IMPROVED STATE ESTIMATION IN AN MPC ALGORITHM BASED ON FUZZY DECISION

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Abstract: Due to the difficulties arising in state estimation in Model Predictive Control (MPC) algorithms, Kalman filtering and dynamic matrix control (DMC) estimation approaches were combined in the current work. Then a weighting average of both estimated states was passed to the algorithm. To determine the weighting coefficient of the mentioned average, a fuzzy supervisor was designed to control the combined estimation. An industrial process 'heavy oil fractionator' was used for simulation. The results demonstrated the improved performance of the approach particularly in better disturbance rejection capability.

Keywords: State Estimation, Predictive Control, Kalman Filter, Fuzzy Supervision, Process Control.

1. INTRODUCTION

Model Predictive Control (MPC) has emerged as a powerful practical control technique during last years. Its strength lies in that it uses step (or impulse) response data which are physically intuitive, and that it can handle hard constraints explicitly through on-line optimization. Various MPC techniques such as Dynamic Matrix Control (DMC) (Cutler and Remaker, 1980), Model Algorithmic Control (MAC) (Rouhani and Mehra, 1982), and Internal Model Control (IMC) (Garcia *et al*., 1989) have demonstrated their effectiveness in industrial applications during the past 20 years (Cutler and Remaker, 1980; Richalet *et al*., 1987; Qin and Badgwell, 2003). One drawback of these 'traditional' MPC techniques has been that, their generalization to more complex cases has been difficult, because they are developed in an unconventional manner using step response models. For example, most of the traditional techniques incorporate feedback into the algorithm in an *ad hoc* way, such as by adding a constant bias term in the prediction of the future outputs. In addition, because of the use of step response models, the traditional techniques are not applicable to integrating systems, which are common in chemical process industries. Lately, there have been efforts to interpret MPC in a state-space framework. This not only permits the use of well-known state-space theorems, but also allows straightforward generalization to more complex cases such as systems with general stochastic disturbances and measurement noise.

Li *et al* (1989) and Navratil *et al* (1988) showed that the step response model can be put into the general state-space model structure and presented an MPC technique using the tools available from stochastic optimal control theory. They showed how open-loop and closed-loop observers can be incorporated into the predictive control framework to improve regulatory control of MPC. Ricker (1990) showed how an MPC algorithm similar to the conventional MPC techniques can be developed based on a general state-space model. Lee *et al* (1992) proposed a state-space MPC technique applicable to general multi-rate sampled-data systems. Bitmead *et al* (1990) presented a lucid and detailed analysis of the basic features inherent in all MPC algorithms from the viewpoint of Linear Quadratic Regulator and Linear Quadratic Gaussian Control theory.

In section 2, we present a state-space model expressed in terms of step response parameters and in next session two state estimation techniques are discussed. In section 4 a fuzzy system for supervising the combined estimator is introduced and in the last section simulation results are shown.

2. STATE-SPACE MPC MODEL

This section demonstrates how the step response data can be put in a standard state-space form for stable and integrating systems (Lee *et al.*, 1994). The conventional step response model was extended to include integrating dynamics in a manner such that all the desirable features of the step response are retained.

2.1 Model form

The model we use in this work is the following state-space model represented by step response coefficients:

$$\boldsymbol{Y}(k) = \boldsymbol{MY}(k-1) + \boldsymbol{S}\Delta\boldsymbol{u}(k-1) + \boldsymbol{T}\Delta\boldsymbol{u}(k-1) \quad (1)$$

$$\boldsymbol{y}(k) = \boldsymbol{NY}(k) \quad (2)$$

$$\hat{\boldsymbol{y}}(k) = \boldsymbol{y}(k) + \boldsymbol{v}(k) \quad (3)$$

Where

$$\boldsymbol{Y}(k) = \left[\boldsymbol{y}_o^T(k), \boldsymbol{y}_1^T(k), \ldots\ldots, \boldsymbol{y}_{n-2}^T(k), \boldsymbol{y}_{n-1}^T(k), \boldsymbol{x}_P^T(k), \boldsymbol{x}_d^T(k)\right]^T \quad (4)$$

