VALIDATION OF STABILITY FOR AN INDUCTION MACHINE DRIVE USING POWER ITERATIONS [1]

Märta Barenthin * **Henrik Mosskull** *,**
Håkan Hjalmarsson * **Bo Wahlberg** *

* *S3 Automatic Control, KTH, SE-100 44 Stockholm, Sweden, (marta.barenthin@s3.kth.se)*
** *Bombardier Transportation, SE-721 73 Västerås, Sweden*

Abstract: This work is an extension of the paper (Mosskull *et al.*, 2003), in which the modelling, identification and stability of an nonlinear induction machine drive is studied. The validation of the stability margins of the system is refined by an improved estimate of the induced L_2 loop gain of the system. This is done with a procedure called power iterations where input sequences suitable for estimating the gain are generated iteratively through experiments on the system. The power iterations result in higher gain estimates compared to the experiments previously presented. This implies that more accurate estimates are obtained as, in general, only lower bounds can be obtained as estimates for the gain. The new gain estimates are well below one, which suggests that the feedback system is stable. The experiments are performed on an industrial hardware/software simulation platform. In this paper we also discuss the power iterations from a more general point of view. The usefulness of the method for gain estimation of nonlinear systems is illustrated through simulation examples. The basic principles of the method are provided.

Keywords: Nonlinear systems, stability analysis, power method, experimental validation, induction machines, drives

1. INTRODUCTION

Our main objective is to *extend and improve* the validation presented in the previous paper (Mosskull *et al.*, 2003), which will be our key reference in this work. Both this paper and the previous are about the closed loop stability properties of a nonlinear induction machine drive for rail vehicle propulsion. A critical issue is to obtain an accurate estimate of the loop gain of the unmodelled nonlinear dynamics of the drive. By definition, the loop gain is the maximum of the the ratio of the L_2 norms of the output and input of the loop. Hence, computing this ratio for any input/output pair gives a lower bound on the gain. However, it is a most difficult problem to find an input which maximizes the ratio for such a nonlinear system. An important contribution of this work is that we present new experimental results where a procedure called power iterations, suggested in (Hjalmarsson, 2004), is used to generate suitable inputs. Briefly, this method uses the system itself in repeated experiments to generate inputs. Hence in this work the validation is done in a new and more structured way than in the previous work, where physical insight of the system was used to make a qualified estimate of the maximizing input. In view of that very little is known about the properties of power iterations for nonlinear systems, another contribution of this paper is to show that this method is potentially useful for estimation of nonlinear system gains. This is shown via the induction machine drive application and a simulation example. Also, the basic principles behind this method are provided.

The paper is organized as follows. Section 2 gives a description of how the induction machine drive is modelled and explains how the small gain theorem can be used to analyze the stability properties of the induction machine drive. This section is a summary of

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the previous work. In Section 3 the power iterations are introduced. The connection to the power method is described as well as how it can be applied when analyzing stability. Experimental results are given in Section 4, which compares our new results using power iterations with the previous results. Section 5 concludes the paper.

2. THE INDUCTION MACHINE DRIVE

This section is a summary of the description of the induction machine drive and the modelling procedure already discussed in the previous work. Therefore, for all details in this section, see (Mosskull *et al.*, 2003). The drive with voltage source inverter, induction machine and RLC-network is shown in Fig. 1.

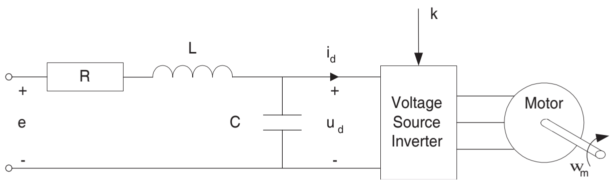

Fig. 1. Induction machine drive.

Here $u_d(t)$ is the DC-link voltage, $i_d(t)$ is the DC-link current and $e(t)$ is the supply voltage. The signal $k(t)$ represents the control of the converter. Furthermore, ω_M denotes the mechanical rotor speed. The nonlinear relation between the voltage and the current is modelled by

$$i_d(t) = G_0(q)u_d(t) + v_{NL}(t) \tag{1}$$

where G_0 is a linear model and v_{NL} is the unmodelled nonlinearity. In fact, v_{NL} is a function of u_d according to

$$v_{NL}(t) = \tilde{g}(u_d(t)). \tag{2}$$

The RLC-circuit is in Laplace domain described by

$$U_d(s) = Z_E(s)E(s) - Z_{DC}(s)I_d(s) \tag{3}$$

where

$$Z_E(s) = \frac{1}{LCs^2 + RCs + 1},$$
$$Z_{DC}(s) = \frac{Ls + R}{LCs^2 + RCs + 1}.$$

Combining equations (1)-(3) gives the feedback system depicted in Fig. 2.

