BOOLEAN SLIDING MODE CONTROL OF MULTILEVEL POWER CONVERTERS

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Abstract: A generic method to build Boolean sliding mode controllers for switching systems, addressed by a previous contribution, basically formulates the synthesis problem as a set of linear matrix inequalities (LMI) defining the parameters of commutation hyperplanes which all intersect on a prescribed sliding manifold. Yet it cannot guarantee the existence of a sliding motion from any initial condition, since it only ensures a local attractivity of the sliding manifold with respect to state trajectories. Aiming at improving this method, the present paper introduces some additional LMI constraints, which eliminate from the admissible solutions the sets of commutation hyperplanes that some state trajectories possibly never intersect. Algorithmic aspects are also considered for the resolution. The efficiency of the proposed step is illustrated by its application to a multilevel power converter.

Keywords: switching systems, multilevel power converters, sliding mode control.

1. INTRODUCTION

A large class of variable structure systems is made of electrical networks including switching components which change the overall topology according to their state. Such systems are widespread in power electronics where static dc--dc converters make an important subclass of them. Multilevel ones, especially, are more and more used in industrial applications, and also studied by many researchers (Schibli, *et al.*, 1998; Cunha and Pagano, 2002). Controlling these systems actually means deciding, at each time, in which mode they have to work. So it comes down to managing the individual states of their different switching devices, which can be represented by Boolean variables. Thus the control is intrinsically a discrete one. For a long time, sliding mode control, which belongs to variable structure control techniques, has been investigated as an appropriate methodology to regulate switching systems and especially power converters (Utkin, 1977; Pinon, *et al.*, 2000). Yet, for this purpose, many works have proposed piecewise continuous control laws, insofar as classical sliding mode design approaches, such as diagonalization or unit vector approach (Edwards and Spurgeon, 1998; Perruquetti and Barbot, 2002), naturally lead to this kind of laws. In the context of switching systems, the actual control inputs of which are Boolean, it is obvious that these laws cannot be applied in an exact way, but require an additional pulse-width modulation (PWM) regulation scheme, the actual control becoming thus periodic. In order to avoid this drawback and to actually act on the switches states themselves in an asynchronous way, which simplifies the control structure, a design methodology has been established by Morvan *et al.* (2004), which directly generates Boolean control actions. For prior related references, see also (Richard *et al.*, 2003). The corresponding approach, based on the structural properties of a generic model proposed by Buisson, *et al.* (2001) for the overall class of switching physical systems, formally characterizes a prescribed sliding manifold as the intersection of as many commutation hyperplanes of the state space as available Boolean control inputs, and allows to determine these hyperplanes by solving a set of linear matrix inequalities (LMI), the expressions of which are given in a generic way. Beyond its general formulation, which makes it straightforwardly applicable to an overall class of systems such as power converters, the main advantage of this method is the Boolean nature of the control laws it generates, which renders a PWM useless. But it cannot guarantee that a sliding motion will be possible from any initial condition, since it only ensures the attractivity of all the hyperplanes with respect to the state trajectories in a local neighbourhood of their common intersection. The present paper proposes some refinement of this method, which concerns the problem statement itself as well as its numerical resolution. Basically, it introduces some additional LMI constraints, which eliminate from the admissible solutions the sets of commutation hyperplanes that some state trajectories in some natural modes possibly never intersect. As for the numerical aspects, algorithmic considerations are developed that allow reducing the domain where the solutions must be sought. In order to illustrate the efficiency of the proposed step, it is applied to a nontrivial case of industrial interest, namely a 3-level dc-dc converter. The paper is organized as follows. In section 2, the highlights of the method reported in (Morvan *et al*., 2004) are recalled. Next, section 3 introduces so-called crossing conditions which complete this method. In section 4, a procedure that simplifies the search for numerical solutions is presented. Eventually, section 5 describes an example of application of the whole step.

