INTERNAL MODEL BASED FRAMEWORK FOR TRACKING AND FAULT TOLERANT CONTROL OF A PERMANENT MAGNET SYNCHRONOUS MOTOR [1]

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Abstract: In this paper an adaptive internal model based control scheme is designed to deal with tracking and input disturbance suppression problems for a permanent magnet synchronous motor. More in detail we show how to design a controller able to guarantee the perfect asymptotic tracking of unknown exogenous trajectories belonging to a certain family, embedding in the regulator the internal model of this family; the theoretical machinery exploited in order to prove the global asymptotical stability of the solution exposed is the nonlinear output regulation theory, specialized for the energy-based port-Hamiltonian formalism.

Keywords: Permanent magnet motor, internal model based control, port-Hamiltonian systems, fault tolerant control

1. INTRODUCTION

In this paper we are interested in solving a tracking problem for a permanent magnet synchronous motor: this is a simple but very significant issue as the tracking of a particular velocity profile is probably the main task to take into account dealing with permanent magnet motors. Moreover the design procedure presented is proved to be able to deal with another important issue: a fault tolerant control design problem taking into account the arise of spurious harmonics in the electrical variables, superimposing to the control inputs. More in detail we show how to design a controller able to guarantee the perfect asymptotic tracking of unknown exogenous trajectories belonging to a certain family and, at the same time, overcome the possible presence of spurious harmonics superimposing to the voltage inputs; the theoretical machinery exploited in order to prove the global asymptotical stability of the solution exposed is the nonlinear output regulation theory (the regulator will embed the internal model of the possible trajectory/fault family) specialized for the energy-based port-Hamiltonian formalism. This formalism is in fact really helpful to describe the problem, starting from an energetic description of the synchronous motor, and to find an elegant solution (for synchronous permanent motor tracking literature see (Shouse and Taylor, 1994*a*), (Shouse and Taylor, 1994*b*), (Shouse and Taylor, 1998), (Zhu *et al.*, 2000), (Dawson *et al.*, 1976), (Ortega *et al.*, 2002) and references therein).

In the next section the permanent magnet synchronous motor model is presented and the tracking problem is stated: a suitable change of coordinates will be introduced in order to obtain an error system again fitting in the port-Hamiltonian framework; it will be shown that the tracking problem can be cast as a regulation and input disturbance suppression problem for the error system and, in Section 3, an internal model based controller is designed in order to globally and asymp-

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totically solve the problem. At the end of Section 3 a remark will point out that the same design procedure can be applied considering a fault tolerant control design problem: hence the proposed scheme can be considered as a comprehensive design framework for tracking and fault tolerant control for a permanent magnet synchronous motor. Section 4 concludes the work with some final remarks.

2. PROBLEM STATEMENT AND PRELIMINARY POSITIONS

Aim of this section is to introduce the model of a permanent magnet synchronous motor and to state the tracking problem that will be addressed in the rest of the paper: the motor should follow a desired velocity profile assuring, at the same time, zero flux current in order to obtain a perfect decoupling between flux and torque generation; obviously this task should be asymptotically achieved despite of the presence of an unknown constant load torque.

In the rest of the paper, the desired velocity profile will be assumed to belong to the class of signals generated by a linear, autonomous and neutrally stable system, usually called *exosystem*. In this set up, for instance, any trajectory obtained by a combination of constant and sinusoidal signals can be modelled. This assumption will allow us to cast the problem of trajectory tracking as a problem of *output regulation* as pointed out more specifically in the following.

Let now introduce the system model: a permanent magnet synchronous motor (in a rotating reference, i.e. the dq frame) can be written as a port-Hamiltonian system with dissipation (see (van der Schaft, 1999), (Ortega *et al.*, 2002)) for the state vector

$$x = M\begin{bmatrix} i_d \\ i_q \\ \omega \end{bmatrix} \quad \text{with} \quad M = \begin{bmatrix} L_d & 0 & 0 \\ 0 & L_q & 0 \\ 0 & 0 & \dfrac{j}{n_p} \end{bmatrix}$$

where i_d and i_q are the stator currents, ω the angular velocity, L_d, L_q the stator inductances, j the inertia momentum and n_p the number of pole pairs. The Hamiltonian function is defined by

$$H(x) = \frac{1}{2}x^{\mathrm{T}}M^{-1}x$$

while $J(x)$, R and g are determined as

$$J(x) = \begin{bmatrix} 0 & L_0 x_3 & 0 \\ -L_0 x_3 & 0 & -\Phi_{q0} \\ 0 & \Phi_{q0} & 0 \end{bmatrix},$$

