AUTONOMOUS TRACTOR-TRAILER BACK-UP MANOEUVERING BASED ON CHANGING TRAILER ORIENTATION

F. Gómez-Bravo*, F. Cuesta and A. Ollero

Departamento de Ingeniería de Sistemas y Automática. Escuela Superior de Ingenieros.
University of Seville.Camino de los Descubrimientos, 41092 Sevilla (Spain)
E-mail: {fede,aollero}@cartuja.us.es

**Departamento de Ingeniería Electrónica, Sistemas Informáticos y Automática. Escuela*
Politécnica Superior, Universidad de Huelva. Carretera de Palos-La Rábida s/n. 21071
Huelva (Spain). E-mail: fernando.gomez@diesia.uhu.es

Abstract: Manoeuvring tractor-trailer vehicles is not a trivial task. This paper presents a method for planning manoeuvres with simple velocity and steering profiles. The method can be easily used in autonomous vehicles and also in a human driver assistance system. A manoeuvre for changing trailer orientation is presented in detail. The practical application of the proposed method has been tested in ROMEO4R autonomous vehicle performing an autonomous parallel parking. *Copyright © 2005 IFAC.*

Keywords: Mobile Robots, Autonomous Parking, Nonholonomic Systems, Tractor-trailer.

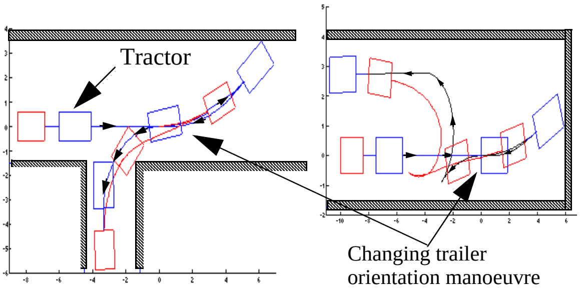
1. INTRODUCTION

Backward manoeuvring with a tractor-trailer system is much more difficult than doing just with a tractor (a car, for instance) both for a human driver or for an autonomous planning and motion control system. The main reason is that for backing up motion the tractor-trailer is an underactuated and unstable system (Lamiraux et al., 1999; Divelbiss and Wen, 1997). In this situation, relative orientation between the tractor vehicle and the trailer plays an important role on planning and motion control of the tractor-trailer systems. Depending on the value of this angle, saturation and uncontrollable situations can appear during backward motion producing the well known “jackknife”.

The tractor-trailer system is also an example of nonholonomic system. Several approaches for path-planning and motion control of these systems have been presented in last ten years (Murray and Sastry, 1993; Lamiraux and Laumont, 1998; Tilbury, 1994; Tilbury et al., 1995; Lamiraux et al., 1999; Divelbiss and Wen, 1997). These approaches have shown good performance for manoeuvring tractor-trailer systems from a tractor position and trailer orientation to another one. However, their performance is lower when they are applied to simpler situations such as just changing only the trailer orientation. Moreover, the application of the aforementioned methods to a human driver assistance system can be quite difficult because the required curvature and velocity profiles are not trivial to be performed by the driver.

The method presented in this paper focuses on simple manoeuvres, in such a way that it could be also easily integrated in a human driver assistance system (Wada et al., 2003). The proposed technique is based on the concept of *restricted manoeuvre* (Gómez-Bravo et al., 2001) which combines canonical manoeuvres and piece-wise path generation, and deals with geometrical constraints easily. Different restricted manoeuvres, with simple steering action, can be designed for changing just the position (x or y), the tractor orientation or the tractor-trailer relative orientation. Moreover, they can be concatenated in order to perform more complex manoeuvres such as an autonomous parallel parking of a tractor-trailer (Cuesta et al., 2004). Due to their simplicity, they can be easily implemented in a hardware with low computational capability like FPGA, microcontrollers, etc.

