FAULT TOLERANT CONTROL DESIGN OF NONLINEAR SYSTEMS USING LMI GAIN SYNTHESIS

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Abstract: In this paper, an active Fault Tolerant Control (FTC) strategy is developed to nonlinear systems described by multiple linear models to prevent the system deterioration by the synthesis of adapted controllers. By considering that Fault Detection, Isolation (FDI) and estimation is realized, the synthesis of an appropriate combination of predesigned gains is performed. The main contribution concerns the design of state feedback gains through $\mathcal{LMI}$ both in fault-free and faulty cases in order to preserve the system performances. For each separate actuator, a robust pole placement is designed by pole clustering. The effectiveness and performances of the method are illustrated in simulation by considering a nonlinear system: a Three-Tank system.

Keywords: Fault Tolerant Control, Nonlinear, Multi-model, LMI, Stability

1. INTRODUCTION

To overcome the limitations of conventional feedback control, new controllers have been developed with accommodation capabilities or tolerance to faults. The objective of Fault Tolerant Control system (FTC) is to maintain current performances closed to desirable performances and preserve stability conditions in the presence of component and/or instrument faults ; in some circumstances reduced performance could be accepted as a trade-off. Accommodation capability of a control system depends on many factors such as the severity of the failure, the robustness of the nominal system, and the actuators redundancy. FTC can be motivated by different goals depending on the application under consideration, for instance, safety in flight control or reliability or quality improvements in industrial processes. Various approaches for FTC have been suggested in the literature (Patton, 1997), (Zhang and Jiang, 2003) but often deal with linear systems. For nonlinear systems, the design of Fault Tolerant controller is far more complicated. Nonlinear systems based on multiple linear models, represents an attractive solution to deal with the control of nonlinear systems (Leith and Leithead, 2000), (Banerjee *et al.*, 1995) or FDI methods as in the chapter nine of (Chen and Patton, 1999) where nonlinear dynamic systems are described by a number of locally linearized models based on the idea of Tagaki-Sugeno fuzzy models or as interpolated multiple linear models (Murray-Smith and Johansen, 1997). Various recent FDI/FTC studies, based on a multiple model method have been developed in order to detect, isolate and estimated an accurate state of a system in presence of faults/failures around an operating point (Maybeck, 1999), (Zhang and Jiang, 2001). Compared to multi-model based reconfigurable control method presented by (Aström *et al.*, 2001), this paper not consider some redundant hardware which is very useful when failures are supposed to occur on the system. In this paper, an active fault tolerant strategy is developed so as to avoid actuator fault effect on nonlinear system where faults are assumed to be incipient, abrupt but not generate a total actuator failure. Under the assumption that the fault is detected, isolated and estimated, the developed method preserves the system performances through an appropriate gain synthesis in faulty case. It is a big difference with robust control which does not deal with a FDI module and does not take into account fault estimation for reconfiguration. Compared to recent work applied to similar nonlinear system (Theilliol *et al.*, 2003*a*), where a multi-model representation is considered, the proposed FTC strategy is not based on an additional control law but on the redesign of appropriate gain in faulty case allowing stability and performances of the system.

This appropriate design is inspired from a previous work made by (Kanev and Verhaegen, 2002) where a FTC strategy is devoted for each actuator. However, this work considers only linear case with a single operating point. Our paper contributes to improve it on real systems with multi-operating points and a relaxed $\mathcal{LMI}$ region for stability. The paper is organized as follows. In section II, an active state space representation is derived from an additive one by underlying the links between them. A global state space representation of nonlinear system is given through a multi-model approach. In section III, we introduce a pole placement by $\mathcal{LMI}$ region and then a gain synthesis for each actuator generate an active global state feedback synthesis. A simulation example is given in section IV to illustrate the proposed method. Finally, concluding remarks are given in the last section.

