# AN ANTI-WINDUP STRATEGY FOR A FLEXIBLE CANTILEVER BEAM

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Abstract: This paper addresses the problem of stability analysis for a flexible cantilever beam which is subject to amplitude limitations in actuator and sensor. The design of anti-windup schemes is considered in order to enlarge the region of stability of the closed-loop system. Based on the modelling of the saturated system as a linear system with dead-zone nested nonlinearities, constructive stability conditions are formulated in terms of linear matrix inequalities. Some discussions about possible extensions based on the use of observer loops are provided. 

Keywords: Anti-windup scheme, nested saturations, stability regions, observer

## 1. EXPERIMENTAL SETUP AND PIEZOELECTRIC MODEL

We consider a flexible cantilever beam, that is clamped at one end and free at the other as shown in Figure 1. Two piezoelectric patches are bonded on this beam. They are placed symmetrically at the root of the beam. One of them is used as sensor and the other is used as actuator.

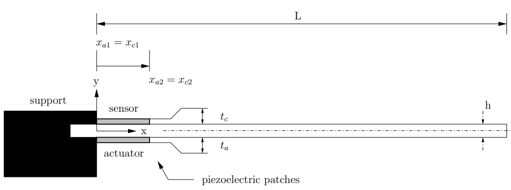


Fig. 1. Flexible cantilever beam

The equipped structure is linked to an experimental device described in Figure 2, allowing a controller implementation on a DSP board. Note that the active reduction of the vibrations of this smart flexible beam under saturated actuator is under very active research (Halim and Moheimani, 2001), (Smith *et al.*, 1994), (Halim and Moheimani, 2002).

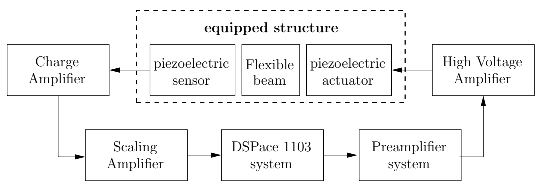


Fig. 2. Experimental setup

The finite-dimensional model is derived from a Partial Derivative Equation (PDE) modelling, namely the Euler-Bernoulli equation (see e.g. (Halim and Moheimani, 2001; Crépeau and Prieur, 2004)). We deduce the finite-dimensional state equations by modal analysis, as done in (Henrion *et al.*, 2004), and by restricting ourselves to the first 12 modal functions only (instead of the infinity modal functions as derived directly from the PDE). This allows to consider a model in the form:

$$\begin{aligned} \dot{x} &= Ax + Bu \\ y &= Cx \end{aligned} \tag{1}$$

where $x$ is the state of the beam evolving in $\Re^{24}$, $y \in \Re$ is the voltage on the piezoelectric sensor, $u \in \Re$ is the voltage applied to the piezoelectric actuator, and $A$, $B$, $C$ are matrices of appropriate dimensions.

### *1.1 The voltage amplifier model*

We use a first-order dynamical model for the voltage amplifier. Let us denote $v$ the input of the voltage amplifier and $u$ its output. We have

$$\tau \dot{u} + u = kv \tag{2}$$

The output voltage range is $\pm 500\ V$, whereas the input voltage range must be $\pm 5\ V$. The numerical values of the constants $\tau$ and $k$ are computed from the technical specifications of the amplifier Model 601C from the Trek Incorporated used at the laboratory SATIE, CNRS-ENS de Cachan, France. More precisely $\tau = 6.51 \times 10^{-5}\ s$ and $k = 100$. But in fact the slew rate of the amplifier is bounded, that is, the maximal output voltage rate, denoted $speed_{ampli}$, is equal to $50 \times 10^6\ Vs^{-1}$. Moreover, the maximal $\dot{u}$ computed with the first order model (2) is $\frac{kv}{\tau} = 500/\tau$, i.e. $7.7 \times 10^6\ Vs^{-1}$ which is less than $speed_{ampli}$. Thus we do not consider a saturation on $\dot{u}$ in our model. Furthermore, we apply a saturation device before the voltage amplifier to garantee the factory setting: $v \in [-5\ V, 5\ V]$.