This paper presents in detail the restricted manoeuvre for changing trailer orientation. It should be stressed that this manoeuvre is very useful for many applications. Fig.1 presents two examples of tractor-trailer manoeuvres in which the change of the trailer orientation allows the vehicle to accomplish properly the navigation task. These situations can appear, for instance, when a reactive navigation strategy is applied: since the environment constraints are not previously known a dead-end or narrow corridor could be found while the tractor-trailer navigates following a wall. The manoeuvre for changing trailer orientation can be combined with a conventional navigation method (to perform reorientation when it is required) or with another restricted manoeuvres (to perform an autonomous parallel parking, for instance).



a) b)

Fig. 1: a) Backing up in narrow corridors; b) Dead-end corridor

The paper is organized as follows: in Section 2 the tractor-trailer kinematic model is introduced. Section 3, presents an overview of different methods for planning tractor-trailer manoeuvres and a comparison with the proposed one. Section 4 is devoted to present the planning scheme which provides manoeuvres for changing the trailer orientation. Section 5 introduces experimental results showing the proposed planning techniques in practice. The paper closes with the conclusions and references.

2. TRACTOR-TRAILER SYSTEM

Tractor-Trailer vehicles consist of a tractor vehicle (similar to a car or cart vehicle) towing a two-wheeled trailer, as illustrated in Fig.2, (Latombe, 1991). The kinematics of these vehicles is usually characterized by the following nonholonomic constraints

$$\begin{aligned} \dot{x}\sin\theta - \dot{y}\cos\theta &= 0 \\ \dot{x}_t\sin\beta - \dot{y}_t\cos\beta &= 0 \end{aligned} \qquad (1)$$

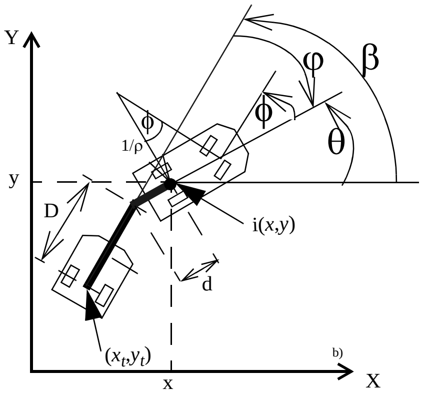


Fig. 2: Tractor-Trailer system

where x and y are the coordinates of point i (the midpoint between the two rear wheels of the tractor), x_t and y_t are the coordinates of the midpoint between the two wheels of the trailer, θ is the orientation of the tractor and β the orientation of the trailer. If φ is the relative orientation of the tractor and the trailer, it is defined as

$$\varphi = \theta - \beta \ . \qquad (2)$$

Then, by means of transformations on (x_t, y_t), the state vector can be represented by $[x, y, \theta, \varphi]$ and the two nonholonomic constraints of Eq. (1) can be written as

$$\begin{bmatrix} -\sin\theta & \cos\theta & 0 & 0 \\ \cos\theta\sin\varphi & \sin\theta\sin\varphi & -D-d\cos\varphi & D \end{bmatrix} \begin{bmatrix} \dot{x} \\ \dot{y} \\ \dot{\theta} \\ \dot{\varphi} \end{bmatrix} = 0\,, \qquad (3)$$

where d is the distance between i and the hooking point and D is the distance between the trailer wheels and the hooking device (see Fig.2).

The kinematic model of this system is

$$\begin{bmatrix} \dot{x} \\ \dot{y} \\ \dot{\theta} \\ \dot{\varphi} \end{bmatrix} = \begin{bmatrix} \cos\theta & 0 \\ \sin\theta & 0 \\ 0 & 1 \\ -\frac{\sin(\varphi)}{D} & \frac{\cos(\varphi)d}{D} + 1 \end{bmatrix} \begin{bmatrix} v(t) \\ v(t)\rho(t) \end{bmatrix} . \qquad (4)$$

The control vector of this model is $[v(t),\ v(t)\rho(t)]^T$, where $v(t)$ is the linear velocity of point i, and $\rho(t)$ the curvature of the path depicted by point i.

