

# ROBUST CONTROL OF TIME DELAY SYSTEMS THROUGH A STRUCTURED UNCERTAINTY APPROACH

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**Abstract:** In this work  $\mu$ -Synthesis technique is utilized to design controllers for uncertain time-delay systems. For this purpose, delay elements are replaced by a linear uncertain block based on pade approximation of the delay elements. This will result in a linear system with nonlinear dependency on an uncertain parameter which lies in an infinite interval. Two general ideas are employed to cope with the difficulties associated with such an irregular dependency. These two ideas are general and can be applied in other cases. A design example and comparison with one of the newest control techniques is also provided. *Copyright 2005 IFAC.*

**Keywords:** Robust control, Structured singular value, Time delay, Uncertainty, Design.

## 1. INTRODUCTION

An important problem associated with robust control methods is the degree of their conservativeness. A general and less conservative technique for handling systems with parametric uncertainties is representing them in the form of systems with structured uncertainty. This representation allows use of well known  $\mu$ -analysis and  $\mu$ -synthesis techniques. Representation of a system appropriately for application of standard control methods is actually a matter of art, because the representation is usually not unique. In this work  $\mu$ -synthesis is employed for control of time-delay systems with uncertainties in delay times and in other parts of system. All delay elements of the system are replaced by a linear fractional transformation (LFT) model based on Pade approximation and accounting for the approximation error. As will be seen, a suitable representation for application of  $\mu$ -synthesis in this case requires coping with two important difficulties. First, treatment of nonlinear dependency of a transfer function to some uncertain parameters. Second, an infinite domain of

changes for an uncertain parameter that is not a barrier for design of a robust controller, but due to weakness of the available method for handling infinite intervals a complementary idea is necessary. These two problems are not restricted to control of pade approximated time-delay systems. They are general problems that can arise in finding suitable representations of many systems for application of several robust control methods, specially  $\mu$ -analysis and  $\mu$ -synthesis.

Control of time delay systems is a very practical control problem that has attracted much interest over the past decades and especially in recent years. The problem is stated in various forms for very different cases which include new works employing advanced theoretical frameworks like infinite dimensional systems in (Azuma et al., 2003) and other methods like the early idea of smith predictors which is still widely used and under research (Zhang et al., 2002; Wang et al., 1994). There are also very different types of delay systems in industrial applications where dedicated works is being done to find better control solutions for them, an example is tele-

operation systems over TCP/IP or UDP/IP computer networks (Shiotsuki et al., 2002). A very important body of research in the field of time-delay systems is based on Lyapunov's second method and using Lyapunov-Krasowskii functionals (Kolmanovskii et al., 1999). Among this large category a new and improved idea is introduced in (Fridman et al., 2001) which provides LMIs for analysis and design and is reported to achieve superior results with respect to other methods (Gao et al., 2003). The results of this work (Fridman et al., 2001) are chosen for a comparison with the results in the current paper where it is shown that in some cases better control solutions can be achieved by using the method presented here.

The main advantages of the control solution achieved in the current work includes: First; the designed controllers is an output feedback controller (not state feedback). Second, the controller is composed only of rational transfer functions (no memory). Third, the proposed method can handle any delay element at every part of the system and with any interval for the uncertain delay time. Forth, software tools for control design are widely available, for example in  $\mu$ -Analysis and Synthesis toolbox of Matlab<sup>®</sup>. Other methods usually don't have these advantages together. For example (Fridman et al., 2001) at least lacks the first one. The drawback of the method in current work with respect to some other methods like that in (Fridman et al., 2001) is the computational complexity of  $\mu$ -design method with respect to LMI technique. However as mentioned before finding new solutions for control of delayed systems is not the only aim of the current work. Coping with the two general representation problems mentioned before are also results of this paper.

The paper is organized as the following, in section 2 some basic ideas about system representation are described. In section 3 the proposed state-space model for replacing a time delay element is derived. Section 4 contains a design example followed by a comparison and conclusions are presented at the end.

## 2. PRELIMINARIES

In this section, two ideas are presented that are utilized in the next section and reviewing them before their application can be helpful.