### *1.2 The piezoelectric actuator*

Let us compute the maximal speed of the piezoelectric patch. A PZT can reach its nominal displacement in approximately 1/3 of its resonant period. The resonant frequency is equal to $f_0 = \frac{N}{L}$, where $N$ is its frequency constant and $L$ is the length of the piezoelectric actuator. For our experimental device (piezoceramic material PIC 151 from Physik Instrumente) we have $N = 1500$ and $L = 2 \times 10^{-2} m$. Thus the resonant period is equal to $4.444 \times 10^{-6}\ s$. Note that, due to the technical specifications recalled above, the mimimal period of the output voltage delivered by the amplifier is $2 \times \frac{1000}{7.7 \times 10^6}$, i.e. $2.6 \times 10^{-4}\ s$. As a conclusion, the speed of the voltage applied to the piezoelectric actuator cannot saturate, since the limit of the speed of the voltage getting from the amplifier is lower than the admissible maximal speed for the piezoelectric actuator.

We can consider a linear model for the piezoelectric actuator and sensor as soon as the voltage which is applied or measured are in $[-500\ V, 500\ V]$. Therefore, thanks to the saturation map of level 500 present in the voltage amplifier, a linear model to describe the behavior of the piezoelectric actuator is sufficient.

The voltage measured through the piezoelectric sensor is collected in a charge amplifier whose voltage input saturates at $50\ V$. We can model this charge amplifier by a saturation map with a level at $50\ V$. Thus, the output signal of the experimental device is obtained through this saturation map.

## 2. PROBLEM STATEMENT

From the description above, the system under consideration is then described by the following equations:

$$\begin{aligned} \dot{x} &= Ax + B sat_{u_0}(u) \\ \dot{u} &= -\frac{1}{\tau}u + \frac{k}{\tau}v \\ y &= sat_{y_0}(Cx) \end{aligned} \tag{3}$$

with $u_0 = 500$ and $y_0 = 50$.

Let us consider the control law computed by a numerical conditioning of the model of the beam and by a low order pole-placement controller design as done in (Henrion *et al.*, 2004). This numerical algorithm yields to the controller dynamics in $\Re^2$:

$$\begin{aligned} \dot{\eta} &= A_c\eta + B_c u_c \\ y_c &= C_c\eta + D_c u_c \end{aligned} \tag{4}$$

where $\eta \in \Re^2$ and $A_c$, $B_c$, $C_c$, $D_c$ are matrices of appropriate dimensions numerically computed in (Henrion *et al.*, 2004). After the dynamic controller we scale the output of the control by mutliplying by 1/100, since we use a high voltage amplifier of gain $k = 100$. Thus, such a controller has been designed such that the closed-loop system resulting from the interconnection conditions

$$v = \frac{y_c}{100}\ ;\ u_c = Cx \tag{5}$$

is asymptotically stable.

Some windup problems arise when the saturation $sat_{u_0}(u)$ occurs and when the previous linear interconnection is replaced by the real interconnection:

$$v = sat_{v_0}(\frac{y_c}{100})\ ;\ u_c = sat_{y_0}(Cx) \tag{6}$$

with $v_0 = 5$.

Hence, from the above description, the complete experimental system reads:

$$\begin{aligned} \dot{x} &= Ax + B sat_{u_0}(u) \\ \dot{u} &= -\frac{1}{\tau}u + \frac{k}{\tau} sat_{v_0}(\frac{C_c}{100}\eta + \frac{D_c}{100} sat_{y_0}(Cx)) \\ \dot{\eta} &= A_c\eta + B_c sat_{y_0}(Cx) \end{aligned} \tag{7}$$