Thus, vectors generated by (4) and satisfying (3) can be used in order to connect configurations by a feasible path.

3. VEHICLE’S MANOEUVRING

3.1 Nonholonomic Constraints

Nonholonomic systems are characterized by nonholonomic constraints, i.e. nonintegrable differential expressions involving velocity and state vectors. These constraints can be found in different kind of systems: wheeled vehicles, due to non slipping condition, finger contact in robot hand grasping, due to rolling contact constraint, free floating robot manipulators, due to angular momentum conservation, etc.

Controllability studies of nonholonomic systems (Laumont, et al., 1994; Latombe, 1991) show that, if the system is fully controllable, any two configurations can be connected by a feasible path, generated by a finite sequence of vectors fields. Based on this property different approaches have been developed by using piece-wise path generation (Laumont et al., 1998; Fortune et al., 1998; Latombe, 1991).

In this paper, manoeuvre design is based on the concept of *restricted manoeuvre* (Gómez-Bravo et al., 2001) which combines both concepts, canonical path and piece-wise path generation. A restricted manoeuvre represents a special case of canonical path in which some variables (independent variables) follow a closed trajectory, meanwhile, other variables (dependent variables) follow up a trajectory which provides a desired increment. Even more, restricted manoeuvres are built up by a sequence of control vectors generated by a set of parametrized piece-wise constant inputs based on heuristic considerations.

3.2 Tractor-trailer Manoeuvring

Several approaches have been proposed for manoeuvring tractor-trailer systems. In Murray and Sastry (1990) a control law based on sinusoidal inputs is presented. In Lamiraux and Laumont (1998) a virtual robot during backward movement is used and a manoeuvre's generation method, based on the concept of canonical paths, is presented. In Tilbury (1994), Tilbury et al. (1995), and Sekhavat and Laumond (1998), the kinematic model is transformed into a chained form and sinusoidal inputs are also considered. In all these methods the computed manoeuvre has to be tested in order to ensure that it is collision-free. If not, a new manoeuvre has to be computed.

Neither of the above refrences pay special attention to changing trailer orientation. As a consequence they can exhibit a poor performance when applied to perform just a trailer reorientation. For instance, Fig.3 presents some of the best results obtained from the above methods when they are applied to obtain this type of manoeuvres. Notice that, even if the spatial requirement of these examples are similar to the ones developed in this article (as will be shown later), the shape of the manoeuvres presents many curvature changes (what can be difficult to be performed by a human driver). Indeed, it depends tremendously on the values of the parameters involved in the generation procedure (Lamiraux et al., 1999; Tilbury, 1994). Moreover, the curvature profile is not as simple as the one of the proposed manoeuvre (as will be shown in the next sections). Even more, since the non existence of collisions in a manoeuvre is evaluated by using an algorithm after the computation of such manoeuvre, the manoeuvre generation procedure in constrained environments can be a difficult iterative task. These reasons motivate the development of a new approach for generating this kind of manoeuvres.

Restricted manoeuvres in car like vehicles (without trailer) have been previously reported in Gómez-Bravo et al. (2001), where manoeuvres for changing the value of each variable, i.e., x, y and θ were presented.

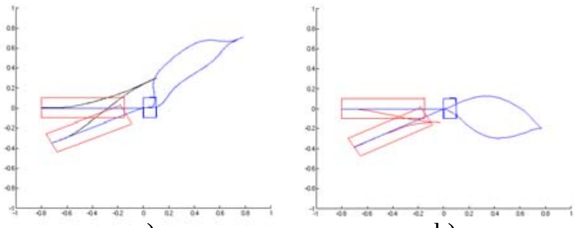

a) b)

Fig. 3: Changing trailer orientation: a) Canonical path; b) Sinusoidal input

Tractor-trailer vehicles are composed of two nonholonomic vehicles. The concept of restricted manoeuvre can be applied to this type of system. However, restricted manoeuvres designed for car vehicles fail when they are applied to tractor-trailer systems because trailer motion was not taken into account.