### 2.1. Block expansion for better use of information about uncertainty

Consider a system with an uncertain block as shown in the figure 1-a. Despite of increment in the uncertainty dimensions, decomposing  $\Delta$  as  $\Delta = \Delta_1 + \Delta_2$  in figure 1-b can be helpful for reducing conservativeness if new bounds on  $H_\infty$  norm of  $\Delta_1$  and  $\Delta_2$  contain more information than the bound on

on  $H_\infty$  norm of  $\Delta$ . In the next section (during subsection 3.2) an uncertainty block will be decomposed in this way to increase efficiency of control design.

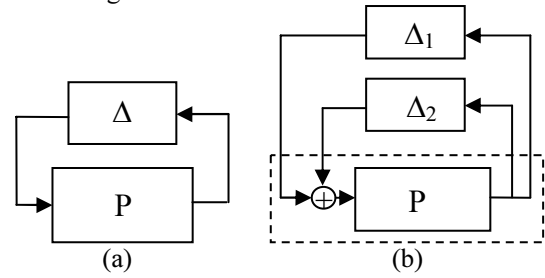


Fig.1. a: original system b: expanded system

*Example 1:* consider the values

$$\Delta_1 = \begin{bmatrix} 0.1 & 0.2 \\ 0 & 1 \end{bmatrix} \delta_1 \quad \Delta_2 = \begin{bmatrix} 0 & 0.3 \\ 0.5 & 0 \end{bmatrix} \delta_2$$

$$P = \begin{bmatrix} 0.3 & 0.1 & -1 \\ 0 & 0.4 & 1 \\ 1 & 0 & 0 \end{bmatrix} \quad (-1 < \delta_1, \delta_2 < 1)$$

where  $\Delta = \Delta_1 + \Delta_2$  connects first two outputs of constant transfer function  $P$  to its first two inputs and we are going to obtain an upper bound on the gain from third input to third output in presence of  $\Delta$ . Without expansion we have  $\|\Delta\| < 1.52$  which result in an upper bound of about 2.88, but application of expansion idea results in an upper bound of about 0.8 which contains more information.

### 2.2. Uncertain parameters with infinite domain of changes

Consider for example the transfer function  $G(s) = H(s) \times a/(s+a)$  where  $a$  belongs to the interval  $[a_0, +\infty)$  and  $H(s)$  is a known transfer function. If  $a_0$  is sufficiently large then the uncertainty in  $G(s)$  can be negligible but its parametric analysis through standard  $H_\infty$  methods encounters problems because center of uncertainty and its norm will become infinite. To solve this problem it can be considered that when  $a$  in  $G(s)$  is large, the uncertainty is in fact in higher regions of frequency domain. In other word the value of uncertain pole is large and is not important. This can provide the idea for handling this type of uncertainty.

Now consider the general case for a transfer function with one uncertain parameter  $G(s, a)$  where the following limit exists.

$$G_f(s) = \lim_{a \rightarrow \infty} G(s, a)$$

(Generalizations to the case of several uncertain parameters can be made, but this is not needed and is ignored here).  $G(s, a)$  can now be presented as

$$\begin{aligned}
G(s, a) &= G(s, \bar{a}) + \Delta_f(s, a) \\
a \in [a_0, +\infty) \quad \bar{a} \in [a_0, a_1] \\
\bar{a} &= \min(a, a_1) \\
\Delta_f(s, a) &= \begin{cases} 0 & a \in [a_0, a_1] \\ G(s, a) - G(s, a_1) & a \in [a_1, +\infty) \end{cases} \quad (1) \\
\|\Delta_f(j\omega, a)\| &< \Delta_{fb}(\omega) = \sup_{a \in [a_1, +\infty)} \|\Delta_f(j\omega, a)\|
\end{aligned}$$

where  $\Delta_f(s)$  is an uncertainty in high frequency region provided that  $a_1$  is sufficiently large. Usually we should have  $\Delta_{fb}(\omega) = \|G(j\omega, a_1) - G_f(j\omega)\|$  if the supremum in calculation of  $\Delta_{fb}$  is not in an extremum with respect to  $a$  and occurs on limits of interval  $[a_1, +\infty)$ . Therefore  $G(s, a)$  can be replaced by an equivalent system with bounded uncertainties as in figure 2 which allows application of known  $H_\infty$  methods. In this figure  $W_f(s)$  is a high-pass filter where its magnitude is an upper bound for  $\Delta_{fb}(\omega)$ . The tighter is this bound the less conservative will be the replacement. The resulting bound on  $\Delta_f$  is  $\|\Delta_f\| < 1$ .