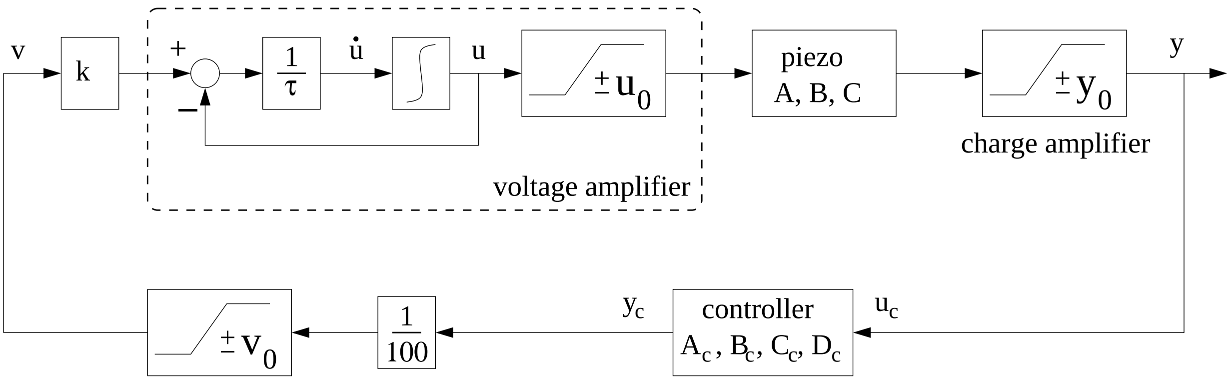


Fig. 3. Complete system

and can be depicted in Figure 3.

In the absence of saturation, the stability of system (7) is directly characterized by the closed-loop matrix $\mathbb{A} = \begin{bmatrix} A & B & 0 \\ \frac{kD_cC}{100\tau} & -\frac{1}{\tau} & \frac{kC_c}{100\tau} \\ B_cC & 0 & A_c \end{bmatrix}$ which is Hurwitz by construction. In the presence of saturation, considering that the exact analytical determination of the basin of attraction of the system is, in general, not possible, we should be concerned with the determination of estimates of this basin, i.e. regions in the state space in which the asymptotic stability of system (7) is guaranteed.

The problem we intend to solve can therefore be summarized as follows.

**Problem 1.** *Given a system where the matrix* $\mathbb{A}$ *is Hurwitz, determine a region of stability* $\mathcal{E}_0$ *for system (7) as large as possible.*

## 3. STABILITY ANALYSIS RESULTS

Remark that the system (7) is a nonlinear system in which the nonlinearities are of nested type. The stability analysis of this system can therefore be carried out from the results proposed in (Tarbouriech *et al.*, 2004). In the sequel we consider these results in order to provide a solution to Problem 1.

Note first that system (7) can be re-written in the following form:

$$\dot{X} = \mathcal{A}_2 X + \mathcal{B}_2 sat_{s_2}\left(\mathcal{A}_1 X + \mathcal{B}_1 sat_{s_1}(\mathcal{C}X)\right) \quad (8)$$

where the state $X = [x' \; u \; \eta']' \in \Re^{n+3}$, $\mathcal{A}_2$, $\mathcal{B}_2$, $\mathcal{A}_1$, $\mathcal{B}_1$ and $\mathcal{C}$ are matrices of appropriate dimensions such that

$$\mathcal{A}_2 = \begin{bmatrix} A & 0 & 0 \\ 0 & \frac{-1}{\tau} & 0 \\ 0 & 0 & A_c \end{bmatrix}, \quad \mathcal{B}_2 = \begin{bmatrix} B & 0 & 0 \\ 0 & \frac{k}{\tau} & 0 \\ 0 & 0 & B_c \end{bmatrix}$$

$$\mathcal{A}_1 = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & \frac{C_c}{100} \\ C & 0 & 0 \end{bmatrix}, \mathcal{B}_1 = \begin{bmatrix} 0 \\ \frac{D_c}{100} \\ 0 \end{bmatrix}, \mathcal{C} = \begin{bmatrix} C & 0 & 0 \end{bmatrix}$$

and the saturation maps are defined with $s_1 = y_0 \in \Re$ and $s_2 = \begin{bmatrix} u_0 \\ v_0 \\ y_0 \end{bmatrix} \in \Re^3$.

Concerning the analysis of the global asymptotic stability of (8) let us particularize the result of (Tarbouriech *et al.*, 2004) as follows.