$$R = \begin{bmatrix} R_s & 0 & 0 \\ 0 & R_s & 0 \\ 0 & 0 & 0 \end{bmatrix}, \; g = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -\dfrac{1}{n_p} \end{bmatrix}$$

with R_s the stator winding resistance, Φ_{q0} a constant term due to interaction of the permanent magnet and the magnetic material in the stator, and $L_0 = L_d n_p / j$. The stator voltages v_d and v_q are the available control inputs while the constant load torque τ_l is an unknown input. Hence the permanent magnet motor can be modelled as a port-Hamiltonian system of the form

$$\dot{x} = [J(x) - R]\frac{\partial H(x)}{\partial x} + g\begin{bmatrix} v_d & v_q & \tau_l \end{bmatrix}^{\mathrm{T}}. \qquad (1)$$

As already announced, the control objective is to make the motor follow an unknown, exogenous, desired velocity trajectory $x_3^{des} = j/n_p \; \omega^{des}$ with, at the same time, zero flux current in order to obtain a perfect decoupling between flux and torque generation (i.e. $i_d^{des} = L_d x_1^{des} = 0$).

Obviously this task should be asymptotically achieved despite of the presence of an unknown constant load torque τ_l.

Let us assume that the desired trajectory profile $h(w)$ is generated by the linear, neutrally stable autonomous system (*exosystem*)

$$\begin{aligned} \dot{w} &= Sw \\ x^{des} &= \begin{bmatrix} x_1^{des} \\ x_2^{des} \\ x_3^{des} \end{bmatrix} = h(w) = \begin{bmatrix} h_1(w) \\ h_2(w) \\ h_3(w) \end{bmatrix} \end{aligned} \qquad (2)$$

where $w \in \mathbb{R}^s$ and S is defined by

$$S = \mathrm{diag}\{S_0, S_1, \ldots, S_\alpha\}, \qquad s = 2\alpha + 1$$

with $S_0 = 0$,

$$S_i = \begin{bmatrix} 0 & \omega_i \\ -\omega_i & 0 \end{bmatrix} \qquad \omega_i > 0 \qquad i = 1, \ldots, \alpha$$

and $w(0) \in \mathcal{W}$, with $\mathcal{W} \subseteq \mathbb{R}^s$ bounded compact set. In this discussion the dimension s of matrix S will be considered known but all characteristic frequencies ω_i are unknown but ranging within known compact sets, i.e. $\omega_i^{\min} \le \omega_i \le \omega_i^{\max}$.

In this set up, the lack of knowledge of desired velocity profile reflects into the lack of knowledge of the initial state $w(0)$ of the exosystem and of the characteristic frequencies. Moreover the profile $h(w)$ will be assumed to be polynomial in w: hence the class of signal considered is no more restricted to a simple purely linear case, but will take into account a polynomial combination of constant and sinusoidal signals with unknown frequencies, amplitudes and phases.

All those assumptions allow us to cast the tracking problem as a problem of *output regulation*, (see (Byrnes *et al.*, 1997*b*), (Gentili and van der Schaft, 2003)) complicated by the lack of knowledge of the matrix S (see (Serrani *et al.*, 2001), (Bonivento *et al.*, 2004*b*)), and suggests to look for a controller which embeds an *internal model* of the exogenous signals, augmented by an adaptive part in order to estimate the characteristic frequencies.

In order to show how the port-Hamiltonian formalism could be really helpful to describe the problem and to find an elegant solution, it is now possible to define a change of coordinates and introduce a new error system again fitting in the port-Hamiltonian framework: we will point out that the tracking problem is now cast as a regulation problem complicated by the presence of exogenous "virtual" disturbance signals. In the next section this error system will be stabilized asymptotically (achieving a perfect asymptotical tracking) with the design of a canonical adaptive internal model unit.