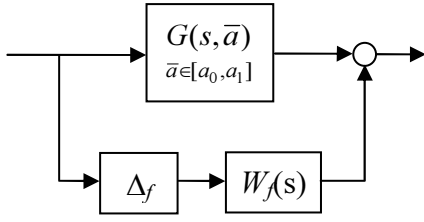


Fig.2. An equivalent system for  $G(s, a)$ .

An idea for selecting  $a_1$  can be overthrowing cut-off frequency of  $W_f(s)$  beyond the bandwidth of system to make the effect of  $\Delta_f$  negligible. It should be mentioned that the way of using filters presented here is essentially different from common use of filters like in frequency weighting or other purposes.

### 3. REPLACING TIME DELAY ELEMENTS BASED ON PADE APPROXIMATION

The exact form of  $m$ th order Pade transfer function for approximating a time delay of value  $\tau$  or in fact  $\exp(-s\tau)$  is shown in (2). Based on this approximation a state-space model will be derived for a delay element in this part which is suitable for control design.

$$\begin{aligned}
P(s, \tau) &= \frac{\sum_{k=0}^m \frac{(-s\tau)^k}{k!}}{\sum_{k=0}^m \frac{(+s\tau)^k}{k!}} = \left(a = \frac{1}{\tau}\right) \quad (2) \\
\frac{s^m - ams^{m-1} + \dots + (-a)^{m-k} (m! / k!) s^k + \dots + (-a)^n n!}{s^m + ams^{m-1} + \dots + (+a)^{m-k} (m! / k!) s^k + \dots + (+a)^n n!}
\end{aligned}$$

### 3.1. Approximate state space model for uncertain time delay

Consider an uncertain proper transfer function

$$\frac{(b_{n-1} + \beta_{n-1})s^{n-1} + \dots + (b_1 + \beta_1)s^1 + (b_0 + \beta_0)}{s^n + (a_{n-1} + \alpha_{n-1})s^{n-1} + \dots + (a_1 + \alpha_1)s^1 + (a_0 + \alpha_0)}$$

where  $a_i$  and  $b_i$  are known parameters and  $\alpha_i$  and  $\beta_i$  are uncertain parameters. This transfer function can be presented in state-space form with  $2n+1$  inputs and outputs where one input-output pair is for transfer function's input and output and two sets of  $n$  input-output pairs are for connection with two real diagonal uncertainty blocks  $\text{diag}(\alpha_0, \alpha_1, \dots, \alpha_{n-1})$  and  $\text{diag}(\beta_0, \beta_1, \dots, \beta_{n-1})$  and dimension of total structured uncertainty block will be  $2n$ . In an all-pass transfer function like Pade approximation of order  $m$

$$P(s) = \frac{P_0(s^2) - sP_1(s^2)}{P_0(s^2) + sP_1(s^2)}$$

where  $P_0$  and  $P_1$  are polynomial functions (with uncertainty in coefficients), dimensions of structured uncertainty block are reduced. It can be shown that a Pade transfer function can be represented in an alternative form of

$$P(s) = (-1)^m \left( 1 - \frac{2Q(s)}{s^m + Q(s) + R(s)} \right) \quad (3)$$

$$\begin{aligned}
Q(s) &= [q_0 \quad q_1 \quad \dots \quad q_{m-1}] \bar{s}^T = \bar{q} \bar{s}^T \\
R(s) &= [r_0 \quad r_1 \quad \dots \quad r_{m-1}] \bar{s}^T = \bar{r} \bar{s}^T \\
\bar{s} &= [1 \quad s \quad \dots \quad s^{m-1}]
\end{aligned}$$