2. PROBLEM STATEMENT

It is assumed that dynamic behaviour of the system operating at different operating points can be approximated by a set of N linear time invariant models. Consider the following state space representation of a nonlinear system around j-th operating point with actuator faults:

$$\begin{cases} x^j_{k+1} &= A_j x^j_k + B_j u^j_k + F_j f_k + \Delta x_j \\ y^j_k &= C_j x^j_k + \Delta y_j \end{cases} \tag{1}$$

where $x^j \in \mathbb{R}^n$ represents the state vector, $u^j \in \mathbb{R}^p$ is the input vector, $y^j \in \mathbb{R}^m$ is the output vector and $f_k \in \mathbb{R}^p$ represents the actuator fault vector. $A_j \in \mathbb{R}^{n\times n}$, $B_j \in \mathbb{R}^{n\times p}$, $C_j \in \mathbb{R}^{m\times n}$ and the fault distribution matrix $F_j \in \mathbb{R}^{n\times f}$ are invariant matrices defined around the j^{th} operating point ($\mathcal{OP}_j$). We consider the terms Δx_j, Δy_j:

$$\begin{aligned} \Delta x_j &= -A_j x^j_e - B_j u^j_e + x^j_e \\ \Delta y_j &= -C_j x^j_e + y^j_e \end{aligned} \tag{2}$$

where (x^j_e, y^j_e, u^j_e) are a family of operating points of the nonlinear plant. As considered in (Johansen *et al.*, 1998), (Murray-Smith and Johansen, 1997) and (Theilliol *et al.*, 2003*b*), a possible model that would be able to catch the full range of operation is made from N weighting local models $\mathcal{OP}_j$ by interpolation functions ρ^j_k. These activation functions $\rho^j_k \quad \forall j \in [1, 2, .., N]$ lie in a convex set $\Omega = \{\rho^j_k \in \mathbb{R}^N, \rho_k = [\rho^1_k \quad ....\rho^N_k]^T, \quad \rho^j_k \geq 0 \quad \forall j \quad and \quad \sum_{j=1}^N \rho^j_k = 1\}$ and these functions are generated via works of (Adam-Medina *et al.*, 2003) and (Theilliol *et al.*, 2003*b*), which permit to generate insensitive residual to faults and some uncertainties. So, activation functions are robust against faults and errors modeling and the dynamic system is well represented. The plant dynamics are formulated as a blended multiple representation as in (Theilliol *et al.*, 2003*a*):

$$x_k = \sum_{j=1}^N \rho^j_k x^j_k \quad y_k = \sum_{j=1}^N \rho^j_k y^j_k \tag{3}$$

The representation considers additive fault representation but there exists multiplicative representation for specific actuator fault as in (Kanev and Verhaegen, 2002). So, consider a local multiplicative actuator fault representation as:

$$\begin{cases} x^j_{k+1} &= A_j x^j_k + B_j (I - \gamma^a) u^j_k + \Delta x_j \\ y^j_k &= C_j x^j_k + \Delta y_j \end{cases} \tag{4}$$

with $\gamma^a \triangleq diag[\gamma^a_1, \gamma^a_2, ..., \gamma^a_p]$, $\gamma^a_1 \in \mathbb{R}$, such that $\gamma^a_i = 1$ represents a total lost, a failure of i-th actuator and $\gamma^a_i = 0$ implies that i-th actuator operates normally. The relation between state space representations (1) and (4) is equivalent to

$$F_j f_k = -B_j \gamma^a u^j_k \tag{5}$$

In closed-loop, the fault occurrence could be detected as described in (Noura *et al.*, 2000) for linear case and (Theilliol *et al.*, 2003*a*) for multi-linear systems and Fault Tolerant Control could be performed via an additional control law as in (Theilliol *et al.*, 2002) which permits to avoid fault on a system based on a state space representation as (1). In these papers, the goal was to synthesize a new control law U_{FTC} with a nominal one u_{nom} and additional one u_{ad}. The term u_{ad} was performed in order to vanish fault on the system. The global control law is obtained by interpolating gains of each local controller (Leith and Leithead, 2000) and is defined as:

$$U_{FTC} = \sum_{j=1}^N \rho^j_k (u^j_{nom} + u^j_{ad}) \tag{6}$$

with $u^j_{nom} = -K^j_{nom} x^j_k$. The gains were only performed for nominal cases and do not take into account fault occurrence. Based on a multiplicative fault representation defined in (4), the new control law u^j_{FTC} must vanish all faults on the system as:

$$u^j_{FTC} = [I - \gamma^a]^+ u^j_{nom} \tag{7}$$

Note that $u^j_{FTC} = [I - \gamma^a]^+ u^j_{nom} = -[I - \gamma^a]^+ K^j_{nom} x^j_k = -K^j_{FTC} x^j_k$. So, without considering total fault, this specific control law in the state space representation (4) leads to:

$$\begin{aligned} B_j(I-\gamma^a)u^j_k &= B_j(I-\gamma^a)(I-\gamma^a)^+ u^j_{nom} \\ &= B_j u^j_{nom} \end{aligned} \tag{8}$$