Remark. It is worth to remark that the error change of coordinates will be suitably defined in order to obtain a damping term in the velocity dynamic: this will be really fundamental in the following, when the asymptotic behaviour of the error system will be studied. $\triangleleft$

Let us define

$$\begin{aligned} \tilde{x}_1 &= x_1 - x_1^{des} \\ \tilde{x}_2 &= x_2 - x_2^{des} - k_3\tilde{x}_3 \\ \tilde{x}_3 &= x_3 - x_3^{des} \end{aligned} \tag{3}$$

where k_3 is a suitably defined gain. Deriving the new coordinates (calling the state $\tilde{x} = (\tilde{x}_1, \tilde{x}_2, \tilde{x}_3)^{\mathrm{T}}$ and the input control vector $v = [v_d \;\; v_q]^{\mathrm{T}}$) we obtain a new error port-Hamiltonian system of the form:

$$\dot{\tilde{x}} = [J(\tilde{x}) - \tilde{R}]\frac{\partial \tilde{H}}{\partial \tilde{x}} + \tilde{g}(v - \tilde{\lambda}(\tilde{x}) - \tilde{\varphi}(\tilde{x})\psi(w)) \tag{4}$$

where the new Hamiltonian is defined by:

$$\tilde{H}(\tilde{x}) = \frac{1}{2}\tilde{x}^{\mathrm{T}} M^{-1}\tilde{x}\,,$$

the input matrix $\tilde{g}$ and the damping matrix $\tilde{R}$ are

$$\tilde{g} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \\ 0 & 0 \end{bmatrix}, \qquad \tilde{R} = \begin{bmatrix} R_s & 0 & 0 \\ 0 & R_s & 0 \\ 0 & 0 & -\dfrac{\Phi_{q0} j}{L_q n_p} k_3 \end{bmatrix}$$

and

$$\tilde{\lambda}(\tilde{x}) + \tilde{\varphi}(\tilde{x})\psi(w) = \begin{bmatrix} \tilde{\lambda}_1(\tilde{x}) + \tilde{\varphi}_1(\tilde{x})\psi_1(w) \\ \tilde{\lambda}_2(\tilde{x}) + \tilde{\varphi}_2(\tilde{x})\psi_2(w) \end{bmatrix} \tag{5}$$

with

$$\begin{aligned} \tilde{\lambda}_1(\tilde{x}) &= -\frac{L_0}{L_q} k_3 \tilde{x}_3^2 \\ \tilde{\varphi}_1(\tilde{x})\psi_1(w) &= -\frac{L_0}{\Phi_{q0}}\tilde{x}_3\left(\dot{h}_3(w) + \frac{\tau_l}{n_p}\right) + \\ &\quad - \frac{L_0}{L_q}k_3\tilde{x}_3 h_3(w) - \frac{L_0}{L_q}\tilde{x}_2 h_3(w) + \\ &\quad - \frac{L_0}{\Phi_{q0}} h_3(w)\left(\dot{h}_3(w) + \frac{\tau_l}{n_p}\right) \\ \tilde{\lambda}_2(\tilde{x}) &= \frac{R_s}{L_q}k_3\tilde{x}_3 + \frac{\Phi_{q0}}{L_q}k_3\tilde{x}_2 + \frac{\Phi_{q0}}{L_q}k_3^2\tilde{x}_3 \\ \tilde{\varphi}_2(\tilde{x})\psi_2(w) &= \frac{L_0}{L_d}\tilde{x}_1 h_3(w) + \frac{\Phi_{q0}}{j/n_p}h_3(w) + \\ &\quad + \frac{R_s}{\Phi_{q0}}\left(\dot{h}_3(w) + \frac{\tau_l}{n_p}\right) + \frac{L_q}{\Phi_{q0}}\ddot{h}_3(w)\,. \end{aligned} \tag{6}$$

It is now immediate to consider the original tracking problem as a stabilization problem with "virtual" input disturbance suppression for the error port-Hamiltonian system (4).

As the velocity trajectory to be tracked $h_3(w)$ is polynomial in w, from (5) and (6) it is straightforward to state that also the disturbance terms $\psi_1(w)$ and $\psi_2(w)$ are polynomial in w and it is possible to solve the problem following (Astolfi *et al.*, 2003) or (Gentili and van der Schaft, 2003), designing a suitable port-Hamiltonian internal model unit.

Let us impose a preliminary control action in order to compensate for the known "virtual" disturbance term $\lambda(\tilde{x})$:

$$v' = \begin{bmatrix} \tilde{\lambda}_1(\tilde{x}) + v'_d \\ \tilde{\lambda}_2(\tilde{x}) + v'_q \end{bmatrix}.$$

It is worth to point out that the controller to be designed will use for feedback the whole error state $\tilde{x}$.

3. INTERNAL MODEL UNIT DESIGN

In this section we are going to design a suitable internal model unit based controller able to globally, asymptotically regulate to the origin the state of (4) despite of the presence of the "virtual" input disturbance (5), (6); this controller represents, obviously, a solution to the original tracking problem.