y(k), u(k) and ***d(k)*** are output, input and disturbance vectors, respectively and $\Delta\boldsymbol{u}(k)$ and $\Delta\boldsymbol{d}(k)$ are the changes in ***u(k)*** and ***d(k)*** from the previous sampling time. The vector ***Y(k)*** represents dynamic states of the system and $\hat{\boldsymbol{y}}(k)$ is the noise-corrupt measurement of ***y(k)*** and:

$$\boldsymbol{M} = \begin{bmatrix} \boldsymbol{0} & \boldsymbol{I}_{n_y} & \boldsymbol{0} & \cdots & \boldsymbol{0} & \boldsymbol{0} & \boldsymbol{0} & \boldsymbol{0} \\ \boldsymbol{0} & \boldsymbol{0} & \boldsymbol{I}_{n_y} & \cdots & \boldsymbol{0} & \boldsymbol{0} & \boldsymbol{0} & \boldsymbol{0} \\ \vdots & \vdots & \vdots & & \vdots & \vdots & \vdots & \vdots \\ \boldsymbol{0} & \boldsymbol{0} & \boldsymbol{0} & \ddots & \boldsymbol{0} & \boldsymbol{I}_{n_y} & \boldsymbol{0} & \boldsymbol{0} \\ \boldsymbol{0} & \boldsymbol{0} & \boldsymbol{0} & & \boldsymbol{0} & \boldsymbol{I}_{n_y} & \boldsymbol{I}_{n_y} & \boldsymbol{0} \\ \boldsymbol{0} & \boldsymbol{0} & \boldsymbol{0} & \cdots & \boldsymbol{0} & \boldsymbol{0} & \boldsymbol{A}_p & \boldsymbol{C}_d \\ \boldsymbol{0} & \boldsymbol{0} & \boldsymbol{0} & \cdots & \boldsymbol{0} & \boldsymbol{0} & \boldsymbol{0} & \boldsymbol{A}_d \end{bmatrix}$$

$$\times (n+1)n_y + dim\{x_d\} \quad (5)$$

$$\boldsymbol{N} = \Big[\overbrace{\boldsymbol{I}_{ny} \quad \boldsymbol{0} \quad \boldsymbol{0} \quad \cdots \quad \boldsymbol{0} \quad \boldsymbol{0} \quad \boldsymbol{0}}^{(n+1)n_y + dim\{x_d\}}\Big] \quad (6)$$

$$\boldsymbol{S} = \begin{bmatrix} \boldsymbol{S}_1 \\ \boldsymbol{S}_2 \\ \vdots \\ \boldsymbol{S}_{n-1} \\ \boldsymbol{S}_n \\ \boldsymbol{S}_{n+1} - \boldsymbol{S}_n \\ \boldsymbol{0} \end{bmatrix}, \quad \boldsymbol{T} = \begin{bmatrix} \boldsymbol{0} \\ \boldsymbol{0} \\ \vdots \\ \boldsymbol{0} \\ \boldsymbol{0} \\ \boldsymbol{B}_d \end{bmatrix} \quad (7)$$

$$\boldsymbol{S}_i = \begin{bmatrix} \boldsymbol{S}_{11,i} & \boldsymbol{S}_{12,i} & \cdots & \boldsymbol{S}_{1n_u,i} \\ \boldsymbol{S}_{21,i} & \boldsymbol{S}_{22,i} & \cdots & \boldsymbol{S}_{2n_u,i} \\ \vdots & \vdots & \ddots & \vdots \\ \boldsymbol{S}_{n_y1,i} & \boldsymbol{S}_{n_y2,i} & \cdots & \boldsymbol{S}_{n_yn_u,i} \end{bmatrix}, i = 1, \ldots, n \quad (8)$$