**Proposition 1.** *(Tarbouriech* et al.*, 2004) If there exist a symmetric positive definite matrix $W$ and diagonal positive matrices $S_1$ and $S_2$ of appropriate dimensions satisfying*[1] *:*

$$\begin{bmatrix} sym(\mathcal{A}_2W + \mathcal{B}_2(\mathcal{A}_1 + \mathcal{B}_1\mathcal{C})W) & \star & \star \\ S_1\mathcal{B}_1'\mathcal{B}_2' - \mathcal{C}W & -2S_1 & \star \\ S_2\mathcal{B}_2' - (\mathcal{A}_1 + \mathcal{B}_1\mathcal{C})W & -\mathcal{B}_1S_1 & -2S_2 \end{bmatrix} < 0 \quad (9)$$

*then the closed-loop (7) (or equivalently (8)) is globally asymptotically stable.*

Unfortunately, for the current system LMI (9) is unfeasible. Hence this sufficient condition does not apply and thus we infer that the closed-loop system (7) is not globally asymptotically stable. In this case, the objective will consist in determining a region of stability, as large as possible, in order to estimate the actual basin of attraction of the closed-loop system. With this aim, we particularize the result of (Tarbouriech *et al.*, 2004) to the present system structure.

**Proposition 2.** *(Tarbouriech* et al.*, 2004) If there exist a symmetric positive definite matrix $W$, matrices $Z_{11}$, $Z_{22}$ and $Y_{21}$, and diagonal positive matrices $S_1$ and $S_2$ of appropriate dimensions satisfying:*

$$\begin{bmatrix} sym(\mathcal{A}_2W + \mathcal{B}_2(\mathcal{A}_1 + \mathcal{B}_1\mathcal{C})W) & \star & \star \\ S_1\mathcal{B}_1'\mathcal{B}_2' - Z_{11} & -2S_1 & \star \\ S_2\mathcal{B}_2' - Z_{22} & -Y_{21} & -2S_2 \end{bmatrix} < 0 \quad (10)$$

$$\begin{bmatrix} W & W\mathcal{C}'_{(i)} - Z'_{11(i)} \\ \star & s_1^2 \end{bmatrix} \geq 0, \quad (11)$$

[1] The symbol $\star$ stands for symmetric blocks. Furthermore, $sym(A) = A + A'$.

$$\begin{bmatrix} W & Z_{11}' & W(\mathcal{C}'\mathcal{B}_1' + \mathcal{A}_1')_{(i)} - Z_{22(i)}' \\ \star & 2S_1 & S_1 B_{1(i)}' - Y_{21(i)}' \\ \star & \star & s_{2(i)}^2 \end{bmatrix} \geq 0 \quad (12)$$
$$i = 1, 2, 3$$

*then the set*

$$\mathcal{E}(W^{-1}, 1) = \{X \in \Re^{n+3}; X'W^{-1}X \leq 1\}$$

*is a region of stability for system (8) or equivalently (7).*

Since the system (3) in closed-loop with (4) is locally asymptotically stable, the LMIs (10)-(12) are feasible. Thus an optimization problem with respect to the "size" of the region of stability can be considered. For instance, the minimization of the trace of $W^{-1}$ can be pursued as follows:

$$\begin{array}{c} \min \ \mathrm{Trace}(M_W) \\ \text{subject to relations } (10), (11), (12) \\ \begin{bmatrix} M_W & I_{n+3} \\ I_{n+3} & W \end{bmatrix} \geq 0 \\ M_w > 0 \end{array} \quad (13)$$

Considering the numerical data, one gets matrices $W$, $Z_{11}$, $Z_{22}$, $Y_{21}$ with the optimal cost: $\mathrm{Trace}(M_W) = 1.41\,10^8$ which corresponds to a very small region of stability for the closed-loop system (the volume of the set $\mathcal{E}(W^{-1}, 1)$ is equal to $2.9\,10^{-11}$).

## 4. ANTI-WINDUP STRATEGY RESULTS

In order to provide better solution to Problem 1 and to avoid the undesirable effects of the windup, or at least to mitigate them, a strategy through anti-windup scheme can be considered.