In order to design the internal model unit as a port-Hamiltonian system, it is worth to recall that main proposition in (Huang, 2001) assures that, as $\psi(w)$ is polynomial in w, then there exists some set of r real numbers $a_0, a_1, \ldots, a_{r-1}$ such that

$$\begin{aligned} L_{Sw}^r\psi(w) &= a_0\psi(w) + a_1 L_{Sw}\psi(w) + \cdots \\ &\quad \cdots + a_{r-1}L_{Sw}^{r-1}\psi(w) \end{aligned} \tag{7}$$

Moreover, still in (Huang, 2001), it is assured the existence of $\hat{\omega}_0 = 0$ and $\hat{\omega}_1, \ldots, \hat{\omega}_{n_k} \in \Omega$ where $r = 2n_k + 1$ and $\Omega = \{l_1\hat{\omega}_1 + \cdots + l_k\hat{\omega}_k \geq 0\,,\, l_1, \ldots, l_k = 0, \pm 1, \pm 2, \ldots\}$, such that

$$\lambda\prod_{l=1}^{n_k}(\lambda^2 + \hat{\omega}_l^2) = \lambda^r - a_0 - a_1\lambda - \cdots - a_{r-1}\lambda^{r-1} \tag{8}$$

From (8) we immediately found out that $a_i = 0$ for i even.

Condition (7) implies that the autonomous system

$$\begin{cases} \dot{w} = Sw \\ u = \psi(w) \end{cases}$$

is immersed by a map $\bar{\rho}(w)$ into the linear observable system (see (Byrnes *et al.*, 1997*a*), (Isidori, 1995)) defined by

$$\begin{cases} \dot{\bar{w}} = \Theta\bar{w} \\ u = \Upsilon\bar{w} \end{cases} \tag{9}$$

where matrices Θ and Υ are defined as $\Theta = \mathrm{diag}\{\tilde{\Theta}, \tilde{\Theta}\}$, $\Upsilon = \mathrm{diag}\{\tilde{\Upsilon}, \tilde{\Upsilon}\}$ with

$$\tilde{\Theta} = \begin{bmatrix} 0 & 1 & 0 & 0 & \cdots & 0 & 0 \\ 0 & 0 & 1 & 0 & \cdots & 0 & 0 \\ \vdots & \vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\ 0 & 0 & 0 & 0 & \cdots & 0 & 1 \\ 0 & a_1 & 0 & a_3 & \cdots & a_{r-2} & 0 \end{bmatrix} \quad \tilde{\Upsilon} = \begin{bmatrix} 1 & 0 & 0 & \cdots & 0 \end{bmatrix}.$$

It is easy to realize that system (9) is equivalent, and therefore immersed, by means of a simple linear transformation, to the linear system

$$\begin{cases} \dot{z} \;=\; \Phi z \\ u \;=\; \Lambda z \end{cases} \tag{10}$$

where matrices Φ and Λ are defined as $\Lambda = \mathrm{diag}\{\tilde{\Lambda}, \tilde{\Lambda}\}$ and $\Phi = \mathrm{diag}\{\tilde{\Phi}, \tilde{\Phi}\}$ with $\tilde{\Lambda} = \begin{bmatrix} 1 & 0 & 1 & 0 & \cdots & 1 & 0 \end{bmatrix}$ and

$$\tilde{\Phi} = \mathrm{diag}\{\Phi_0, \Phi_1, \ldots, \Phi_k\}$$

with $\Phi_0 = 0$ and

$$\Phi_i = \begin{bmatrix} 0 & \hat{\omega}_i \\ -\hat{\omega}_i & 0 \end{bmatrix} \qquad \hat{\omega}_i > 0 \qquad i = 1, \ldots, n_k \,.$$

The linear transformation is defined by $z = T\xi$ with

$$T^{-1} = \begin{bmatrix} \Lambda^{\mathrm{T}} & \Phi^{\mathrm{T}}\Lambda^{\mathrm{T}} & \cdots & \Phi^{\mathrm{T}^{r-1}}\Lambda^{\mathrm{T}} \end{bmatrix} .$$

In the end we can write again the system to regulate (4) with the "virtual" input disturbance (5), (6) as

$$\dot{\tilde{x}} = [J(\tilde{x}) - \tilde{R}]\frac{\partial \tilde{H}}{\partial \tilde{x}} + \tilde{g}(v' - \tilde{\varphi}(\tilde{x})\Lambda z) \,. \tag{11}$$