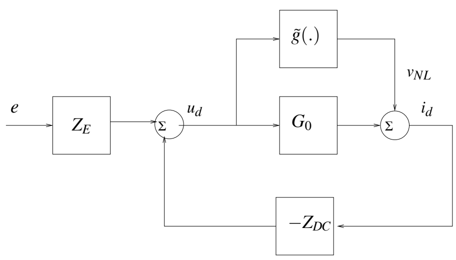

Fig. 2. Model of the induction machine drive.

The goal of the previous work and of this paper is to validate if this unmodelled nonlinearity causes instability. In this paper, however, the validation is done in a more structured way and with a new procedure, the so-called power iterations that will be introduced in the next section. As discussed in the previous work, the transfer function $-Z_{DC}(q)/(1+G_0(q)Z_{DC}(q))$ is stable and the stability of the system in Fig. 2 is equivalent to stability of the system in Fig. 3.

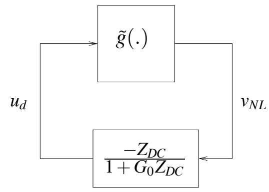

Fig. 3. Equivalent closed loop system.

The approach taken is inspired by the work in (Schoukens *et al.*, 2002), on identification of the stability of feedback systems in the presence of nonlinear distortions. Denoting the output from the linear filter in Fig. 3 by $w(t)$, we have

$$w(t) = -\frac{Z_{DC}(q)}{1 + G_0(q)Z_{DC}(q)} v_{NL}(t). \tag{4}$$

If

$$||w|| \leq \beta ||u_d|| + \alpha, \tag{5}$$

where $0 \leq \beta < 1$, the small-gain theorem implies that the closed loop system is stable, see (Khalil, 2002). The constant α can be used to model off-sets and external L_2 signals, see (Ljung, 2001). The goal of the power iterations discussed in the next section is thus to get a more accurate estimate for β than in the previous paper.

3. THE POWER ITERATIONS

In this section we discuss a procedure, the so-called power iterations, to estimate β in equation (5). This estimate can then be used to infer stability, as discussed in the previous section. The L_2-gain β of the system in Fig. 3 is formally defined as

$$\beta = \sup_{u \neq 0} \frac{||w||}{||u||} \tag{6}$$

Here the L_2-norm is used, which for a vector $x \in \mathbb{R}^n$ is defined as

$$||x|| = \left[\sum_{i=1}^{n} |x(i)|^2 \right]^{1/2}. \tag{7}$$

The problem is of course to find the input sequence which gives the maximal gain. For linear systems, the worst case input signal is a sinusoid corresponding to the maximum gain of the system and the problem therefore simplifies to finding this frequency. For nonlinear models, however, this might not be the case. From (6) it is clear that an estimate which is a lower bound for the gain will be obtained by computing the ratio $||w||/||u||$ for any input u. However, it is not at all clear what input should be used in order to get an estimate close to the actual gain. In this paper we will apply a method where iterative experiments on the system are used to obtain a gain estimate. The iterations produce monotonically increasing gain estimates for LTI systems and the convergence point can be made arbitrarily close to the system gain by using long enough experiments. In (Hjalmarsson, 2004) it is illustrated, by way of an example, that power iterations also may be useful for certain nonlinear systems. The method is called power iterations due to its close connection with the power method, used in linear algebra to iteratively compute an approximation of the largest eigenvalue of a symmetric matrix. The power method appears in many standard books on matrix analysis, *e.g.* (Golub and van Loan, 1983). We will now describe how the iterations work in the linear case.