where some elements of  $\bar{p}$  and  $\bar{q}$  are zero and total count of nonzero elements in  $\bar{r}$  and  $\bar{q}$  is  $m$ . A state-space realization for  $P(s)$  can be derived as

$$\begin{aligned}
\dot{x} &= Ax + B_u u \\
y &= C_y x + D_{yu} u
\end{aligned}$$

$$\begin{aligned}
A &= \begin{bmatrix} 0_{m-1 \times 1} & I_{m-1 \times m-1} \\ -\bar{q} - \bar{r} & \end{bmatrix} & B_u &= \begin{bmatrix} 0_{m-1 \times 1} \\ 1 \end{bmatrix} \\
C_y &= -2(-1)^m \bar{q} & D_{yu} &= (-1)^m
\end{aligned}$$

allowing for uncertainty in nonzero elements of  $\bar{r}$  and  $\bar{q}$ , pushing uncertain parameters out of main model in a feedback path from a new output  $z$ , through a diagonal real block of uncertain parameters  $\Delta_p$  to a new input  $w$ , the above state space realization can be extended to

$$\begin{aligned}
\dot{x} &= Ax + B_u u + B_w w \\
y &= C_y x + D_{yu} u + D_{yw} w \\
z &= C_z x
\end{aligned} \quad (4-1)$$

$$w = \Delta_p z \quad , \quad \Delta_p = \text{diag}(\delta_{p_1}, \dots, \delta_{p_m}) \quad (4-2)$$

$$B_w = \begin{bmatrix} 0_{m-1 \times 1} \\ 1 \end{bmatrix} [1_{1 \times m}] = B_u 1_{1 \times m} \quad (4-3)$$

$$C_z = I_{m \times m} \quad , \quad D_{yw} = -2 \times \text{sign}(\bar{q}^T)$$

where in (4-3) “sign” is element-wise sign function and “1” means a matrix with all of its elements equal to one.

### 3.2. Improvement in uncertainty handling

The state-space model (4) is derived to handle uncertainty in coefficient values  $(m!/k!)a^{m-k}$ ,  $k=1\dots m$  in equation (2) independently. However these uncertainties all depend on uncertainty in a single value  $a$  which is assumed to lie in a bounded interval. This dependency is nonlinear and can not be handled directly. However any of the coefficients can be decomposed to sum of a term with linear dependency on  $\delta_a$  (deviation of  $a$  from its central or mean value) and another term with nonlinear dependency on  $a$  but with minimized largest absolute value in the domain of changes for  $a$  so that

$$\begin{aligned} \delta_{p_i} &= b_i \delta_a + \delta_{r_i} \\ \Delta_p &= \delta_a \Lambda_b + \Delta_r \\ \Lambda_b &= \text{diag}(b_1, b_2, \dots, b_m) \\ \Delta_r &= \text{diag}(\delta_{r_1}, \delta_{r_2}, \dots, \delta_{r_m}) \end{aligned} \quad (5)$$

According to subsection 2.1, expansion of  $m \times m$  uncertainty block  $\Delta_p$  to a  $2m \times 2m$  block diagonal uncertainty composed of  $\delta_a \Lambda_b$  and  $\Delta_r$  will provide a better condition for handling uncertainty. Now we are going to show that due to special structure of state space model in (4) the dimensions of new uncertainty block can be decreased. Combination of (5) with (4-1,2,3) results in

$$\begin{aligned} \dot{x} &= Ax + B_u u + B_u 1_{1 \times m} (\delta_a \Lambda_b + \Delta_r) C_z x \\ y &= C_y x + D_{yu} u + D_{yw} (\delta_a \Lambda_b + \Delta_r) C_z x \end{aligned} \Rightarrow$$

$$\begin{aligned} \dot{x} &= Ax + B_u u + B_u \delta_a 1_{1 \times m} \Lambda_b C_z x + B_u 1_{1 \times m} \Delta_r C_z x \\ y &= C_y x + D_{yu} u + \delta_a D_{yw} \Lambda_b C_z x + \Delta_r C_z x \end{aligned} \Rightarrow$$