The anti-windup techniques consist in taking into account the effects of saturation in a second stage after previous design performed disregarding the actuator limitations. The idea is to introduce some control modification, active when the saturation occurs, in order to recover, as much as possible the properties induced by the previous design carried out for the unsaturated system. In particular, anti-windup schemes have been successfully applied in order to avoid, or at least to minimize the windup of the integral action in PID controllers, largely applied in the industry. In this case, most of the related literature focuses on the performance improvement in the sense of avoiding large and oscillatory transient responses (see, among others, (Åström and Rundqwist, 1989)). Furthermore, a special attention has been paid to the influence of the anti-windup schemes in the stability and the performances of the closed-loop system (see, for example, (Barbu *et al.*, 2000), (Kothare and Morari, 1999)). Several results on the anti-windup problem are concerned with achieving global stability properties. Since global results cannot be achieved for open-loop unstable linear systems in the presence of actuator saturation, local results have to be considered. It is then important to note that the basin of attraction is modified by the anti-windup loop.

Recently, some constructive conditions are proposed both to determine suitable anti-windup gains and to quantify the region of stability of linear systems subject to amplitude saturation actuator (see, among others, (Cao *et al.*, 2002), (Gomes da Silva Jr. and Tarbouriech, 2003), the ACC03 Workshop "T-1: Modern Anti-windup Synthesis" in the ACC04 (Session FrP04 "Anti-windup") or the invited session in IFAC NOLCOS04). In this paper we focus our attention on a linear system with amplitude nested saturations on actuator and sensor representing a flexible cantilever beam. We are then interested in the design of the suitable anti-windup gains in order to ensure the closed-loop stability for regions of admissible initial states as large as possible.

For anti-windup purpose, the set of measured variables of this system are the following:

$$v \ ; \ y_c \ ; \ y \quad (14)$$

Thus, the idea consists in adding the term:

$$E_c(sat_{v_0}(\frac{y_c}{100}) - \frac{y_c}{100}) \quad (15)$$

in the dynamics of the controller. More complex anti-windup loops will be discussed in Section 5.

The problem we intend to solve can therefore be summarized as follows.

**Problem 2.** *Determine the gain $E_c$, and a region of stability $\mathcal{E}_0$, for the closed-loop system (7) with the anti-windup term (15), as large as possible.*

Considering the controller with an anti-windup term:

$$\begin{aligned} \dot{\eta} &= A_c\eta + B_c u_c + E_c(sat_{v_0}(\frac{y_c}{100}) - \frac{y_c}{100}) \\ y_c &= C_c\eta + D_c u_c \end{aligned} \quad (16)$$

leads to the following closed-loop system:

$$\begin{aligned} \dot{x} &= Ax + Bsat_{u_0}(u) \\ \dot{u} &= -\frac{1}{\tau}u + \frac{k}{\tau}sat_{v_0}(\frac{C_c}{100}\eta + \frac{D_c}{100}sat_{y_0}(Cx)) \\ \dot{\eta} &= A_c\eta + B_c sat_{y_0}(Cx) \\ &\quad + E_c\left(sat_{v_0}(\frac{C_c}{100}\eta + \frac{D_c}{100}sat_{y_0}(Cx)) \right. \\ &\qquad \left. - \frac{C_c}{100}\eta - \frac{D_c}{100}sat_{y_0}(Cx))\right) \end{aligned} \quad (17)$$

We rewrite (17) as

$$\dot{X} = \mathcal{A}_2 X + \tilde{\mathcal{B}}_2 sat_2\left(\mathcal{A}_1 X + \mathcal{B}_1 sat_1(\mathcal{C}X)\right) \quad (18)$$

where $X$, $\mathcal{A}_2$, $\mathcal{A}_1$, $\mathcal{B}_1$ and $\mathcal{C}$ are defined as in Section 3 and

$$\tilde{\mathcal{B}}_2 = \begin{bmatrix} B & 0 & 0 \\ 0 & \frac{k}{\tau} & 0 \\ 0 & 0 & B_c \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} E_c \, [0\ 1\ 0]$$

The following proposition provides a solution to Problem 2.