2. BACKGROUND

Sliding mode control uses a high-speed switched control law to drive a system's state trajectory onto a specified manifold, and then maintain it on this manifold for all subsequent time. The control action is a state feedback which is discontinuous across the so-called sliding manifold, and which renders it invariant (at least locally) with respect to the state trajectories. Besides, once the sliding motion has begun, its dynamics only depends on the sliding manifold itself, and not on the actual control ; thus the system behaves as if it was subject to a smooth equivalent control. In the particular case of switching systems, the generic design methodology established by Morvan *et al*. (2004) is dedicated to square systems with Boolean control inputs, whose state equations are, in addition, supposed to be affine in the control variables, such as defined by (1).

$$\begin{cases} \dot{X} = A(\rho) X + B(\rho) U \\ Y = D X \end{cases} \qquad \begin{cases} \dim(X) = n \\ \dim(\rho) = \dim(Y) = m \\ A(\rho) = A_0 + \sum_{i=1}^{m} \rho_i A_i \\ B(\rho) = B_0 + \sum_{i=1}^{m} \rho_i B_i \end{cases} \tag{1}$$

In the above equations, Boolean vector ρ represents the actual control input, whereas U denotes a constant input vector. Moreover, D is a full rank matrix.

Most usual dc-dc power converters can be modelled by such a model as (1) where, in general, each component of ρ allows to control the state of a pair of switching devices that commutate simultaneously. For this class of systems, the generic expression (2) of a Boolean sliding mode control law was first proposed by Richard *et al*., (2003). It allows a regulation objective to be achieved, which consists in maintaining the system output Y around a prescribed reference Y_c.

$$\forall i \in \{1,\ldots,m\},$$
$$\rho_i = \frac{1+\operatorname{sgn}\left[S_i(X)\right]}{2} = \frac{1+\operatorname{sgn}\left[Q_i^T (D X - Y_c)\right]}{2} \tag{2}$$

where $\begin{cases} \operatorname{sgn}(x) = -1 \text{ if } x \in \left]-\infty, 0\right[\\ \operatorname{sgn}(x) = +1 \text{ if } x \in \left[0, +\infty\right[\end{cases}$

The square matrix $Q = [Q_1 \quad Q_2 \quad \cdots \quad Q_m]$, which must be non-singular, is a design parameter whose dimension equals the number m of control inputs. In the state space, each equation $S_i(X) = 0$, where $i \in \{1,\ldots,m\}$, characterizes an hyperplane across which one of the Boolean control inputs commutates. The sliding manifold is defined as the intersection of those m hyperplanes. It exactly coincides with the $n-m$ dimensional affine subspace where the control objective is satisfied.

In this framework, the whole design process simply amounts to the search for an appropriate regular matrix Q, namely a set of m independent vectors Q_i such that the resulting control laws given by (2) ensure a local invariance of the sliding manifold $S = \bigcap_{i=1}^{m} S_i$ with respect to state trajectories. In order to solve this so-called reachability problem, Morvan *et al*., (2004) have proposed a *sufficient* condition for the local invariance of S, which consists in the existence of a point $X_0 \in \mathbb{R}^n$ such that:

$$D X_0 - Y_c = 0 \tag{3}$$

and

$\forall i \in \{1,\ldots,m\}$, for any of the 2^{m-1} possible combinations of ρ_j values with $j \neq i$,

$$\begin{cases} Q_i^T D \left[\left(A_0 + \sum_{\substack{j=1 \\ j \neq i}}^{m} \rho_j A_j + A_i \right) X_0 + \left(B_0 + \sum_{\substack{j=1 \\ j \neq i}}^{m} \rho_j B_j + B_i \right) U \right] < 0 \\ Q_i^T D \left[A_0 + \sum_{\substack{j=1 \\ j \neq i}}^{m} \rho_j A_j X_0 + B_0 + \sum_{\substack{j=1 \\ j \neq i}}^{m} \rho_j B_j U \right] > 0 \end{cases} \tag{4}$$

Note that for a given i and a given set of ρ_j values, the couple of linear matrix inequalities displayed above expresses the invariance condition of the i th switching hyperplane around point X_0, provided that this set of ρ_j values is applied.

Thus, given point X_0, the synthesis problem has been transformed into a decoupled set of LMI concerning the parameters of m switching hyperplanes. Each solution provides a set of such hyperplanes, whose common intersection satisfies the control objective and is made locally invariant with respect to state trajectories.