Nevertheless, one of the advantages of the restricted manoeuvres is that they can be concatenated in order to perform more complex manoeuvres. Thus, restricted manoeuvres for changing only the value of x, y, θ or φ in a tractor-trailer system can be obtained by composing a sequence of two manoeuvres (Cuesta et al., 2004). The next section is devoted to design a restricted manoeuvre for changing only the relative orientation of the trailer, i.e., φ.

4. Changing Trailer Orientation

4.1 Manoeuvre Design Approach

The method applied in this paper makes use of a set of piece-wise constant inputs in order to design the manoeuvre. Thus, a sequence of velocity vectors which accomplish relation (3) is applied to the system. Once the manoeuvre is finished, the tractor vehicle has arrived again to its initial configuration, however the relative orientation of the tractor and the trailer, φ, has changed.

The initial configuration is assumed to be $(0,0,0,\varphi_0)$ without lost of generality, where φ_0 is the initial tractor-trailer angle. The manoeuvre described in this section consists of moving the vehicle from $(0,0,0,\varphi_0)$ to $(0,0,0,\varphi_0 + \Delta\varphi)$ (Gómez-Bravo et al., 2002). Then, the trajectory proposed to change φ is defined by the points P0 → P1 → P2 → P3 → P0, as shown in Fig.4.

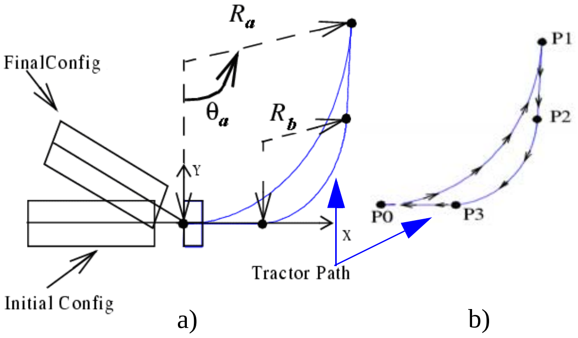
Thus, the piece-wise manoeuvre can be obtained integrating the kinematic model as

(P0 → P1): $[\dot{x}, \dot{y}, \dot{\theta}, \dot{\varphi}]^T = \Omega_2$

(P1 → P2): $[\dot{x}, \dot{y}, \dot{\theta}, \dot{\varphi}]^T = \Omega_1$

(P2 → P3): $[\dot{x}, \dot{y}, \dot{\theta}, \dot{\varphi}]^T = \Omega_3$

(P3 → P0): $[\dot{x}, \dot{y}, \dot{\theta}, \dot{\varphi}]^T = \Omega_1$



a) b)

Fig. 4: a) Reorientation manoeuvre; b) tractor path

If the velocity vector is equal to Ω_2 the tractor vehicle will describe a forward arc of circumference of curvature radius R_a (moving from P0 to P1). When the velocity vector is equal to Ω_1 the tractor describes a backward straight segment (moving from P1 to P2). Also, when velocity vector is equal to Ω_3 the tractor vehicle describes a backward arc of circumference of curvature radius R_b. Vectors Ω_1 ,Ω_2 and Ω_3 satisfying equations (3) were introduced in Gómez-Bravo et al. (2002).

Therefore, the conditions for the restricted manoeuvre are $\Delta_T x = 0$, $\Delta_T y = 0$, $\Delta_T \theta = 0$, where Δ_T stands for the increment of the variable at the end of the manoeuvre with respect to its initial value. Then, the relations which provide with the values of the restricted manoeuvre are

$$\begin{aligned} l_1 &= \theta_a R_a, & l_2 &= (R_b - R_a)\tan\frac{\theta_a}{2} \\ l_3 &= -R_b\theta_a, & l_4 &= l_2 \end{aligned} \qquad (5)$$

where l_1 represents the longitude of the path between P0 and P1, l_2 the longitude of the path between P1 and P2, l_3 the longitude of the path between P2 and P3 and l_4 the longitude of the path between P3 and P0.