**Proposition 3.** *If there exist a symmetric positive definite matrix $W$, matrices $Z_{11}$, $Z_{22}$, $Z_{33}$ and $Y_{21}$, and diagonal positive matrices $S_1$ and $S_2$ of appropriate dimensions satisfying relations (11), (12) and*

$$\begin{bmatrix} sym(\mathcal{A}_2 W + \mathcal{B}_2(\mathcal{A}_1 + \mathcal{B}_1\mathcal{C})W) & \star & \star \\ S_1\mathcal{B}_1'\mathcal{B}_2' - Z_{11} & -2S_1 & \star \\ S_2\mathcal{B}_2' - Z_{22} + \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} Z_{33}' \begin{bmatrix} 0 & 0 & 1 \end{bmatrix} & -Y_{21} & -2S_2 \end{bmatrix} < 0 \quad (19)$$

*then the set*

$$\mathcal{E}(W^{-1}, 1) = \{X \in \Re^{n+3}; X'W^{-1}X \leq 1\}$$

*is a region of stability for the system (18) or equivalently (17) with the anti-windup gain*

$$E_c = Z_{33} \begin{bmatrix} 0 & 1 & 0 \end{bmatrix} S_2^{-1} \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}$$

*Proof.* It suffices to replace $\mathcal{B}_2$ by $\tilde{\mathcal{B}}_2$ in Proposition 2. In this case, one obtains:

$$\begin{bmatrix} sym(\mathcal{A}_2 W + \mathcal{B}_2(\mathcal{A}_1 + \mathcal{B}_1\mathcal{C})W) & \star & \star \\ S_1\mathcal{B}_1'\mathcal{B}_2' - Z_{11} & -2S_1 & \star \\ S_2\mathcal{B}_2' - Z_{22} + S_2 \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} E_c' \, [0\ 0\ 1] & -Y_{21} & -2S_2 \end{bmatrix} < 0 \quad (20)$$

By considering the change of variable

$$Z_{33} = E_c \begin{bmatrix} 0 & 1 & 0 \end{bmatrix} S_2 \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} = E_c S_{22}$$

we rewrite (20) as (19), which is linear in the new variables. This concludes the proof. □

The same kind of convex problem as stated in Section 3 can be used and tested with the numerical data to obtain matrices $W$, $Z_{11}$, $Z_{22}$, $Y_{21}$, $Z_{33}$, $E_c$. By comparing the size of the regions of stability (issued from Propositions 2 and 3), it follows that the size of the region of stability obtained by using the anti-windup strategy is 22% larger than the one obtained without it.

## 5. DISCUSSION

### *5.1 Numerical issues*

For numerical limitations, the solutions of the optimization Problems like (13) have been computed after order reduction of the matrices $A$, $B$ and $C$ of system (1). This reduction has been done by a modal truncation i.e. by considering only the first fourth modal functions. After this reduction, the order of the system (7) is equal to 11. This reduction is lawful since, for flexible structure, with a small number of smart actuators and sensors, the first modal functions contains the main part of the mechanical energy as remarked in (Tliba and Abou-Kandil, 2003). This reduction has been also checked on numerical simulations, where the complete experimental system has been simulated with 12 modal functions and with the anti-windup gain computed with only 4 modal functions. The initial condition has been chosen in the region of stability issued from Proposition 3 but it is not in the region of stability determined in Proposition 2 (without anti-windup). Moreover it represents physically the controlled beam drop test with an impulsion, i.e. an initial deformation of the beam with an initial speed. See the output of this system (i.e. the voltage given by the charge amplifier) on Figure 4.

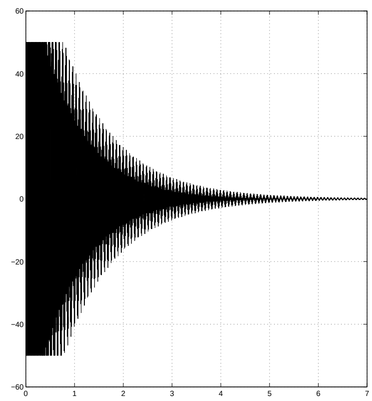


Fig. 4. Simulation of the evolution of the output

### *5.2 Anti-windup observer-based strategy*

In order to take into account the saturation of the plant input, or equivalently, of the output of the voltage amplifier, a second anti-windup term can be considered. However, it should be noticed that the variable $u$ is not available for measurement. In this case, an observer of the voltage amplifier part can be considered as follows:

$$\dot{\hat{u}} = -\frac{1}{\tau}u + \frac{k}{\tau}v + L_u(sat_{u_0}(\hat{u}) - sat_{u_0}(u)) \quad (21)$$

with the error

$$\epsilon_u = \hat{u} - u \quad (22)$$

From (21) and (22), the anti-windup strategy consists in adding the following term in the dynamics of the controller:

$$F_c(sat_{u_0}(\hat{u}) - \hat{u}) \quad (23)$$

Thus, the closed-loop system in $\Re^{n+4}$ reads:

$$\begin{array}{rcl}\dot{x} &=& Ax + Bsat_{u_0}(u)\\ \dot{u} &=& -\frac{1}{\tau}u + \frac{k}{\tau}sat_{v_0}(\frac{C_c}{100}\eta + \frac{D_c}{100}sat_{y_0}(Cx))\\ \dot{\eta} &=& A_c\eta + B_c sat_{y_0}(Cx)\\ && +E_c(sat_{v_0}(\frac{C_c}{100}\eta + \frac{D_c}{100}sat_{y_0}(Cx)) - (\frac{C_c}{100}\eta\\ && +\frac{D_c}{100}sat_{y_0}(Cx))\\ && +F_c(sat_{u_0}(u+\epsilon_u) - (u+\epsilon_u))\\ \dot{\epsilon}_u &=& -\frac{1}{\tau}\epsilon_u + L_u(sat_{u_0}(u+\epsilon_u) - sat_{u_0}(u))\end{array} \quad (24)$$

In this case, one has to determine the anti-windup gains $E_c$ and $F_c$, and the observer gain $L_u$.

By the same way, if one wants also to use the charge amplifier, that is, the output saturation $sat_{y_0}(Cx)$, since the variable $x$ is not measured, one cannot use directly the difference $sat_{y_0}(Cx) - Cx$ but an estimate of this one. For this, an observer of the state of the smart system can be considered as follows:

$$\dot{\hat{x}} = Ax + Bsat_{u_0}(u) + L_x(sat_{y_0}(C\hat{x}) - sat_{y_0}(Cx)) \quad (25)$$

with the error

$$\epsilon_x = \hat{x} - x \quad (26)$$

As previously, from (25) and (26), the anti-windup strategy consists in adding the following part in the dynamics of the controller:

$$F_c(sat_{y_0}(C\hat{x}) - \hat{x}) \quad (27)$$

In this case, the closed-loop system in $\Re^{2n+3}$ reads:

$$\begin{array}{rcl}\dot{x} &=& Ax + Bsat_{u_0}(u)\\ \dot{u} &=& -\frac{1}{\tau}u + \frac{k}{\tau}sat_{v_0}(\frac{C_c}{100}\eta + \frac{D_c}{100}sat_{y_0}(Cx))\\ \dot{\eta} &=& A_c\eta + B_c sat_{y_0}(Cx)\\ && +E_c(sat_{v_0}(\frac{C_c}{100}\eta + \frac{D_c}{100}sat_{y_0}(Cx)) - (\frac{C_c}{100}\eta\\ && +\frac{D_c}{100}sat_{y_0}(Cx))\\ && +F_c(sat_{y_0}(Cx + C\epsilon_x) - (Cx + C\epsilon_x))\\ \dot{\epsilon}_x &=& A\epsilon_x + L_x(sat_{y_0}(Cx + C\epsilon_x) - sat_{y_0}(Cx))\end{array} \quad (28)$$

In this case, one has to determine the anti-windup gains $E_c$ and $F_c$, and the observer gain $L_x$.

Note that if we are able to ensure the asymptotic stability of (24) or (28), the convergence of the observers will be ensured, since in both cases the errors $\epsilon_u$ or $\epsilon_x$ will converge towards the origin. Some preliminary results addressing this last problem are proposed in (Tarbouriech and Garcia, 2005). A complete description of the conditions in the case of (24) or (28) will be presented in an extended version of the paper.

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