The exosystem is now defined by (10): the dimension of the characteristic matrix Φ (i.e. $\beta = 2r = 2(2n_k + 1)$) is still assumed to be known, but again, all characteristic frequencies $\hat{\omega}_i$ will be unknown but ranging within known compact sets, i.e. $\hat{\omega}_i^{\min} \leq \hat{\omega}_i \leq \hat{\omega}_i^{\max}$.
The regulator to be designed will embed the internal model of the exogenous disturbance: this internal model unit is designed according to the procedure proposed in (Nikiforov, 1998) (*canonical internal model*); obviously, as the exosystem matrix Φ is not known, the "classical" approach will be augmented with an adaptive mechanism in order to obtain an estimate of this matrix (i.e. an estimate of the characteristic frequencies).
Given any Hurwitz matrix F and any matrix G such that (F, G) is controllable, denote by Y the unique matrix solution of the Sylvester equation

$$Y\Phi - FY = G\Gamma$$

where $\Gamma = \tilde{\varphi}(\tilde{x})\Lambda$ and define $\Psi := \Gamma Y^{-1}$.
As Φ is not know, it is impossible to calculate the solution of the Sylvester equation Y and hence Γ: hence an estimation mechanism is needed. To this aim, let introduce an adaptive internal model unit as

$$\begin{cases} \;\dot{\xi} \;=\; (F + G\hat{\Psi})\xi + N(\tilde{x}) \\ \dot{\hat{\Psi}}_{ij} \;=\; \psi_{ij}(\xi, \tilde{x}) \quad i = (1,2)\,,\; j = (1, \cdots, \beta) \end{cases} \tag{12}$$

where $\hat{\Psi}_{ij}$ represents the (ij)-th element of matrix $\hat{\Psi}$ and set the control law as

$$v' = \hat{\Psi}\xi + v_{st} \tag{13}$$

where $N(\tilde{x})$ and u_{st} are additional terms to be designed later. The adaptation law $\psi(\xi, \tilde{x})$ will be designed in order to assure that, asymptotically, the internal model unit will provide a control action able to overcome all "virtual" disturbances.
Defining the changes of coordinate

$$\begin{aligned} \chi \;\;\; &= \xi - Yz - A\tilde{x} \\ \tilde{\Psi}_{ij} \;&=\; \hat{\Psi}_{ij} - \Psi_{ij} \qquad i = (1,2)\,, \quad j = (1, \cdots, \beta) \end{aligned}$$

where matrix A is such that $A\tilde{g} = G$, system (11) with controller (12) becomes

$$\begin{cases} \;\dot{\tilde{x}} \;=\; [J(\tilde{x}) - \tilde{R}]\dfrac{\partial \tilde{H}}{\partial \tilde{x}} + \tilde{g}\tilde{\Psi}\xi + \tilde{g}\Psi\xi + \tilde{g}v_{st} - \tilde{g}\Gamma z \\ \;\dot{\chi} \;=\; (F + G\hat{\Psi})\xi + N(\tilde{x}) - Y\Phi z - A\dot{\tilde{x}} \\ \dot{\tilde{\Psi}}_{ij} \;=\; \psi_{ij}(\xi, \tilde{x}) \qquad i = (1,2)\,, \quad j = (1, \cdots, \beta)\,. \end{cases} \tag{14}$$

Note that

$$\begin{aligned} \dot{\tilde{x}} \;=\; & [J(\tilde{x}) - \tilde{R}]\frac{\partial \tilde{H}}{\partial \tilde{x}} + \tilde{g}\tilde{\Psi}\xi + \tilde{g}\Psi(\xi - Yz - A\tilde{x}) + \\ & + \tilde{g}\Psi A\tilde{x} + \tilde{g}v_{st} = \\ = \;& [J(\tilde{x}) - \tilde{R}]\frac{\partial \tilde{H}}{\partial \tilde{x}} + \tilde{g}\tilde{\Psi}\xi + \tilde{g}\Psi\chi + \tilde{g}\hat{\Psi}A\tilde{x} + \\ & - \tilde{g}\tilde{\Psi}A\tilde{x} + \tilde{g}v_{st}\,. \end{aligned}$$