As previously defined in (6), the model probability is viewed as a scheduled variable in the synthesis of the controller and the global control law is defined as:

$$U_{FTC} = \sum_{j=1}^{N} \rho^j_k u^j_{FTC} \tag{9}$$

with u^j_{FTC} the output of each local controller defined around each operating point. In order to synthesize state feedback for FTC ensuring both active control in multi-model philosophy and quadratic stability, use of $\mathcal{LMI}$ provide a well toolbox for these purposes. Total failures are not consider in this paper but only partial ones. We attract attention that the system has to be observable in all around each operating points and we will consider now only nonlinear plant defined around Equilibrium Points ($\mathcal{EP}$) i.e. $\Delta x_j = 0$.

3. FAULT TOLERANT CONTROL ON MULTIPLE OPERATING POINTS

3.1 Pole clustering

In the synthesis of control system, some desired performances should be considered in addition to stability. In fact, classical stability conditions do not deal with transient responses of the closed-loop system. In contrast, a satisfactory transient response can be guaranteed by confining its poles in a prescribed region. For many real problems, exact pole assignment may not be necessary: it suffices to locate the closed-loop poles in a prescribed subregion in the complex plane. We will discuss about pole clustering by introducing the following $\mathcal{LMI}$-based representation of stability regions.

Definition 1. LMI stability region (Chilali and Gahinet, 1996). A subset D of the complex plane is called an LMI region if there exist a symmetric matrix $\alpha = [\alpha_{kl}] \in \mathbb{R}^{n\times n}$ and a matrix $\beta = [\beta_{kl}] \in \mathbb{R}^{n\times n}$ such that

$$D = \{z \in C : f_D(z) < 0\} \tag{10}$$

where the characteristic function $f_D(z)$ is given by

$$f_D(z) = [\alpha_{kl} + \beta_{kl} z + \beta_{lk}\bar{z}]_{1\le k,l\le n} \tag{11}$$

(f_D is valued in the space of $n \times n$ Hermitian matrices). □

Moreover, $\mathcal{LMI}$ regions are convex and symmetric with respect to the real axis since for any $z \in \mathcal{D} f_{\mathcal{D}}(\bar{z}) = \overline{f_{\mathcal{D}}(z)} < 0$. Then, a matrix A has all its eigenvalue in $\mathcal{D}$, if and only if there exists a symmetric matrix P such that (Chilali and Gahinet, 1996):

$$\begin{aligned} M_{\mathcal{D}}(A,P) &= \alpha \otimes P + \beta \otimes (AP) + \beta^T \otimes (AP)^T \\ &= [\alpha_{kl}P + \beta_{kl}AP + \beta_{lk}PA^T]_{1\le k,l\le m} \end{aligned} \tag{12}$$

with $M = [\mu_{kl}]_{1\le k,l\le n}$ means that M is an $n \times n$ matrix (respectively, bloc matrix) with generic entry (respectively bloc) μ_{kl}. Note that $M_{\mathcal{D}}(A,P)$ in (12) and $f_D(z)$ in (11) are related by the substitution $(P, AP, PA^T) \leftrightarrow (1, z, \bar{z})$. It is easily seen that $\mathcal{LMI}$ regions are convex and symmetric with respect to real axis. Specifically, the circular $\mathcal{LMI}$ region D is considered:

$$D = \{x + jy \in C : (x+q)^2 + y^2 < r^2\} \tag{13}$$

centered at $(-q, 0)$ with radius $r > 0$, where the characteristic function $f_D(z)$ is given by:

$$f_D(z) = \begin{pmatrix} -r & \bar{z}+q \\ z+q & -r \end{pmatrix} \tag{14}$$

Therefore, this circular region puts a lower bound on both the exponential decay rate and the damping ratio of the closed-loop response, and thus is very common in practical control design. It is obvious that well chosen $\mathcal{LMI}$ region is needed for ensuring stability and good results: the parameters q, r have to be defined by the engineer.