$\boldsymbol{S}_{lm,i}$ is the *i*th step response coefficient relating the *m*th input to the *l*th output, n_u and n_y are the number of inputs and outputs, respectively. $\boldsymbol{A}_P$ is a diagonal matrix of the following form:

$$\boldsymbol{A}_P = \begin{bmatrix} a_1 & & & \\ & \ddots & & \\ & & \ddots & \\ & & & a_{n_y} \end{bmatrix} \quad (9)$$

$$a_i = \begin{cases} 0 & \text{if } i\text{th output is a stable (non-integrating) output} \\ 1 & \text{if } i\text{th output is an integrating output} \end{cases} \quad (10)$$

It is assumed here, after n time steps, all the effects of stable dynamics settle and the step responses of non-integrating and integrating outputs remain constant and increase with a constant slope respectively. It is assumed that all the eigenvalues of $\boldsymbol{A}_d$ lie strictly reside the unit disk making the disturbance stable (except for the integrating dynamics already present in ***M***).

3. STATE ESTIMATION

In this section, two state estimation techniques for the step response model (1-3) is presented.

3.1 Optimal estimator form

For the system (1-3), the optimal estimator (i.e Kalman filter) based on the measurements at time *k* is most conveniently expressed in the following two-step form:

Model prediction:

$$\boldsymbol{Y}(k \mid k-1) = \boldsymbol{MY}(k-1 \mid k-1) + \boldsymbol{S}\Delta\boldsymbol{u}(k-1) \quad (11)$$

Correction based on measurements:

$$\boldsymbol{Y}(k \mid k) = \boldsymbol{Y}(k \mid k-1) + \boldsymbol{K}\{\hat{\boldsymbol{y}}(k) - \boldsymbol{NY}(k \mid k-1)\} \quad (12)$$

The notation $\boldsymbol{Y}(l \mid m)$ represents the estimate of ***Y(l)*** based on the measurements up to time *m*. *k* is the optimal filter gain that can be calculated from (Astrom and Wittenmark, 1984):

$$\boldsymbol{K} = \sum\nolimits_s \boldsymbol{N}^T\left(\boldsymbol{N}\sum\nolimits_s \boldsymbol{N}^T + \boldsymbol{V}\right)^{-1} \quad (13)$$

Where the $((n+1)\times n_y + dim\{x_d\}\times(n+1)\times n_y + dim\{x_d\})$ matrix $\sum_s$ is the steady-state solution (i.e. the asymptotic solution as $k\to\infty$) of the following Riccati difference equation:

$$\boldsymbol{\Sigma}(k) = \boldsymbol{M\Sigma}(k-1)\boldsymbol{M}^T - \boldsymbol{M\Sigma}(k-1)\boldsymbol{N}^T\left(\boldsymbol{N\Sigma}(k-1)\boldsymbol{N}^T + \boldsymbol{V}\right)^{-1}\times \boldsymbol{N\Sigma}(k-1)\boldsymbol{M}^T + \boldsymbol{TWT}^T \quad (14)$$

In the general case of the state space models, a similar Riccati equation should be solved to obtain the Kalman filter gain, even of much larger dimension.

3.2 Constant output disturbance

For stable processes, most conventional MPC algorithms use the same form of feedback ,based on comparing the current measured process output $\boldsymbol{y}_k^m$ to the current predicted output $\boldsymbol{y}_k$:

$$\boldsymbol{b}_k = \boldsymbol{y}_k^m - \boldsymbol{y}_k \quad (15)$$

The bias $\boldsymbol{b}_k$ term is added to the model for use in subsequent predictions. This form of feedback is equivalent to assuming that a step disturbance enters at the output and remains constant for all future time (Lee *et al*., 1994, Morari and Lee, 1991). Muske and Rawlings (1993) analyzed this assumption in the context of the Kalman filter; the corresponding filter gain for a system with augmented outputs is :

$$\boldsymbol{K} = [\boldsymbol{0}\ \boldsymbol{I}]^T \quad (16)$$

which means no feedback for the process state estimates and identity feedback for the output disturbances. The assumption of constant output disturbance is called "DMC scheme" (Qin and Badgwell, 2003).