Choosing $v_{st} = -\hat{\Psi}A\tilde{x} - K\tilde{g}^{\mathrm{T}}\tilde{x}$ with the gain matrix K defined as

$$K = \begin{bmatrix} k_1 & 0 \\ 0 & k_2 \end{bmatrix},$$

it is possible to write

$$\dot{\tilde{x}} = [J(\tilde{x}) - \tilde{R} - \tilde{g}K\tilde{g}^{\mathrm{T}}]\frac{\partial \tilde{H}}{\partial \tilde{x}} + \tilde{g}\tilde{\Psi}(\xi - A\tilde{x}) + \tilde{g}\Psi\chi\,.$$

Defining now a vector Δ containing every element of matrix Ψ (and defining suitably $\hat{\Delta}$ and $\tilde{\Delta}$) as

$$\Delta = \begin{bmatrix} \Psi_{11} & \cdots & \Psi_{1\beta} & \Psi_{21} & \cdots & \Psi_{2\beta} \end{bmatrix}^{\mathrm{T}}\,, \tag{15}$$

it is possible to define a suitable matrix $\Pi(\tilde{x}, \xi)$ such that

$$\Pi(\tilde{x}, \xi)\tilde{\Delta} = \tilde{g}\tilde{\Psi}(\xi - A\tilde{x}),$$

and to write

$$\dot{\tilde{x}} = [J(\tilde{x}) - \tilde{R} - \tilde{g}K\tilde{g}^{\mathrm{T}}]\frac{\partial \tilde{H}}{\partial \tilde{x}} + \Pi(\tilde{x}, \xi)\tilde{\Delta} + \tilde{g}\Psi\chi\,. \tag{16}$$

Concentrate now on the χ-dynamic in order to design suitably the update term $N(\tilde{x})$:

$$\begin{aligned} \dot{\chi} \;=\; & (F + G\hat{\Psi})\xi + N(\tilde{x}) - FYz - G\Gamma z + \\ & -\; A\Big\{[J(\tilde{x}) - \tilde{R}]\frac{\partial \tilde{H}}{\partial \tilde{x}} + \tilde{g}\hat{\Psi}\xi - \tilde{g}\hat{\Psi}A\tilde{x} + \\ & -\; \tilde{g}K\tilde{g}^{\mathrm{T}}\tilde{x} - \tilde{g}\Gamma z\Big\} = \\ & =\; F\chi + FA\tilde{x} + N(\tilde{x}) - A[J(\tilde{x}) - \tilde{R}]\frac{\partial \tilde{H}}{\partial \tilde{x}} + \\ & +\; A\tilde{g}\hat{\Psi}A\tilde{x} + A\tilde{g}K\tilde{g}^{\mathrm{T}}\tilde{x}. \end{aligned}$$

Choosing

$$\begin{aligned} N(\tilde{x}) \;=\; & -FA\tilde{x} + A[J(\tilde{x}) - \tilde{R}]\frac{\partial \tilde{H}}{\partial \tilde{x}} - A\tilde{g}\hat{\Psi}A\tilde{x} + \\ & -\; A\tilde{g}K\tilde{g}^{\mathrm{T}}\tilde{x} \end{aligned}$$

we obtain

$$\dot{\chi} = F\chi\,. \tag{17}$$

As all dynamics of (14) have been investigated, it is now possible to design an adaptation law for $\hat{\Psi}^{\mathrm{T}}$: assume then updating functions $\varphi_{ij}(x,\xi)$ such that

$$\dot{\hat{\Delta}} = -\Pi(\tilde{x},\xi)^{\mathrm{T}}\frac{\partial \tilde{H}}{\partial \tilde{x}}\,.$$

With this in mind it is immediate to write the $\tilde{\Delta}$-dynamic as

$$\dot{\tilde{\Delta}} = \dot{\hat{\Delta}} - \dot{\Delta} = -\Pi(\tilde{x},\xi)^{\mathrm{T}}\frac{\partial \tilde{H}}{\partial \tilde{x}}\,. \tag{18}$$

Consider now equations (16) with (17) and (18). This new system identifies an interconnection described by:

$$\dot{\bar{x}} = [\bar{J}(\bar{x}) - \bar{R}]\frac{\partial H_x(\bar{x})}{\partial \bar{x}} + \Lambda(\chi)$$

with

$$\bar{x} = \begin{bmatrix}\tilde{x} & \chi & \tilde{\Delta}\end{bmatrix}^{\mathrm{T}}\,,$$

the Hamiltonian $H_x(\bar{x})$ defined by

$$H_x(\bar{x}) = \tilde{H}(\tilde{x}) + \frac{1}{2}\chi^{\mathrm{T}}\chi + \frac{1}{2}\tilde{\Delta}^{\mathrm{T}}\tilde{\Delta}\,,$$