3.2 Control law synthesis in fault-free case

Let consider the state space representation (4) of nonlinear system defined around the Equilibrium Points $\mathcal{EP}_j$, $\forall j = 1, 2, .., N$

$$\begin{cases} x^j_{k+1} = A_j x^j_k + B^i_j (I-\gamma^a) u^j_k \\ y^j_k = C_j x^j_k \end{cases} \tag{15}$$

with $i = 1, 2, .., p$ the actuators for each $\mathcal{EP}_j$. Consider the matrix representing total faults in all actuators but the i-th:

$$B^i_j = [0, .., 0, b^i_j, 0, ..., 0] \tag{16}$$

and $B_j = [b^1_j, b^2_j, ..., b^p_j,]$ with $b^i_j \in \mathbb{R}^{n\times 1}$. It is assumed that each column of B_j is full column rank whatever the $\mathcal{EP}_j$. The pairs $(A_j, b^i_j), \forall i = 1, ..., p$ are assumed to be controllable for all $\forall j = 1, ..., N$. Let $\mathcal{D}$, a $\mathcal{LMI}$ region defining a disk with a center $(-q, 0)$, and a radius r with $(q + r) < 1$ for defining pole assignment in the unit circle. Assume that for each B^i_j, there exist matrices $X_i = X_i^T > 0$ and Y_i, $\forall j = 1, ..., N, \forall i = 1, 2, ..., p$ such as:

$$\begin{pmatrix} -rX_i & qX_i + (A_jX_i - B^i_jY_i)^T \\ qX_i + A_jX_i - B^i_jY_i & -rX_i \end{pmatrix} < 0 \tag{17}$$

It can be noticed that if $q = 0$ and $r = 1$, the previous equation (17) is equivalent to solve a classical quadratic stability problem.

Based on the assumptions that for each $\mathcal{OP}_j$ each pairs (A_j, b^i_j) are controllable, it is possible to find a Lyapunov matrices $X_i > 0$ and state-feedback K_i with $Y_i = K_i X_i$ and finally form a global state-feedback gain K_{nom}.

Theorem 1. Consider the system (15) in fault-free case ($\gamma^a = 0$) defined for all $\mathcal{EP}_j$, $j = 1, 2, .., N$: it is possible to develop a mixing of pre-designed state-feedback gains matrices $K_i = Y_i X_i^{-1}$ for each actuator i with $i = 1, 2, .., p$ such that (17) holds for all $j = 1, ..., N$. The state feedback control for each operating point is given by:

$$\begin{aligned} u_k^j &= u_{nom}^j = -(\sum_{i=1}^{p} G_i Y_i)(\sum_{i=1}^{p} X_i)^{-1} x_k^j \\ &= -Y X^{-1} x_k^j = -K_{nom} x_k^j \end{aligned} \quad (18)$$

with $\sum_{i=1}^{p} G_i Y_i = Y$, $X = \sum_{i=1}^{p} X_i$ and $G_i = B_j^{i+} B_j^i$ is matrix that has zeros everywhere except in entry (i, i) where it has a one. The general control law for all $\mathcal{EP}_j$ could be defined as:

$$U_{nom} = \sum_{j=1}^{N} \rho_k^j u_{nom}^j \quad (19)$$

□

Proof:
Summation of (17) for $i = 1, 2, .., p$ gives for one equilibrium point j

$$\sum_{i=1}^{p} \begin{pmatrix} -rX_i & qX_i + (A_j X_i - B_j^i Y_i)^T \\ qX_i + (A_j X_i - B_j^i Y_i) & -rX_i \end{pmatrix} < 0 \quad (20)$$

related to the quadratic $\mathcal{D}$-stability in a prescribed $\mathcal{LMI}$ region as in (Chilali and Gahinet, 1996). Next, denote $X = \sum_{i=1}^{p} X_i$ (with $X = X^T > 0$) to obtain

$$\begin{pmatrix} -rX & qX + (A_j X - \sum_{i=1}^{p} B_j^i Y_i)^T \\ qX + (A_j X - \sum_{i=1}^{p} B_j^i Y_i) & -rX \end{pmatrix} < 0 \quad (21)$$