Kalman filter gives unbiased estimate of states. however, in the presence of modeling errors , the final control signals will lead to biased outputs. In other words, although Kalman filter can help state space MPC algorithms to give better *regulatory* responses, the *tracking* response might be worse than conventional algorithms. One way to encounter this problem is using the above DMC approach. Clearly in this case we don't have the disturbance rejection characteristic of Kalman filter and *regulatory* responses will descend.

There is a simple idea for having a trade-off between both approaches. Because the DMC estimator does not add any computational cost to the algorithm, states are driven from both estimators and then a weighting average of them is given to the algorithm. So some aspects of both mentioned properties (setpoint tracking and disturbance rejection) are obtained.

Consider that $\boldsymbol{Y}_{kf}$ and $\boldsymbol{Y}_{dmc}$ are estimated states of Kalman and DMC estimation techniques, respectively. The estimated state that will be given to the algorithm is:

$$\boldsymbol{Y} = \alpha\, \boldsymbol{Y}_{kf} + (1-\alpha)\, \boldsymbol{Y}_{dmc} \quad (17)$$

Which α is a tuning parameter in [0 1]. If α is increased, effect of Kalman estimator in computing ***x(k)*** is more than DMC estimator and consequently better regulation will result and vice-versa.

4. FUZZY SUPERVISOR

A fuzzy supervisor was designed to take mean and variance of recent outputs as input and computed α based on a fuzzy decision rule base. The basic concept behind of this idea is that, when the mean value of recent samples of one output is in a distant from desired setpoint, tracking of that output is weak and vice versa. On the other hand, large standard deviation of one output signal means that the controller can not reject disturbances well.
Defining the following variables:

$diff_k = |\text{mean}(\boldsymbol{y}_k) - stp_k|$ **:** difference between the mean value and the corresponding setpoint of each output.
var_k **:** variance of each output signal, $k=1,2,\ldots,n$

Where n is the number of outputs. Considering 100 sampled sequence of each output for computing $mean(\boldsymbol{y}_k)$ and var_k. The membership functions for input and output variables are shown in figures 1 to 3.

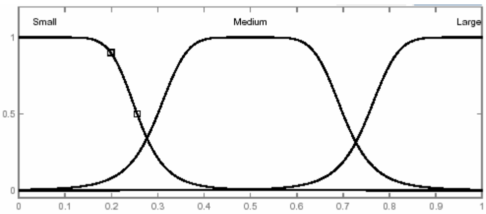

Fig.1. Membership function for $diff_k$

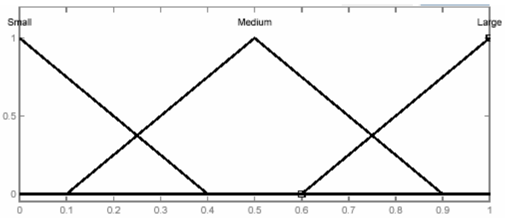

Fig.2. Membership function for var_k

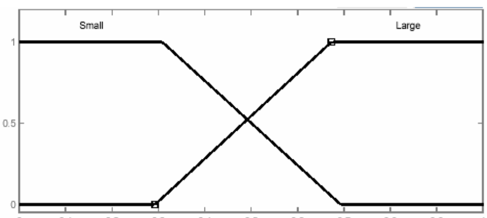

Fig.3. Membership function for α

Range of variations for $diff_k$ and var_k are selected for the special plant that will be used in simulation section and may differ in other problems. The fuzzy supervisor has a *mamdani* rule base.