3.1 The L_2-gain of Finite Data Discrete LTI Systems

This section deals with the L_2-gain of finite data discrete linear time invariant (LTI) systems. Consider the linear filter

$$G(q) = \sum_{k=0}^{\infty} g(k) q^{-k} \tag{8}$$

where $g(k)$ denotes the impulse response. In practical experiments, the input to the system (8) is always a sequence of *finite* length. Denote this finite sequence of length N by $u_N = (\, u(1) \;\; u(2) \;\; \ldots \;\; u(N) \,)^T$. The output, which also is a finite sequence of length N, is denoted by $w_N = (\, w(1) \;\; w(2) \;\; \ldots \;\; w(N) \,)^T$. In matrix representation the relation between the finite input and the output therefore is

$$w_N = \mathbf{G} u_N \tag{9}$$

where

$$\mathbf{G} = \begin{pmatrix} g(0) & 0 & 0 & 0 & \ldots & 0 \\ g(1) & g(0) & 0 & 0 & \ldots & 0 \\ g(2) & g(1) & g(0) & 0 & \ldots & 0 \\ \vdots & \ddots & \ddots & \ddots & & \\ g(N-1) & \ldots & & g(2) & g(1) & g(0) \end{pmatrix} \tag{10}$$

Now we would like to obtain an estimate of the L_2-gain of the system (8) using the definition (6). An input sequence of finite length gives an underestimation of the gain since it holds that

$$\frac{||w||}{||u||} \geq \frac{||w_N||}{||u_N||} \tag{11}$$

Thus for the system (8) with a finite input sequence of length N a lower bound for the L_2-gain can be calculated according to

$$\max_{u_N} \frac{||w_N||}{||u_N||} = ||\mathbf{G}|| = \overline{\sigma}(\mathbf{G}) = \sqrt{\lambda_{max}(\mathbf{G}^*\mathbf{G})} \tag{12}$$

Here $\overline{\sigma}$ denotes the maximum singular value and λ_{max} the eigenvalue of largest modulus. Therefore what we now would like to calculate is the largest eigenvalue of the matrix $\mathbf{G}^*\mathbf{G}$ and the corresponding eigenvector, which is the input sequence that maximizes the gain calculated in (12). A procedure to estimate the largest eigenvector is the power method described in the next section.

3.2 The Power Method

The goal of this section is to estimate the largest eigenvalue of $\mathbf{G}^*\mathbf{G}$. Therefore define the symmetric matrix

$$A = \mathbf{G}^*\mathbf{G} \tag{13}$$

and let λ_{max} be the eigenvalue of A with largest modulus and v_{max} its corresponding eigenvector with length 1. The power method can be used to find an estimate of v_{max} and λ_{max} in the following way. Starting with vector v_0, calculate $v_{k+1} = \frac{Av_k}{||Av_k||}$ for $k = 0, 1, 2, \ldots$. The iterations converge to v_{max} and an approximation of the largest eigenvalue is $\lambda_{max} \approx v_k^T A v_k$. For convergence the initial vector v_0 must have a component in the direction of v_{max}. In the experiments treated in Section 4 we will chose a white input sequence as initial vector to ensure this property.

3.3 The Power Iterations

This section is a practical implementation of the power method described in Section 3.2. Consider a system with input $u(t)$ and output $w(t)$. The power iteration algorithm is as follows:

(1) Let $k = 0$ and select an arbitrary input sequence $\{u_k\}_{t=1}^{N}$ with $\|u_k\|_2 = \eta$.
(2) Perform experiments where the input sequence $u_k(t)$ is applied to the system and use (2) and (4) to calculate the output $w_k(t)$.
(3) Calculate the gain $\hat{\beta} = \frac{\|w_k\|_2}{\eta}$.
(4) Let $u_{k+1}(t) = \frac{w_k(-t)}{\hat{\beta}}$.
(5) Let $k = k + 1$ and go to Step 2.

The reversal of time in Step 3 is a mere technicality that has to do with multiplying with the adjoint matrix, compare with equation (13). Particular to nonlinear systems is that the gain may be dependent on the norm of the input signal. Therefore, in Step 1 different norms η may give rise to more or less accurate gain estimates. Here, as an illustration of the power iteration, we first present a linear example. In this example we also consider the case where a disturbance is added to the output.

Example 1. *Consider a linear system with magnitude plot shown in Fig. 4.*

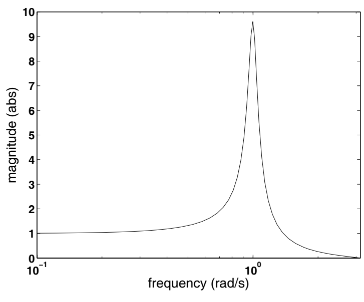

Fig. 4. Magnitude plot of the frequency response in Example 1.