According to Eq. (5) and Fig. 4, the manoeuvre is defined by the values of (R_a, R_b, θ_a); therefore each value of these three parameters is related with an increment of φ .

It can be shown, by numerical integration of equation (4) with different vehicle configurations, that the selection of values of (R_a, R_b, θ_a) which accomplishes $R_a < R_b \le R_{min}$, $0 < \theta_a < \frac{\pi}{2}$, and $-\frac{\pi}{4} < \varphi_0 < \frac{\pi}{4}$, generates a positive increment of φ (i.e. the trailer rotates clockwise) (Gómez-Bravo, 2001).

Moreover, if the vehicle follows the manoeuvre in reverse (i.e. P0 → P3 → P2 → P1 → P0) a negative increment of φ is obtained (i.e. the trailer rotates counterclockwise).

In the next section, techniques for choosing (R_a, R_b θ_a) according to spatial constraints are presented. Such methods have been experimentally validated for a wide range of tractor-trailer configurations. For instance, figures presented in Sections III and IV correspond to a system whose length and width are 9 and 2m, respectively, with d = 0 and D = 8m (see Fig. 2). Results with a shorter system and with $d \ne 0$ are presented in Section 5.

4.2 Collision Avoidance

Different values of (R_a, R_b, θ_a) can produce the same increment of φ , i.e., a desired $\Delta\varphi$ can be achieved by different manoeuvres, as shown in Fig.5. However, each manoeuvre has different longitude. Even more, different collision-free space is required in order to perform each one.

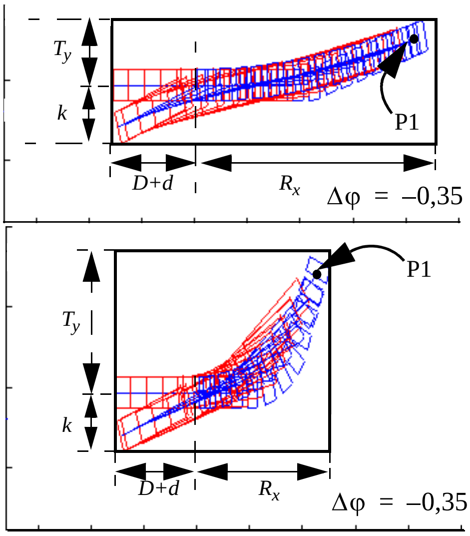


Fig. 5: Two different manoeuvres for the same $\Delta\varphi$

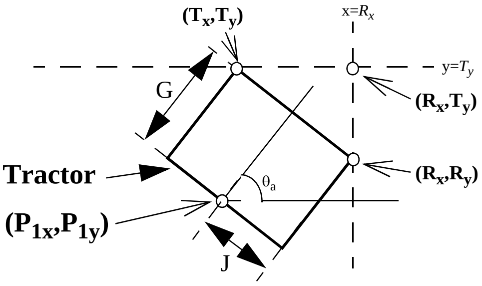


Fig. 6: Determination of R_x and T_y from the tractor at P1.

According to Fig.5 and Fig.6 the size of the collision-free polygons (height T_y+k and width R_x+D+d) is characterized by the point P1 (one of the points defined by the tractor during the manoeuvre, see Fig.4 and Fig.5). This relation can be written as

$$\begin{aligned} R_x &= P_{1x} + J\sin(\theta_a) + G\cos\theta_a \\ T_y &= P_{1y} + J\cos(\theta_a) + G\sin(\theta_a) \end{aligned} . \qquad (6)$$

Coordinates of P1 are related with the manoeuvre parameters according to the expression

$$P1_x = R_a\sin\theta_a \qquad P1_y = R_a(1-\cos\theta_a) \ . \qquad (7)$$

Thus, the height and width of the collision-free polygon are determined by the parameters (R_a, θ_a); observe that k depends only on φ and is the same in both manoeuvres.