5. SIMULATION

The chosen model corresponds to a distillation column. The model which is referred as 'heavy oil fractionator' is described by Prett and Morari (1987) and has been widely used to try different control strategies for distillation columns (Camacho and Bordons, 1999).

The process shown in figure 4 and has three variables that have to be controlled: The top and side product compositions and the bottom temperature. The related setpoints are 0.5, 0.3 and 0.1, respectively. In the 100 step simulation running of this work, top draw ($Y_1(s)$) setpoint was changed to 0.4 at 50^{th} step. The manipulated variables are the top draw rate, the side draw rate and the bottom reflux duty. The bottom temperature must be kept within limits fixed by operational constraints. The top end point had to be maintained within the minimum and maximum value of -0.5 and 0.5. The manipulated variables were also constrained as follows: all draws were kept within hard minimum and maximum bounds of -0.5 and 0.5. The bottom reflux duty was also constrained by -0.5 and 0.5. The maximum allowed slew rates for all manipulated variables were of 0.05 per minute. The following relation can then describe the dynamics of the process:

$$\begin{bmatrix} Y_1(s) \\ Y_2(s) \\ Y_3(s) \end{bmatrix} = \begin{bmatrix} \frac{4.05e^{-27s}}{1+50s} & \frac{1.77e^{-27s}}{1+50s} & \frac{5.88e^{-27s}}{1+50s} \\ \frac{5.39e^{-27s}}{1+50s} & \frac{5.72e^{-27s}}{1+60s} & \frac{6.9e^{-27s}}{1+40s} \\ \frac{4.38e^{-27s}}{1+33s} & \frac{4.42e^{-27s}}{1+44s} & \frac{7.2}{1+19s} \end{bmatrix} \begin{bmatrix} U_1(s) \\ U_2(s) \\ U_3(s) \end{bmatrix} \quad (18)$$

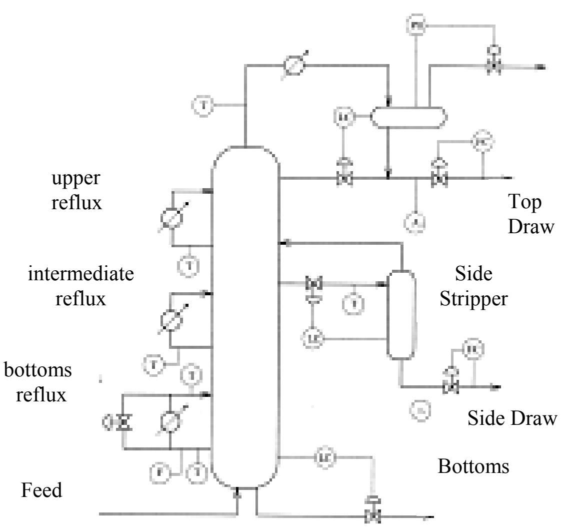

Fig.4. Heavy oil fractionator

Where $U_1(s)$, $U_2(s)$ and $U_3(s)$ correspond to the top draw , side draw and bottom reflux duties and $Y_1(s)$, $Y_2(s)$ and $Y_3(s)$ correspond to the top end point composition, side end point compositions and bottom reflux temperature respectively.

The upper reflux duty is considered to act as unmeasurable disturbance. The small signal dynamic load model for the upper reflux duty is given by the following transfer function:

$$\begin{bmatrix} Y_{p1}(s) \\ Y_{p2}(s) \\ Y_{p3}(s) \end{bmatrix} = \begin{bmatrix} \frac{1.44e^{-27s}}{1+40s} \\ \frac{1.83e^{-15s}}{1+20s} \\ \frac{1.26}{1+32s} \end{bmatrix} \begin{bmatrix} U_1(s) \\ U_2(s) \\ U_3(s) \end{bmatrix} \quad (19)$$

This model is considered as the internal model which is used by the controller to compute manipulated variables. Some gain perturbations are applied to the model based on table (9.4) of Maciejowski (2002) and this representation of the system is considered as the actual plant model, so there is a mismatch between the plant and the model. First model is used to generate an step response model of the system and then is converted to state space model for State-Space algorithm.