We see that the maximum gain occurs at $\omega \approx 1$ *rad/s. As initial input, a white noise sequence of length* $N = 200$ *and variance* 1 *is applied. Then,* 10 *power iterations are performed. The gain estimate versus iteration number is shown in Fig. 5. We see that the gain increases monotonically to a lower bound for the true gain of the system. The gap to the true gain can be made arbitrarily small by increasing the experiment time suitably.*

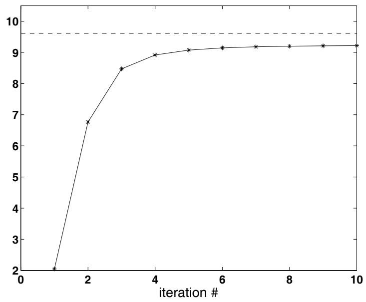

Fig. 5. The gain in Example 1 versus power iteration number (solid line). The true gain of the system is marked with dashed line.

The output in the last iteration is shown in Fig. 6. Clearly, this is approximately a sinusoid with frequency around 1 *rad/s.*

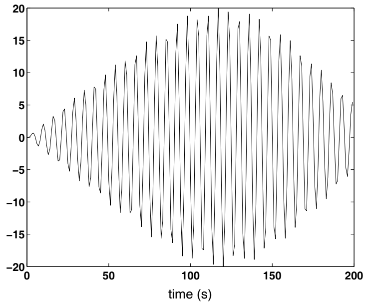

Fig. 6. The output in Example 1 after 10 power iterations.

Now consider the case where a white disturbance with variance 2 *is added to the output, which means that the signal to noise ratio is below one. In Fig. 7 the gain versus iteration number is plotted. The output in the last iteration as well as the disturbance is plotted in Fig. 8. We see that despite the disturbance the gain converges to the lower bound for the true gain of the system.*

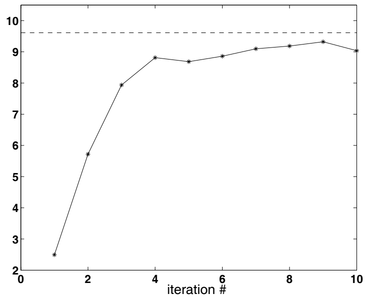

Fig. 7. The gain in Example 1 versus power iteration number (solid line) in case of an output disturbance. The true gain of the system is marked with dashed line.

For nonlinear systems there is at present no theoretical support for power iterations. They produce, of course, lower bounds to the induced gain but nothing is known about their accuracy. However, simulation examples in (Hjalmarsson, 2004) indicate that the method produces useful results for at least some nonlinear systems. An interesting observation is that if the iterations do not converge to a sinusoid, the system must be nonlinear. Here, we present another simulation example

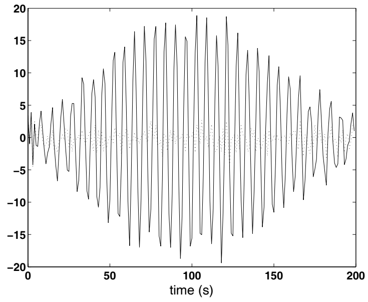

Fig. 8. Solid line: the output in Example 1 in case of an output disturbance after 10 power iterations. Dotted line: the output disturbance.

that further encourages the use of power iterations for nonlinear system.

Example 2. *Consider the nonlinear system with input v and output w defined by*

$$w = \Delta_\varphi(v) \quad \Leftrightarrow \quad \begin{cases} \dot{z} = w, & z(0) = 0 \\ w = \varphi(v - z), \end{cases} \tag{14}$$

where $\varphi(x)$ *is defined by*

$$\varphi(x) = \begin{cases} x - \mathrm{sign}(x), & |x| > 1 \\ 0, & |x| \leq 1 \end{cases} \tag{15}$$

The system is depicted in Fig. 9.

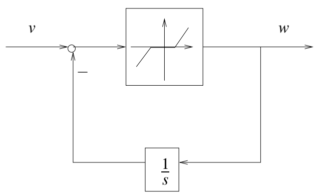

Fig. 9. The nonlinear system in Example 2.

The L_2*-gain from input to output is no greater than* 1*, see (Jönsson and Megretski, 2000). A white signal of length* $N = 500$ *and with variance* 1 *is applied as the initial input. Then,* 35 *power iterations are performed. The gain estimate versus iteration number is shown in Fig. 10. We see that the gain increases monotonically to a lower bound for the true gain of the system.*

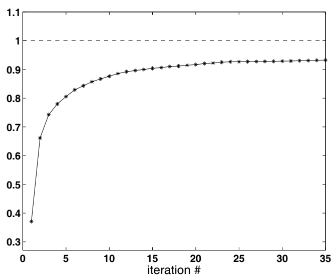

Fig. 10. The gain in Example 2 versus power iteration number (solid line). The true gain of the system is marked with dashed line.