$$\begin{aligned} \dot{x} &= Ax + B_u u + \begin{bmatrix} B_u 1_{1 \times m+1} & 0_{m \times 1} \end{bmatrix} \times \Delta_e \times \\ & \quad \begin{bmatrix} I_{m \times m} & \Lambda_b 1_{m \times 1} & 0_{m \times 1} \end{bmatrix}^T C_z x \\ y &= C_y x + D_{yu} u + \begin{bmatrix} D_{yw} & 0 & 1 \end{bmatrix} \times \Delta_e \times \\ & \quad \begin{bmatrix} I_{m \times m} & \Lambda_b 1_{m \times 1} & 0_{m \times 1} \end{bmatrix}^T C_z x \end{aligned}$$

$$\Delta_e = \begin{bmatrix} \Delta_r & 0_{m \times 1} & 0_{m \times 1} \\ 0_{1 \times m} & \delta_a & 0 \\ 0_{1 \times m} & 0 & \delta_a \end{bmatrix}$$

This later set of equations can be represented as

$$\begin{aligned} \dot{x} &= Ax + B_u u + B_{w_1} w_1 \\ y &= C_y x + D_{yu} u + D_{yw_1} w_1 \\ z_1 &= C_{z_1} x \end{aligned} \quad (6-1)$$

$$w_1 = \Delta_e z_1 \quad (6-2)$$

$$\begin{aligned} B_{w_1} &= [B_u 1_{1 \times m+1} \quad 0_{m \times 1}] \\ C_{z_1} &= [I_{m \times m} \quad \Lambda_b 1_{m \times 1} \quad 0_{m \times 1}]^T C_z \\ D_{yw_1} &= [D_{yw} \quad 0 \quad 1] \end{aligned} \quad (6-3)$$

Therefore dimensions of new uncertainty block  $\Delta_e$  are reduced to  $m+2 \times m+2$ . When  $m=1$ , in (2) becomes an empty matrix and this makes possible to reduce dimensions of  $\Delta_e$  from  $3 \times 3$  to  $2 \times 2$ . Summarizing the result of this subsection, equations in (6) present a state-space realization for time delay and contain extra input  $w_1$  and output  $z_1$  suited for less conservative modeling of uncertainty in delay time  $\tau$  ( $=1/a$ ) as a real diagonal block from  $z_1$  to  $w_1$ . Matrices  $B_{w_1}$  or  $C_{z_1}$  can be normalized so that each diagonal element of uncertainty block lies in  $[-1, +1]$ . This can be done for example by writing  $\Delta_e = \Lambda_e \Delta_{en} = \Delta_{en} \Lambda_e$  where diagonal matrix  $\Delta_{en}$  contains the normalized elements and  $\Lambda_e$  is a diagonal matrix of constant coefficients that should be absorbed  $B_{w_1}$  in  $C_{z_1}$  or (or both). In the following it is assumed that this normalization is performed on model (6).

### 3.3. Modeling approximation error as uncertainty

It is known that  $m$ th order Pade approximation  $P_m(s)$  deviates from delay transfer function  $\exp(-s\tau)$  as frequency increases. In other word the approximation error transfer function  $E_m(s) = \exp(-s\tau) - P_m(s)$  has a high-pass nature. A good idea for modeling approximation error by uncertainty is the solution of figure (3). Weighting function  $W_d(s)$  is a high-pass filter where its magnitude is an upper bound for magnitude of  $E_m(s)$  over all frequencies. A tighter bound can reduce conservativeness. By increasing the order of Pade approximation  $m$ , cut-off frequency of  $E_m(s)$  and  $W_d(s)$  can be increased. Intuitively, if the cut-off frequency of  $W_d(s)$  becomes larger than control system bandwidth, then it can be stated that effect of approximation error is negligible because it belongs to regions outside of working bandwidth. It should be clear that a bound on  $\Delta_d$  is  $\|\Delta_d\| < 1$ .