Now, denote the l-th row of the matrix Y_i as Y_i^l, $i = 1, ..., p$ et $l = 1, .., p$, i.e.

$$Y_i^l = G_l Y_i \quad (22)$$

Therefore,

$$\sum_{i=1}^{p} B_j^i Y_i = \sum_{i=1}^{p} [0, .., 0, b_j^i, 0, ..., 0] Y_i^i = B_j \sum_{i=1}^{p} Y_i^i \quad (23)$$

leading to

$$\sum_{i=1}^{p} B_j^i Y_i = B_j (\sum_{i=1}^{p} G_i Y_i) \quad (24)$$

Thus, taking $Y = \sum_{i=1}^{p} G_i Y_i$, equation (24) becomes

$$\sum_{i=1}^{p} B_j^i Y_i = B_j Y \quad (25)$$

which, substituted into $\mathcal{LMI}$ (21), finally makes

$$\begin{pmatrix} -rX & qX + (A_j X - B_j Y)^T \\ qX + (A_j X - B_j Y) & -rX \end{pmatrix} < 0 \quad (26)$$

for $\mathcal{EP}_j$, $j = 1, 2, .., N$. By multiplying each $\mathcal{LMI}$ (26) by ρ_k^j and summing all of them, we obtain

$$\begin{pmatrix} -rX & qX + \sum_{j=1}^{N} \rho_k^j (A_j X - B_j Y)^T \\ qX + \sum_{j=1}^{N} \rho_k^j (A_j X - B_j Y) & -rX \end{pmatrix} < 0 \quad (27)$$

it is equivalent to

$$\begin{pmatrix} -rX & qX + (A(\rho) X - B(\rho) Y)^T \\ qX + (A(\rho) X - B(\rho) Y) & -rX \end{pmatrix} < 0 \quad (28)$$

with $A(\rho) = \sum_{j=1}^{n} \rho_k^j A_j$ and $B(\rho) = \sum_{j=1}^{n} \rho_k^j B_j$. Hence quadratic $\mathcal{D}$-stability is ensured by solving (27) and $Y = K_{nom} X$ quadratically stabilizes the system (15) by solving (28) with a state feedback law $u_{nom}^j = -Y X^{-1} x_k^j$. ◇

Remark1: It could be noticed that gain synthesis through multiple operating point with such $\mathcal{LMI}$ consideration provide only one single gain for all $\mathcal{OP}$ due to Bilinear Matrix inequality ($\mathcal{BMI}$) problem in term $(2, 1)$ of $\mathcal{LMI}$ (17). However, other system such piecewise linear system could use the same approach with a multiple gain synthesis as in (Ozkan *et al.*, 2003).

3.3 Active Fault Tolerant Control design

As indicated in equation (8) and based on the previous synthesis control law, the FTC method can be developed in this section where only actuator faults are considered under assumptions that fault occurrence and fault magnitude γ^a are known.

Theorem 2. Consider the system (15) in faulty case ($\gamma^a \neq 0$) coupled with regulators with gains $K_i = Y_i X_i^{-1}$ for all equilibrium point $j = 1, ..., N$ and for each actuator i with $i = 1, 2, .., p$. Let introduce the set of indexes of all actuators that are not completely lost, i.e.
$\Theta \triangleq \{i : i \in (1, 2, .., p), \gamma_i^a \neq 1\}$. The control action is

$$u_k^j = u_{FTC}^j = -(I - \gamma^a)^+ \big(\sum_{i \in \Theta} G_i Y_i \big(\sum_{i \in \Theta} X_i\big)^{-1} x_k^j\big) \quad (29)$$

where $G_i = B_j^{i+} B_j^i$, applied to the faulty system allows to constrain pole placement in prescribed $\mathcal{LMI}$ region. □