Different experiments were implemented; First, using a DMC estimation approach for updating the states of the system, and then, by using a Kalman filter, the disturbance rejection capability of the controller was shown. In both cases prediction and control horizons were considered to be equal to 15 and 5 respectively. The weighting matrices used in the quadratic programming were the same for both cases.

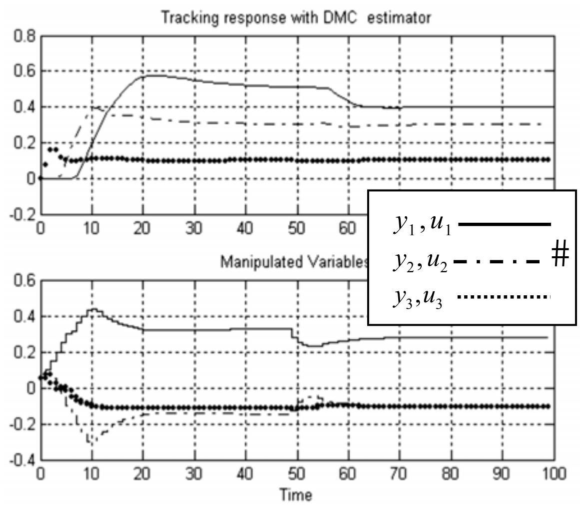

Fig.5. Tracking response using DMC estimator

As can be seen in figure 6 despite of having a model for unmeasured disturbance, in the primary controller design, acceptable action in the face of disturbance was not taken place, but controller had a good setpoint tracking.

In the second design, Kalman filter leads to much better disturbance rejection, but the tracking response of the controller was descended. As mentioned above, this is due to mismatch between plant and the model.