the skew-symmetric interconnection matrix $\bar{J}(\bar{x})$ defined by:

$$\bar{J}(\bar{x}) = \begin{bmatrix} J(\tilde{x}) & 0 & \Pi(\tilde{x},\xi) \\ 0 & 0 & 0 \\ -\Pi(\tilde{x},\xi)^{\mathrm{T}} & 0 & 0 \end{bmatrix}\,,$$

the positive-definite damping matrix $\bar{R}$ defined as:

$$\bar{R} = \begin{bmatrix} \tilde{R} + \tilde{g}K\tilde{g}^{\mathrm{T}} & 0 & 0 \\ 0 & -F & 0 \\ 0 & 0 & 0 \end{bmatrix}\,,$$

and $\Lambda(\chi)$ defined by:

$$\Lambda = \begin{bmatrix}\tilde{g}\Psi\chi & 0 & 0 & 0\end{bmatrix}^{\mathrm{T}}\,.$$

It is now possible to state the following proposition assuring the solution of the tracking problem considered.

Proposition 1. Consider a permanent magnet synchronous motor described by equation (1) and an exogenous trajectory profile to be tracked defined by (2). Defining the state tracking error $\tilde{x} = x - x^{des}$, the error state feedback control law generated by the internal model unit:

$$\begin{cases} \dot{\xi} \;=\; (F + G\hat{\Psi})\xi - FA\tilde{x} + A[J(\tilde{x}) - \tilde{R}]\dfrac{\partial \tilde{H}}{\partial \tilde{x}} + \\ \quad -\; A\tilde{g}\hat{\Psi}A\tilde{x} - A\tilde{g}K\tilde{g}^{\mathrm{T}}\tilde{x} \\ \dot{\hat{\Delta}} \;=\; -\Pi(\tilde{x},\xi)^{\mathrm{T}}\dfrac{\partial \tilde{H}}{\partial \tilde{x}} \\ v \;=\; \tilde{\lambda}(\tilde{x}) + \hat{\Psi}\xi - \hat{\Psi}A\tilde{x} - K\tilde{g}^{\mathrm{T}}\tilde{x} \end{cases}$$

with Δ defined in (15), matrix A such that $A\tilde{g} = G$, F an Hurwitz gain matrix and G such that (F,G) is controllable, assures, provided that k_3 and the gain matrices F and K are suitably defined according to the constructive proof, that the tracking problem is globally, asymptotically solved, i.e.

$$\lim_{t\to\infty} x = x^{des}$$

Proof. Considering $H_x(\bar{x})$ as a Lyapunov function, the proof (remembering that F is an arbitrary Hurwitz matrix and K and arbitrary gain matrix) is immediate considering the time derivative of this Lyapunov function:

$$\begin{aligned} \dot{H}_x \;&=\; -\frac{\partial^{\mathrm{T}} H_x}{\partial \bar{x}}\bar{R}\frac{\partial H_x}{\partial \bar{x}} + \frac{\partial^{\mathrm{T}}\tilde{H}}{\partial \tilde{x}}\tilde{g}\Psi\chi = \\ &=\; -\frac{1}{L_d}(R_s + k_1)\tilde{x}_1^2 - \frac{1}{L_q}(R_s + k_2)\tilde{x}_2^2 + \\ &+\; \frac{\Phi_{q0}j}{L_q n_p}k_3\tilde{x}_3^2 + \begin{bmatrix}\dfrac{\tilde{x}_1}{L_d} & \dfrac{\tilde{x}_2}{L_q}\end{bmatrix}\Psi\chi + \chi^{\mathrm{T}}F\chi\,. \end{aligned}$$

Choosing the gain k_3 such that $\dfrac{\Phi_{q0}j}{L_q n_p}k_3 < 0$, as there exist two constants $\eta_{\Psi 1} \in \mathbb{R}$ and $\eta_{\Psi 2} \in \mathbb{R}$ such that

$$\begin{bmatrix}\dfrac{\tilde{x}_1}{L_d} & \dfrac{\tilde{x}_2}{L_q}\end{bmatrix}\Psi\chi \le \eta_{\Psi 1}\|\tilde{x}_1\|\|\chi\| + \eta_{\Psi 2}\|\tilde{x}_2\|\|\chi\|\,,$$

it is possible to use Young's inequality [3] designing properly F and K and assuring that there exist $\eta_{\tilde{x}_1} \in \mathbb{R}^+$, $\eta_{\tilde{x}_2} \in \mathbb{R}^+$, $\eta_{\tilde{x}_3} \in \mathbb{R}^+$ and $\eta_{\chi} \in \mathbb{R}^+$ such that

$$\begin{aligned} \dot{H}_x \;&\le\; -\eta_{\tilde{x}_1}\|\tilde{x}_1\|^2 - \eta_{\tilde{x}_2}\|\tilde{x}_2\|^2 + \\ &\quad -\eta_{\tilde{x}_3}\|\tilde{x}_3\|^2 - \eta_{\chi}\|\chi\|^2 \le 0\,. \end{aligned} \tag{19}$$