Proof: Applying the new control law (29) to the faulty system (15), leads to the following equation

$$B_j (I - \gamma^a) u_k^j = B_j \Gamma^a (\sum_{i \in \Theta} G_i Y_i)(\sum_{i \in \Theta} X_i)^{-1} x_k^j \quad (30)$$

with

$$\Gamma^a = \begin{pmatrix} I_{p-h} & 0 \\ 0 & O_h \end{pmatrix} \quad (31)$$

Γ^a is a diagonal matrix which contains only entries zero (representing total faults) and one (no fault). But here $h = 0$, which is the number of actuators completely lost, due to the fact that only the set Θ is considered. Since $B_j\Gamma^a = \sum_{i\in\Theta} B_i^j$ models only the actuators that are not completely lost, then performing the summations in the proof of Theorem (1) over the elements of Θ shows that $(\sum_{i=1}^p G_iY_i)(\sum_{i=1}^p X_i)^{-1}$ is the state-feedback gain matrix for the faulty system $(A_j, \sum_{i\in\Theta} B_i^j, C_j)$. $\diamond$

The control law in equation (29) implies that

$$u_k^j = u_{FTC}^j = -K_{FTC}x_k^j \tag{32}$$

with

$$K_{FTC} = (I-\gamma^a)^+\sum_{i\in\Theta} G_iY_i(\sum_{i\in\Theta} X_i)^{-1} \tag{33}$$

The global control law U_{FTC} of the system is realized as:

$$U_{FTC} = \sum_{j=1}^N \rho_k^j u_{FTC}^j \tag{34}$$

4. APPLICATION

4.1 Process description

The approach presented in this paper has been applied to the well known three tanks benchmark as in (Theilliol *et al.*, 2003*a*). As all the three liquid levels are measured by level sensors, the output vector Y is $[l_1 \quad l_2 \quad l_3]^T$. The control input vector is $U = [q_1 \quad q_2]^T$. The goal is to control the system around three equilibrium points with $\Delta x_j = 0$. Thus, 3 linear models have been identified around each of these equilibrium points and the operating conditions are given in Table (1). The linearized system is described by a discrete

Table 1.

Operating point	n°1	n°2	n°3
$y_e^j(m)$	0.20; 0.15; 0.175	0.50; 0.15; 0.325	0.50; 0.405; 0.45
u_e^j $(m^3/s)\times 10^{-5}$	1.7509; 4.0390	4.6324; 1.1574	2.4761; 6.9787

state space representation with a sampling period $Ts = 1s$. For each $\mathcal{OP}$, each control matrix pair (A_j, b_j^i) is controllable. Controllers have been designed for levels l_1 and l_2 to track reference input vector $Y^r \in \mathbb{R}^2$. Nominal controllers have been designed through Theorem (1), leading to two state feedback gain matrices K_1, K_2 (due to 2 actuators) for all the three $\mathcal{OP}$ in order to achieve satisfying tracking performances. The simulation of actuator faults on the system does not affect the controllability and observability of the system.

4.2 Results and comments

Simulations have been performed such as the 3 operating conditions described in Table (1) are reached and weighting functions for each local model are presented in figure (1) always close to the dynamic behaviour of the nonlinear system according to the considered operating regimes. Figure (2) shows the time history of the outputs

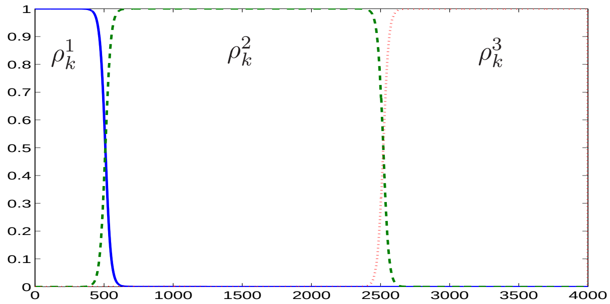

Fig. 1. Weighting functions

with respect to set-point changes occur at time instant $500s$ and after at time instant $2500s$. In the simulation, gaussian noises $(N(0, 1e^{-4^2}))$ are added to each output signal. The reference inputs correspond to step changes for l_1 and l_2. The