4.3 Planner module

The planner module proposed to compute a feasible manoeuvre is shown in Fig.7. This module provides with the control inputs ($v(t)$ and $\rho(t)$) which generate the manoeuvre, by taking into account four values: initial conditions (φ_0), desired increment in φ and the environment constraint (R_x, T_y).

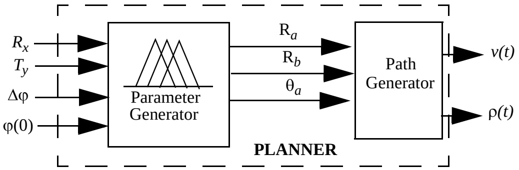


Fig. 7: Manoeuvre planner

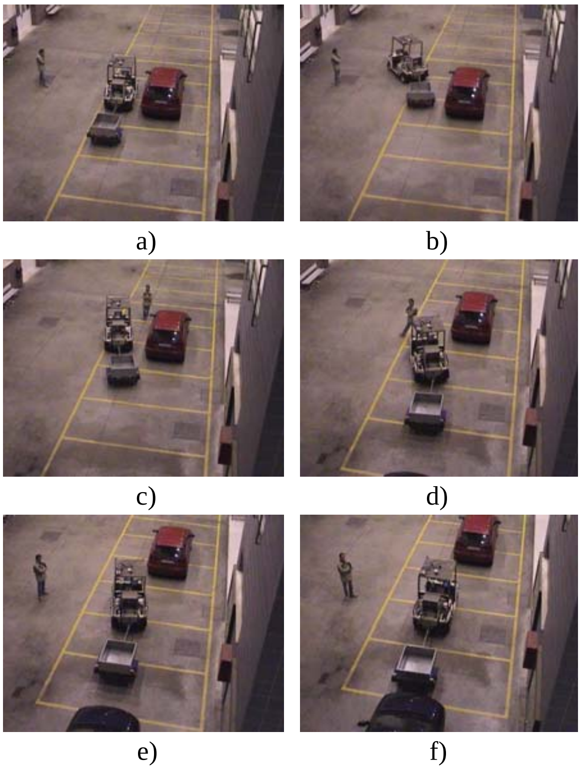
The parameters which define the manoeuvre, (R_a, R_b, θ_a), are related to R_x, T_y, $\Delta\varphi$ and φ_0 by means of a nonlinear mapping. To apply the method in real time this nonlinear mapping has been implemented by using a fuzzy system corresponding to the inverse mapping from (R_a, R_b, θ_a, φ_0) to (R_x, T_y, $\Delta\varphi$) which is simpler to compute.

Once (R_a, R_b, θ_a) has been established, the planner system supplies a velocity and a curvature profile (i.e. the control inputs which generate the desired manoeuvre) by using a path generator module (see Fig.7). This planner module provides with a suitable manoeuvre according to a specific space constraint.

5. EXPERIMENTAL RESULTS

The method presented in the above sections has been implemented in the autonomous vehicle ROMEO4R (see Fig.8), a four-wheeled vehicle with Ackerman steering. It is an electrically powered vehicle designed and built at the Univ. of Sevilla (Ollero et al., 1999). Namely, autonomous parallel parking of ROMEO4R with trailer has been performed (see the videos at www.esi2.us.es/~fcuesta/videos/parking.htm).

Fig. 8: ROMEO4R vehicle with trailer.



a) b) c) d) e) f)

Fig. 9: Parallel parking of ROMEO4R with trailer.

The first step of the parking manoeuvre consists of changing trailer orientation by using a reorientation manoeuvre as shown in Fig.9 and Fig.10. The desired $\Delta\varphi$ is provided by a global planning architecture that takes into account the rest of restricted manoeuvre to accomplish the parking task without collisions (Cuesta et al., 2004). Thus, the resulting target increment for this parking place is $\Delta\varphi = 0{,}27\text{rad}$. Then, a collision-free reorientation manoeuvre is computed by the planning module of Fig. 7 taking into account that the tractor-trailer is located at 0.42m from the parking line, the length of the parking place is 7.8m, the existence of a virtual wall at 3.49m at the left hand side of the vehicle and another one in front of it at 4.82m (see Fig.10), i.e., with P1y = 1.62m, P1x = 3.21m. The obtained manoeuvre is (R_a = 4m, θ_a= 0.93rad, R_b = 3m). The fuzzy inverse mapping was computed off-line for φ_0 =0rad, $0.2<\Delta\varphi<0.9$rad, $0.5<T_y<4$m and $1<R_x<7$m which represent usual values for manoeuvring in real environments with ROMEO4R.