The output in the last iteration is shown in Fig. 11. Clearly, this is neither a sinusoid nor a white signal.

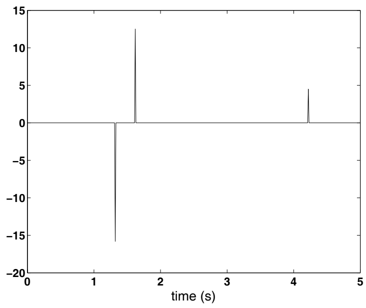

Fig. 11. The output in Example 2 after 35 power iterations.

4. EXPERIMENTAL RESULTS

The aim of this section is to evaluate the stability margins of the induction machine drive through experiments. In this paper, we use a new approach for doing this, namely power iterations. These new results are presented in Section 4.2. However, to facilitate the comparison of the new results in this paper and the previous ones in (Mosskull *et al.*, 2003), we summarize in Section 4.1 the experimental results previously presented.

All experiments have been done on an industrial hardware-in-the-loop simulator at Bombardier Transportation, Sweden. An indication of the operating conditions are given by the following data:

DC-link capacitance	$C = 0.004$ F
DC-link inductance	$L = 0.005$ H
DC-link resistance	$R = 0.04$ Ohm
Nominal DC-link voltage	1700 V
Nominal motor speed	29 Hz

The resonance frequency of the input filter is given by $\omega_0 = 1/\sqrt{LC}$ and the damping factor by $\zeta = R\sqrt{C}/(2\sqrt{L})$. The damping factor for the system in the example will be around 0.02 and the resonance frequency equals 35.5 Hz.

The properties of the drive depend *e.g.* on the motor speed and torque load. Here we have evaluated the system at 100 different motor speeds between 5.8 and 47.85 Hz at equidistant increments. The torque has been set to zero.

The linear transfer functions G_0 are estimated in the frequency range 10 to 180 Hz. The model error $v_{NL}(t)$ is calculated in open-loop from the measured voltage and current. The gain from u_d to w is then estimated as

$$\hat{\beta} = \frac{\|w\|_2}{\|u_d\|_2}. \quad (16)$$

4.1 Previous Experimental Results

This section is a summary of the experimental results in (Mosskull *et al.*, 2003), where power iterations were not used. Instead, the choice of input sequence was done with physical insight of the system rather than in a systematic way. By studying the magnitude plot of the linear transfer function $Z_{DC}/(1 + G_0 Z_{DC})$ given in Fig. 12 we can assume that an input signal with energy around the frequency which corresponds to the maximum gain of the linear transfer function will give high gain. Therefore, as a qualified estimate of the maximizing input signal, sinusoidal signals with frequency equal to the resonance frequency were used in the previous work to excite the system.

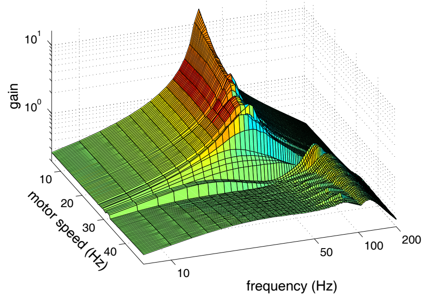

Fig. 12. Magnitude plot of $\frac{Z_{DC}}{1+G_0 Z_{DC}}$ as a function of motor speed. The high gain peaks around the resonance frequency of the transfer function occur at low motor speed.

Fig. 13 shows the experimental results of the estimated stability gains for zero torque as a function of motor speed. This result shows that the gains are well below one. It should be noticed that the controller of the motor has been carefully designed to optimize stability and performance.

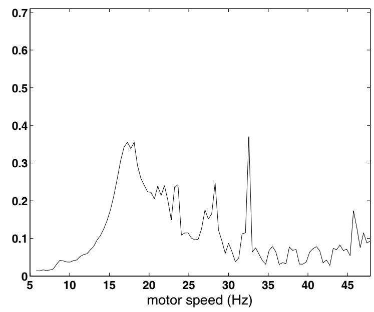

Fig. 13. Estimated stability gains $\hat{\beta}$ for zero torque as a function of motor speed for sinusoidal input.