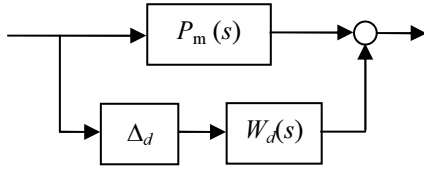


Fig.3. modeling approximation error by uncertainty

### 3.4. Boundedness of uncertainties

A delay interval of  $\tau \in (0, T]$  results in an infinite domain of changes for  $a$ , namely  $a \in [1/T, +\infty)$ . This causes problems in parametric analysis of uncertainty through standard  $H_\infty$  methods. A solution for this difficulty is extending method in subsection 2-2 to contain the idea in subsection 3-3. Denoting  $m$ th order Pade approximation of time delay  $1/a$  by  $P_m(s, a)$ . This can be done by writing

$$P_m(s, a) = P_m(s, \bar{a}) + \Delta_f(s, a)$$

$$a \in [\frac{1}{T}, +\infty) \quad \bar{a} \in [\frac{1}{T}, \frac{1}{T_1}]$$

$$\bar{a} = \min(a, \frac{1}{T_1})$$

$$\Delta_f(s, a) = \begin{cases} \exp(-\frac{s}{a}) - P_m(s, a) & a \in [\frac{1}{T}, \frac{1}{T_1}] \\ \exp(-\frac{s}{a}) - P_m(s, \frac{1}{T_1}) & a \in [\frac{1}{T_1}, +\infty) \end{cases}$$

or equivalently

$$\Delta_f(s, a) = \exp(-\frac{s}{a}) - P_m\left(s, \min(a, \frac{1}{T_1})\right)$$

$$\|\Delta_f(j\omega, a)\| < \Delta_{fb}(\omega) = \sup_{a \in [\frac{1}{T}, +\infty)} \|\Delta_f(j\omega, a)\|$$

In comparison to systems of figures 2 and 3, an uncertain transfer function with bounded uncertainty that can replace time delay is shown in figure 4. Again  $W_f(s)$  is a high-pass filter and its magnitude should be an upper bound for  $\Delta_{fb}(\omega)$ . A tighter bound results in a less conservative model. It should be indicated that cut-off frequency of  $W_f(s)$  depends on the value of  $T_1$ . Smaller values of  $T_1$  result in larger cut-off frequencies. It is possible to calculate first  $W_d(s)$  in subsection 3-3 and then to select  $T_1$  such that magnitude of  $W_d(s)$  becomes an upper bound for  $\Delta_{fb}(\omega)$ . In this manner the resulting  $W_d(s)$  is a choice for  $W_f(s)$ .

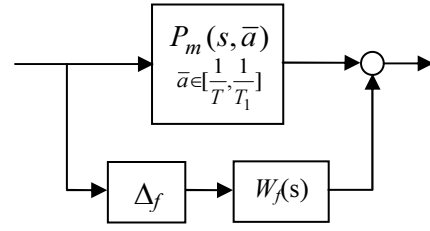


Fig.4. Replacement for time delay with bounded domain of parameter changes

### 3.5. State-space model of uncertain time delay for replacing delay elements

Combining the results of subsections 3-2 and 3-4, the state space realization (6) should be used for  $P_m$  in figure 4. The result will be an LFT that can replace time delay elements anywhere in an interconnection of LTI systems. For example in figure 5 the idea is applied to a series connection of a time delay with an uncertain rational transfer function. New uncertainty blocks like  $\Delta_m$  in figure 5 should be augmented to those of delay elements (one or multiple  $\Delta_e$  and  $\Delta_f$  blocks) to form the overall structured uncertainty block.  $\mu$ -synthesis control design method can be used for the resulting state space model.