Hence for LaSalle invariance principle, system's trajectory are asymptotically captured by the largest invariant set characterized by $\dot{H}_x = 0$; then by (19) the system will asymptotically converge to

$$\lim_{t\to\infty}(\tilde{x}_1, \tilde{x}_2, \tilde{x}_3, \chi) = (0,0,0,0)\,.$$

In other words the tracking problem is solved and the proposition proved. ◁

Remark. Electrical motors can be subject to some asymmetries (e.g. due to some electrical or mechanical faults) that comport the arise of spurious harmonics in the electrical variables. It is possible to model these effects as sinusoidal signals $\zeta(t)$ superimposed to the controlled input. For this reason equation (4) can be rewritten as

$$\dot{\tilde{x}} = [J(\tilde{x}) - \tilde{R}]\frac{\partial \tilde{H}}{\partial \tilde{x}} + \tilde{g}(v - \tilde{\lambda}(\tilde{x}) - \tilde{\varphi}(\tilde{x})\psi(w) + \zeta)$$

where

$$\begin{aligned} \dot{z} \;&=\; \Xi z \\ \zeta \;&=\; \kappa(z)\,, \end{aligned}$$

κ is a polynomial map, $z \in \mathbb{R}^q$, Ξ is defined by

$$\Xi = \mathrm{diag}\{\Xi_0, \Xi_1, \ldots, \Xi_\beta\}\,, \qquad q = 2\delta + 1$$

with $\Xi_0 = 0$,

$$\Xi_i = \begin{bmatrix} 0 & \varpi_i \\ -\varpi_i & 0 \end{bmatrix} \qquad \varpi_i > 0 \qquad i = 1,\ldots,\delta$$

and $z(0) \in \mathcal{Z}$, with $\mathcal{Z} \subseteq \mathbb{R}^q$ bounded compact set; again, the class of possible fault effects considered, take into account a polynomial combination of constant and sinusoidal signals with unknown frequencies, amplitudes and phases.

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[3] Note that, even if Ψ depends on the state $\tilde{x}$, the procedure is not inficiate since the relation is linear.

With slightly simple modification an input disturbance suppression problem has been cast in the problem solved in Section 3 (actually the original tracking problem has been cast into a regulation and input disturbance problem by the error change of coordinates (3)). Hence, following the same solution procedure, a controller of the form presented in (12) can be found in order to satisfy the tracking objective even in case of disturbances, acting on the input of our system, belonging to the fault family described above.

It is important to stress that, since Proposition 1 still holds, the same solution scheme explained in Section 3 can be used to design a controller which embeds an *internal model* of this fault family in order to generate a control action which compensate for the presence of any of such faults, regardless their entity, achieving also the perfect tracking. The proposed scheme can be said to be an implicit fault tolerant control as discussed in (Bonivento *et al.*, 2004*a*) or (Bonivento *et al.*, 2004*b*).

Hence the proposed scheme can be considered as a comprehensive design framework for tracking and fault tolerant control for a permanent magnet synchronous motor. $\triangleleft$

4. CONCLUSIONS

In this paper an adaptive internal model based control scheme is presented to deal with tracking and input disturbance suppression problems for a permanent magnet synchronous motor. More in detail in Section 2 the port-Hamiltonian model of the motor is presented, the problem is stated and a suitable change of coordinates is introduced to define an error system: this makes it possible to cast the tracking problem into a regulation and input disturbance suppression problem.

In Section 3 a controller able to guarantee the perfect asymptotic tracking of unknown exogenous trajectories belonging to a certain family, embedding the internal model of this family, is presented; the theoretical machinery exploited in order to prove the global asymptotical stability of the solution exposed is the nonlinear output regulation theory, specialized for the energy-based port-Hamiltonian formalism.

The same design procedure can be applied considering a fault tolerant control design problem, taking into account the arise of spurious harmonics in the electrical variables, superimposing to the control inputs.

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