties in each mode be given as follows:

$$
\mathbf{A}(1) = \begin{bmatrix} 1.50 & 0.50 & 1.00 \\ -1.00 & 0.00 & 1.00 \\ 0.50 & 0.00 & 1.00 \end{bmatrix}, \mathbf{D_B}(1) = \begin{bmatrix} 0.0100 \\ 0.2000 \\ 0.1000 \end{bmatrix},
$$

$$
\mathbf{W}(1) = \begin{bmatrix} 0.30 & 0.00 & 0.10 \\ 0.00 & 0.04 & 0.00 \\ 0.10 & 0.00 & 0.30 \end{bmatrix}, \mathbf{B}(1) = \begin{bmatrix} 1.00 & 1.00 & 0.10 \\ 6.00 & 3.00 & 0.10 \\ 0.00 & 2.0 & 0.10 \end{bmatrix},
$$

$$
\mathbf{D_A}(1) = \begin{bmatrix} 0.01 \\ 0.20 \\ 0.12 \end{bmatrix}, \mathbf{E_A}(1) = \begin{bmatrix} 0.20 & 0.10 & 0.01 \end{bmatrix}.
$$

$$
\mathbf{A}(2) = \begin{bmatrix} 1.00 & 0.50 & 1.00 \\ -1.00 & 0.00 & 1.10 \\ 0.20 & 0.00 & 1.00 \end{bmatrix}, \mathbf{D_B}(2) = \begin{bmatrix} 0.20 \\ 0.30 \\ 0.10 \end{bmatrix},
$$

$$
\mathbf{W}(2) = \begin{bmatrix} 0.10 & 0.00 & 0.10 \\ 0.10 & 0.00 & 0.04 \\ 0.02 & 0.10 & 0.30 \end{bmatrix}, \mathbf{B}(2) = \begin{bmatrix} 0.20 & 0.20 & 0.80 \\ 0.40 & 0.00 & 0.20 \\ 0.00 & 0.40 & 0.20 \end{bmatrix},
$$

$$
\mathbf{D_A}(2) = \begin{bmatrix} 0.13 \\ 0.10 \\ 0.10 \end{bmatrix}, \mathbf{E_A}(2) = \begin{bmatrix} 0.03 & 0.01 & 0.02 \end{bmatrix}.
$$

The purpose is to design a robust state feedback controller such that the closed-loop system is regular, impulse free and robustly stochastically asymptotically stable in mean-square sense. To this purpose, we choose $\mathbf{R}(1)$ and $\mathbf{R}(2)$ as follows:

$$
\begin{aligned}
\mathbf{R}(1) &= \begin{bmatrix} 1.00 & -1.00 & 0.00 \end{bmatrix}^\top, \\
\mathbf{R}(2) &= \begin{bmatrix} 2.00 & -2.00 & 0.00 \end{bmatrix}^\top.
\end{aligned}
$$

For the computation, let us choose the following: $\varepsilon_A(1) = \varepsilon_B(1) = 0.50, \varepsilon_A(2) = \varepsilon_B(2) = 0.30$, and solving the LMIs (18), one get:

$$
\mathbf{P}(1) = \begin{bmatrix} 0.8222 & -0.3384 & -0.1677 \\ -0.3384 & 0.9386 & -0.1124 \\ -0.1677 & -0.1124 & 0.5221 \end{bmatrix},
$$

$$
\mathbf{P}(2) = \begin{bmatrix} 0.6563 & -0.2597 & -0.0463 \\ -0.2597 & 0.5679 & -0.1004 \\ -0.0463 & -0.1004 & 0.6319 \end{bmatrix},
$$

$$
\mathbf{H}(1) = \begin{bmatrix} 1.6718 & 0.7682 & -0.0680 \end{bmatrix},
$$

$$
\mathbf{H}(2) = \begin{bmatrix} -0.4637 & -0.3872 & -0.1549 \end{bmatrix},
$$

$$
\mathbf{X}(1) = \begin{bmatrix} -2.9726 & 0.0000 & -0.0000 \\ -2.9359 & -0.8778 & 0.0000 \\ -2.4813 & -2.2914 & -0.8632 \end{bmatrix},
$$

$$
\mathbf{X}(2) = \begin{bmatrix} 0.1503 & 0.0000 & 0.0000 \\ -0.2988 & -0.2397 & 0.0000 \\ -0.8808 & -1.0837 & -0.6485 \end{bmatrix}.
$$

which gives the following gain matrices:

$$\mathbf{K}(1) = \begin{bmatrix} -8.4407 & -2.1099 & -0.5746 \\ -7.2516 & -2.3635 & -0.4385 \\ -9.0251 & -5.5756 & -1.9324 \end{bmatrix},$$

$$\mathbf{K}(2) = \begin{bmatrix} -0.2419 & -0.5324 & -0.1861 \\ -0.9121 & -1.0039 & -0.2768 \\ -1.5565 & -1.9136 & -1.0839 \end{bmatrix}.$$

6. CONCLUSION

In this paper, one studied the problem of stability and stabilizability for stochastic singular systems with both Markovian jumps and Wiener process. Sufficient strict LMI condition for the stability has been presented, and without requiring the regularity assumption, LMI approach has also been developed to design stabilizing state feedback controller which guarantees that the closed-loop system is regular, impulse free as well as stochastically asymptotically mean-square stable. The robustness problem was tackled too.

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