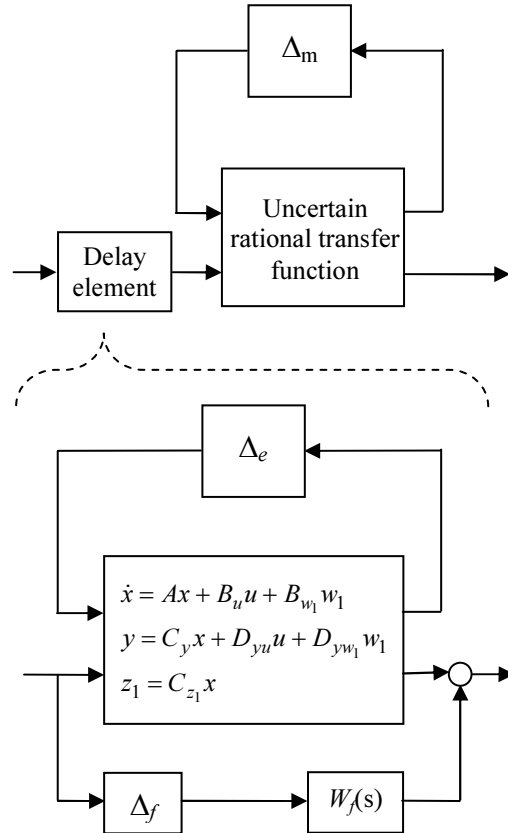


Fig.5. Application of resulting model for time delay 4. A DESIGN EXAMPLE AND COMPARISON

In this subsection control design for an example uncertain system with the following transfer function is considered where  $|\delta_1| < 1$ ,  $|\delta_2| < 1$  and delay time  $\tau$  is between 0 and 1.

$$G(s) = \frac{2.07s + 2.4 + \delta_1(0.065s + 0.037)}{s^2 + 1.06s - 0.072 + \delta_2(0.15s + 0.15)} e^{-\tau s}$$

using first order Pade approximation and applying method of subsection 3-4 an uncertain state space model is calculated. A  $W_f(s)$  filter is designed as below and a value of 0.4 for  $T_1$  is achieved.

$$W_f(s) = \frac{6.04s}{s + 14.57}$$

Connection of the obtained state space model with a realization for rational part of  $G(s)$  results in a block composition of type that is shown in figure (5). Robust stability control design using Matlab  $\mu$ -synthesis toolbox for the total system results in an upper bound for structured singular value  $\mu < 0.805$  which guarantees system stability subjected to the uncertainties.

Now we are going to design a controller by using the method of (Fridman et al., 2001) for comparison. Although the method of (Fridman et al., 2001) is for state feedback design (the  $\mu$ -synthesis used here has the important advantage that it is an output feedback design method), but a comparison will show effectiveness of the method presented in this paper.

To design a controller based on the method of (Fridman et al., 2001) a delayed state feedback (part III-B) with delay time  $\tau=1$  is designed for rational part of  $G(s)$  which have resulted in an upper bound for system gain of  $g=1.89$  viewed from input and output connections from  $G(s)$  to uncertainty block  $\text{diag}(\delta_1, \delta_2)$ . This result can not guarantee system's stability in presence of uncertainty.

For a direct comparison of our method with above result of  $g=1.89$ , system's structured singular value in our method can be calculated when structured uncertainty block  $\text{diag}(\delta_1, \delta_2)$  is replaced by a complex unstructured  $2 \times 2$  block with its norm limited to one. This results in a value of 1.29 which is still less than  $g=1.89$  and shows strictly the advantage of method in this paper for this example. However other examples were found that method of (Fridman et al., 2001) can generate better results. The conditions under which one of methods generate better results depends on nature of both methods and is not still completely clear for the authors, but it is a question for further research.

## 5. CONCLUSIONS

An Linear fractional transformation (LFT) uncertain model was presented to replace uncertain time delay elements in a system. The resulting model is suitable for application of techniques like  $\mu$ -synthesis. Derivation of the LFT model requires application of some additional ideas including: First; uncertainty block expansion for reducing conservativeness. Second, a solution for handling uncertain parameters belonging to infinite intervals. Some way of reducing uncertainty block dimensions was also applied. These ideas are related to general problems and can be applied in many other cases. The LFT model of uncertain time delay was utilized for control design using  $\mu$ -synthesis method and comparison was made to show the efficiency of method. Advantages and drawbacks of the presented method where also reviewed in the introduction section.

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