The control architecture is based on performing manoeuvres with discontinuity in the value of the curvature. For this purpose a new sequential hybrid control structure has been applied (Gómez-Bravo et al., 2003). This architecture considers stopping the vehicle during manoeuvring in order to change the steering angle when curvature's discontinuity appears. Even more this architecture also combines reactive and planned navigation to improve manoeuvre performance. Fig.9 shows the execution of the computed manoeuvre. First, the reorientation manoeuvre is performed, as shown in Fig.9a-c (see also Fig.10). Second, the parking manoeuvre is completed (Fig.9d-e). Fig.11a-b show the evolution of the velocity and curvature during the manoeuvre, respectively. Notice that, during backward motion, the curvature control signal stabiliz-

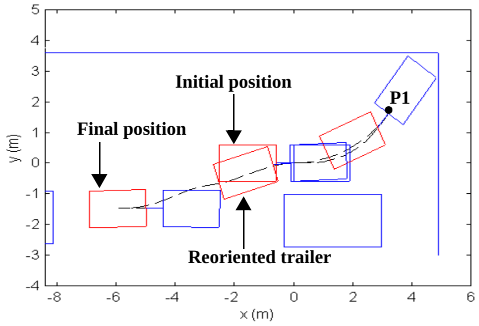
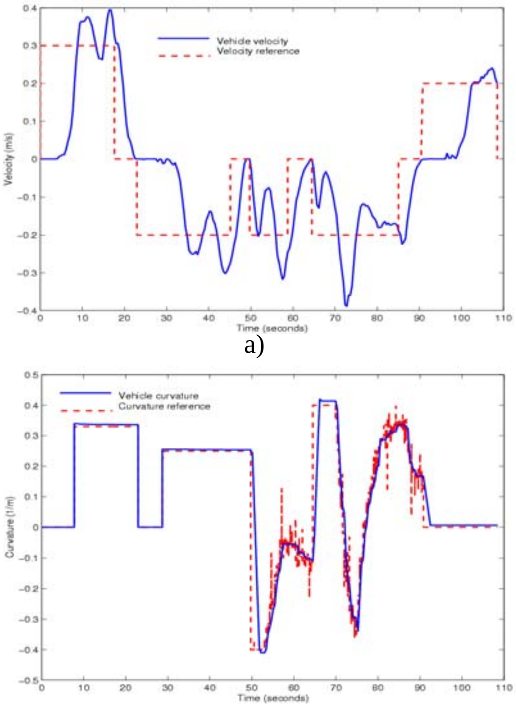


Fig. 10: Parallel parking of ROMEO4R with trailer (cartesian description)



a)

b)

Fig. 11: a) Tractor velocity; b) tractor curvature

es the system and compensates for noise at the angle measurement.

6. CONCLUSIONS

Manoeuvring tractor-trailer vehicles is not a trivial task. A method for planning manoeuvres with simple velocity and steering profiles has been presented. Thus it can be used both in a human driver assistance system and in an autonomous planning and motion control system. The proposed technique takes into account the geometrical constraints of the environment and has low computational requirements. A manoeuvre for changing trailer orientation has been presented in detail. The practical application of the proposed method has been tested with the ROMEO4R autonomous vehicle performing an autonomous parallel parking.

ACKNOWLEDGMENTS

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REFERENCES

Cuesta, F., F. Gómez-Bravo, A. Ollero (1998). A combined planned/reactive system for motion control of vehicles manoeuvres. *MC98*, Sept, France. pp. 303-308.