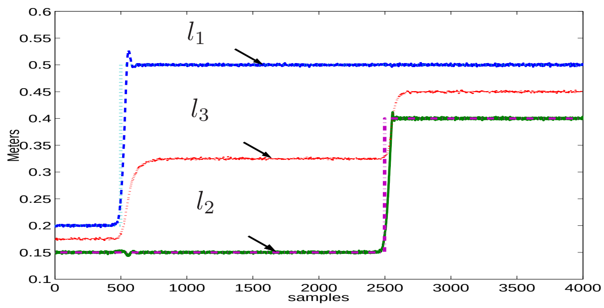

Fig. 2. System outputs in fault-free case with a nominal control law

consequence of an actuator fault is illustrated in figure (3). A gain degradation of pump 1 (clogged or rusty pump) equivalent to 80% loss of effectiveness is supposed to occur at time instant 1000s. Consequently, the dynamic behaviour of the other levels is also affected by this fault and control system tries to cancel the static error created by the corrupted input. Consequently, the real output is different from the reference input and the control law is different from its nominal value. Since an actuator fault acts on the system as a perturbation, and in spite of the presence of an integral controller, the system outputs can not reach again their nominal values. In the same

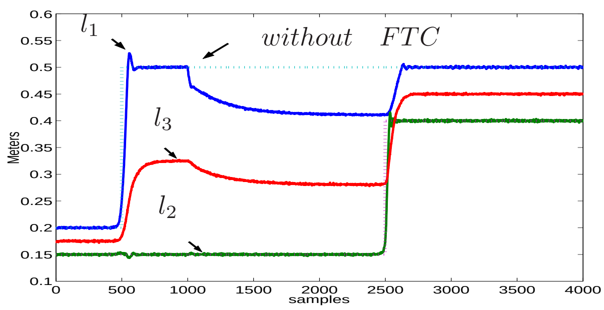

Fig. 3. System outputs with pump degradation and a nominal controller

way, the actuator Fault Tolerant Control method's ability to compensate faults is illustrated in the presence of the same fault. Once the fault is isolated and simultaneously estimated, a new control law (34) is computed in order to reduce the fault effect on the system. Indeed, since the effect of an actuator fault is quite similar to the effect of a perturbation, the system outputs reach again their nominal values, as illustrated in figure (4). A time delay between fault occurrence and fault compensation equals to 10 samples is considered in our simulation. Computation of the tracking

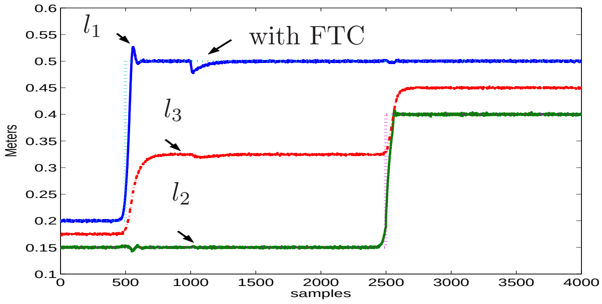

Fig. 4. System outputs with pump degradation and a FTC controller

error norm in fault-free case, in faulty case without and with FTC underlines the performances of this approach as seen in Table (2). With FTC, the tracking error norm for output l_1 is a bit larger than with the fault-free case but it still widely smaller than the one without FTC. The actuator Fault Tolerant Control is able to maintain performances as close as possible to nominal ones and to ensure the closed-loop stability despite the presence of instruments malfunction.

Table 2.

Error norm	Fault-free case	Actuator fault	
		without FTC	with FTC
e_{l1}	1.1063	3.3365	1.1176

5. CONCLUSION

The method developed in this paper emphasises the importance of the active Fault Tolerant Control of nonlinear systems based on multi-model representation. This method is suitable for actuator faults on the whole operating range of the system. A robust controller is designed for each separate actuator through an $\mathcal{LMI}$ pole placement in fault-free case and faulty case. It allows the system to continue operating safely, to avoid stopping it immediately and to ensure stability. The synthesis of this active state feedback control takes into account the information provided by FDI scheme. The performances and the effectiveness of this active Fault Tolerant Control based on multiple model approach have been illustrated in this simulation example. Futures works will deal with total failures, restructuration and will improve the limits of the strategy.

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