4.2 New Experimental Results Using Power Iterations

The purpose of this contribution is to use a more systematic approach to finding the maximum gain. Therefore, the method of power iterations, described in Section 3, is now used to calculate the maximum gain of the loop in Fig. 3. A white initial input disturbance sequence of DC-link voltage $u_d(t)$ with zero mean and variance 1 multiplied with amplitude 84.8 V was used. All input sequences applied to the system have been normed so that they have the same norm as the input sequence used in the experiments described in Section 4.1. To avoid tripping the system, the amplitude of the input sequence was limited to 200 V. The number of iterations in the power method for each motor speed was 19.

In Fig. 14, the gains after zero and 19 iterations for all motor speeds are shown. Note that the gain after zero iteration is the gain from a white noise input. The gain is below one for all speeds and therefore the small gain theorem implies that we have stability margin. In Fig. 15 the gain versus iteration number for the motor speeds 7.5, 34 and 44 Hz are shown. We see that after a few iterations the gain converges at speeds 34 and 44 Hz. However, for motor speed 7.5 Hz, the gain does not change in the iterations. This is the case for all speeds lower than 30 Hz.

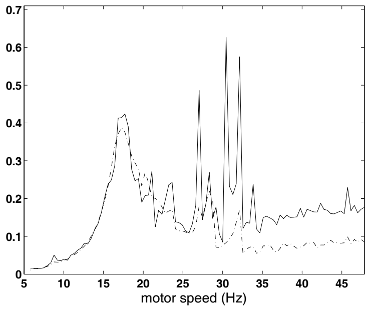

Fig. 14. Estimated stability gains $\hat{\beta}$ calculated with power iterations as a function of motor speed. The gain after zero iteration (white noise input) is the dash-dotted line and the solid line is the gain after 19 iterations.

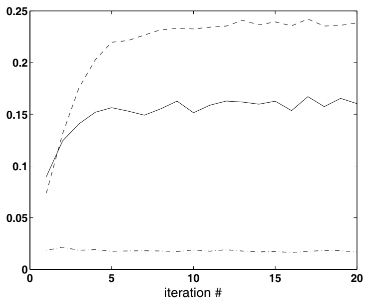

Fig. 15. Estimated stability gains $\hat{\beta}$ versus iteration number in the power method. The motor speeds are 7.5 (dash-dotted), 34 (dashed) and 44 (solid) Hz.

We see, comparing Fig. 14 and 13, that at low motor speeds, the power iterations give about the same maximum gain compared to the case where the input is a single sinusoid with frequency equal to the resonance frequency of the linear filter in Fig. 12. We also see that for low motor speeds the gain does not change, no matter the number of iteration. This implies that the gain of the system at these operating points is the same, whatever the choice of input. A most likely explanation for this is that the model error $v_{NL}(t)$, see Section 2, is small in comparison to disturbances in the system. These disturbances are *e.g.* due to measurement offsets. At higher motor speeds however, the power iterations give input sequences that result in higher gain. This in turn suggests that the model error is large in comparison to the disturbances at high speeds.

We conclude that for low frequencies, the power method does not result in higher gain than what was presented in (Mosskull *et al.*, 2003). We believe that this is due to the dominating stator frequent disturbances at these operating points. For high motor speeds however, we see that the input sequences generated by the power iterations give higher gain. These input sequences have a much broader frequency spectrum than sinusoidal signals.

Since there is no theoretical proof that power iterations converge to the true maximum gain for nonlinear systems and since there is an input amplitude constraint in the experiments described in this paper one cannot be sure that the true maximum gain of the loop in Fig. 3 has been reached. However one can conclude that at high motor speeds the power iterations result in a *higher* gain than the inputs used in the previous work to excite the system.

5. CONCLUSIONS

Here, the work presented in (Mosskull *et al.*, 2003) on the validation of stability margins for a nonlinear induction machine drive is extended and improved. The gain of the unmodelled nonlinear dynamics has been estimated in a new and more structured way using power iterations, introduced in (Hjalmarsson, 2004) and further treated in this paper. Then, the small gain theorem is applied to verify the stability of the system. The experimental results suggest that the system in Fig. 3 is stable, since the gain is well below one. In the previous work single sinusoids were used as input signals to estimate the gain. This work, however, shows that for high motor speeds input sequences with broader frequency spectrum than single sinusoids result in higher gain. Therefore, power iterations are especially useful for estimating the gain at high motor speeds. For these motor speeds the input obtained with the power iterations in this paper results in higher gain than the input signal used in the previous paper. Future work includes continued investigation on the application of the power method to generate the maximizing input of nonlinear systems.

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