Cuesta, F., F. Gómez-Bravo, A.Ollero (2004). Parking Manoeuvres of Industrial-Like Electrical Vehicles With and Without Trailer. *IEEE Trans. on Industrial Electronics: Special Session on Automotive Electronic Systems*. **Vol. 51**, No. 2, pp 257-263, April.

Divelbiss, A.W., T.J. Wen (1997). Trajectory Tracking Control of a Car-Trailer System. *IEEE Trans. on Control Systems Technology*, **Vol 5**, No 3, pp 269-278.

Gómez-Bravo, F., (2001), Planificación de maniobras en sistemas robóticos no holónomos. Aplicaciones en robots móviles. *Ph.D. Thesis*, University of Seville.

Gómez-Bravo, F., F. Cuesta, A. Ollero, (2001) Parallel and diagonal parking in nonholonomic autonomous vehicles, *Engineering Applications of Artificial Intelligence*, **Vol. 14**, No. 4. pp 419-434, August.

Gómez-Bravo, F., F. Cuesta, A. Ollero. (2002)Intelligent System for Parallel Parking in Cars and Tractor-Trailers. *3rd International ICSC Symposium on Engineering of Intelligent Systems EIS2002*. 24-27, Spain.

Gómez-Bravo, F., F. Cuesta, A. Ollero (2003). A new sequential hybrid control structure for car and tractor-trailer parallel parking. *SYROCO03*. Polonia. 605-610.

Fortune, S.G., L.E. Wilson (1988). Planning Constrained Motion. *American Journal of Math*, pp. 497-516.

Lamiraux, F., J.P. Laumont, (1998) A practical approach to feedback control for a mobile robot with trailer. *1998 IEEE ICRA*, pp 3291-3296.

Lamiraux, F., Sekhavat, S. Laumond, J.-P. (1999). Motion planning and control for Hilare pulling a trailer. *IEEE Trans. on Robotics and Automation*, **Vol. 15**, No 4.

Latombe, J.C. (1991), *Robot Motion Planning* . Kluwer Academic Publishers, pp. 424-425.

Laumont, J.P., P.E. Jacobs, M. Taix, M. Murray (1998). A Motion Planner for Nonholonomic Mobile Robots. *IEEE Trans. on Rob. and Autom.*, **Vol 10**, N 5, 577-593.

Lizarraga, D., P. Morin, (1998) Exponential stabilization of certain configuration of general N-Trailer system. *MC98*, France. pp.221-232.

Murray, R., S.S. Sastry (1990) Steering Nonholonomic System Using Sinusoids, *Proc. of the IEEE Int. Conf. on Decision and Control*, 1136-1141.

Murray, R., S.S. Sastry (1993) Nonholonomic motion planning: Steering using sinusoids. *IEEE Trans. on Automatic Control*, **Vol 38**, No 5, 700-716.

Ollero, A., B.C. Arrue, J. Ferruz, G. Heredia, F. Cuesta, F. López-Pichaco, C. Nogales (1999), Control and Perception Components for Autonomous Vehicle Guidance. Application to the Romeo Vehicles. *Control Engineering Practice*. **Vol. 07**, N. 10. pp 1291-1299.

Sekhavat, S., and J. P. Laumond (1998), Topological property for collision-free nonholonomic motion planning: The case of sinusoidal inputs for chained form systems, *IEEE Trans. Rob. Autom*, **Vol.14**, pp 671–680.

Tilbury, D. (1994), Exterior Differential Systems and Nonholonomic Motion Planning. *Ph.D. Thesis*, University of California at Berkeley.

Tilbury, D., R. Murray, S. Sastry (1995), Trajectory Generation for the N-Trailer Problem using Goursat Normal Form. *IEEE T. Autom. Control*, **40:** (5) 802-819.

Wada, M., K.S. Yoon, and H. Hashimoto (2003). Development of Advanced Parking Assistance System. *IEEE Trans. on Industrial Electronics*, **Vol. 50**, No. 1.