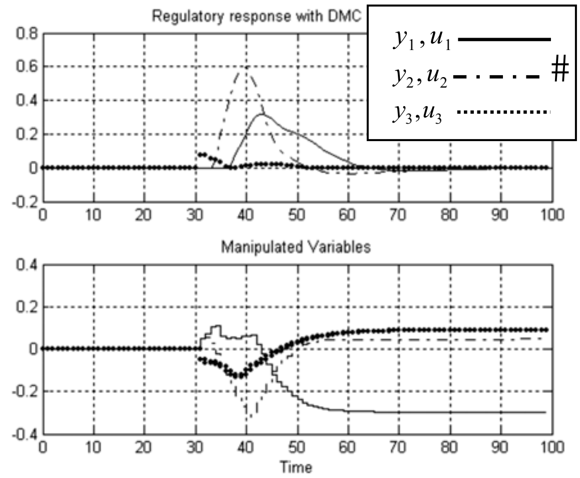

Fig.6. Regulatory response using DMC estimator

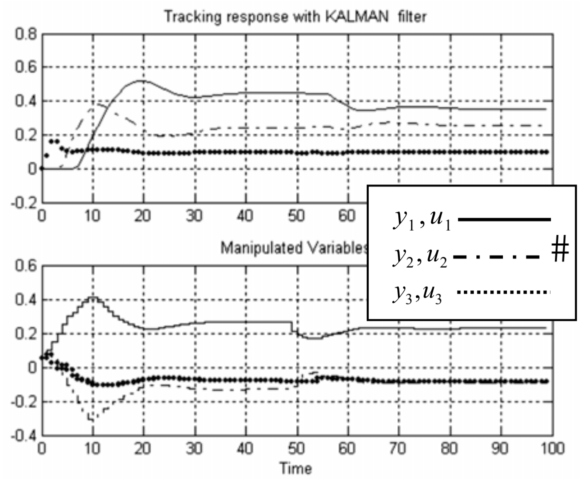

Fig.7. Tracking response using Kalman filter

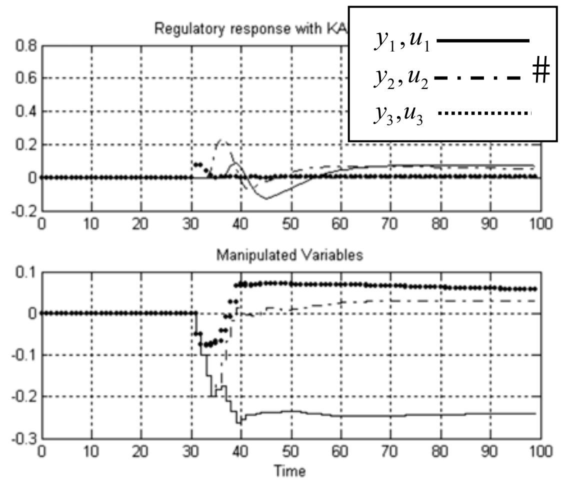

Fig.8. Regulatory response using Kalman filter

In the last simulation, both mentioned estimators were used and α was computed by the fuzzy supervisor. When setpoint tracking was descending ($diff_k$ was being large), supervisor decreased α and when disturbance rejection was poor (var_k was large), α was increased by fuzzy supervisor and therefore, much better responses were achieved.

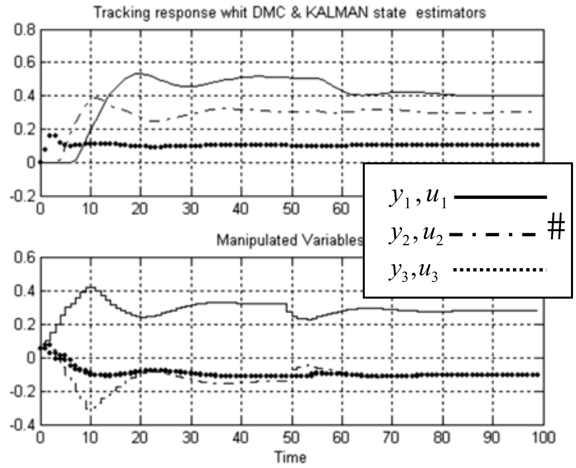

Fig.9. Tracking response using combined estimation

6. CONCLUTION

Although Kalman filter as the best choice in state estimation procedures, can help state space MPC algorithms to provide better *regulatory* responses, the *tracking* response might be worse than conventional algorithms due to plant/model mismatch. In order to reduce this problem in model predictive controllers, Kalman filter and DMC estimators were combined. A fuzzy supervisor was designed to combine the above estimations. Heavy oil fractionator was chosen as an industrial process to examine the validity of the proposed combination. It was noticed that, implementation of this approach needed approximately the same computational efforts as that in the case of using Kalman filter alone. Using the proposed combination showed much better disturbance rejection capability as well as improving the controller performance.

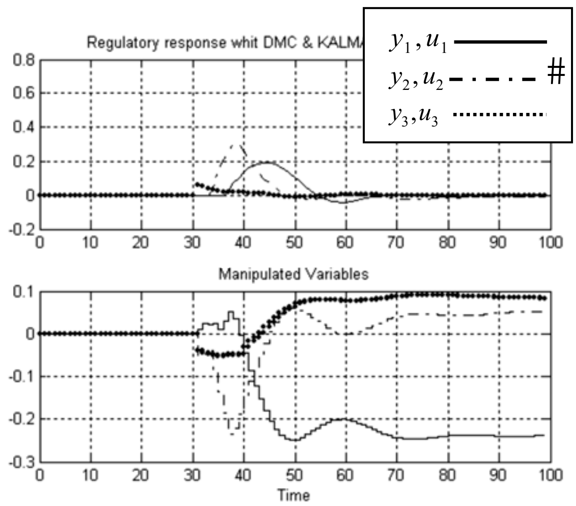

Fig.10. Regulatory response using combined estimation

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