Yet it should be underlined that the invariance of the sliding manifold is only guaranteed around a single point, for the choice of which, besides, no criterion has been proposed so far, except in the trivial case where equation (3) admits of a unique solution. But the actual extent of the state space region where the sliding manifold is attractive to state trajectories is not mastered.

3. CROSSING CONDITIONS

Using such a control law as provided by the approach recalled in section 2, the only way to ensure that a sliding motion will take place is to choose initial conditions inside the state space region where the sliding manifold is attractive with respect to state trajectories. In control applications where the initial conditions cannot be fixed freely, knowing the location of the compulsory initial state, the control should ideally be built such that the resulting attractivity region of the sliding manifold include this point. Now it implies taking into account the initial state of the system to be controlled in the design process itself, which is not done in the method described in (Morvan *et al.*, 2004). Since the only tuning parameter for the control law is matrix Q, a first idea to improve the method would consist in trying and control the boundaries of the attractivity region through an appropriate choice of this matrix among the possible solutions of (3-4). But it proves to be a difficult process, since this region is defined in a complex way for a given solution Q, as can be seen in (Morvan *et al.*, 2004). As a consequence, for the sake of formal simplicity, the idea of controlling this region has been abandoned.

To improve the design method even so, we propose, instead, to complete it with additional constraints which, given some specified initial state, increase the chances to generate a sliding motion whatever the location of this initial state may be (inside as well as outside the attractivity region). Those new constraints will be called crossing conditions. Indeed, they allow to ensure that switching hyperplanes will necessarily be intersected by the state trajectories in finite time starting from the prescribed initial point, despite the existence either of equilibrium points or of asymptotic directions of divergence in the various possible operating modes. Introducing these crossing conditions together with the reachability ones in the control design, a sliding motion is not really guaranteed, but some cases where such a behaviour cannot occur are directly eliminated.

Let now formalize this notion of crossing conditions. There exist 2^m possible combinations for the values of the m Boolean control inputs ρ_i where $i \in \{1,\ldots,m\}$. A specific operating mode of the system can be associated with each of these combinations. Moreover, this mode can be referred to in an unambiguous way using an integer $r \in \{0,1,\ldots,2^m-1\}$, which corresponds to the decimal value of the binary sequence $[\rho_1,\ldots,\rho_m]$ defining this mode. The sequence of Boolean inputs associated with mode r will therefore be denoted by $[\rho_1(r),\ldots,\rho_m(r)]$ from now on.

According to the generic expression (2) of the state feedback control laws, in a given point X of the state space, the value of a specific control input ρ_i depends on the location of this point with respect to the hyperplane associated with the control:

$$\begin{cases} \rho_i = 1 & \Leftrightarrow \quad S_i(X) > 0 \\ \rho_i = 0 & \Leftrightarrow \quad S_i(X) < 0 \end{cases} \tag{5}$$

In other words, the following property holds in each point X:

$$\left[2\rho_i(X) - 1\right] S_i(X) = \left|S_i(X)\right| \tag{6}$$

Hence can be deduced the validity domain of any operating mode referred to by integer r:

$$D(r) = \left\{ \begin{array}{l} X \in \mathbb{R}^n \,\middle|\, \forall i \in \{1,\cdots,m\}, \\ \left[2\rho_i(r) - 1\right] S_i(X) > 0 \end{array} \right\} \tag{7}$$

As for the complementary domain, where the previous mode is not applied, it is defined as :

$$\overline{D(r)} = \left\{ \begin{array}{l} X \in \mathbb{R}^n \,\middle|\, \exists i \in \{1,\cdots,m\}, \\ \left[2\rho_i(r) - 1\right] S_i(X) < 0 \end{array} \right\} \tag{8}$$

In order to prevent the state from evolving without intersecting any switching hyperplane, it is necessary to guarantee that in each mode r, all the state trajectories converge towards domain $\overline{D(r)}$. The crossing conditions can thus be expressed under the generic form:

$$\forall r \in \{0,1,\ldots,2^m-1\},\ \lim_{t\to+\infty} X_r(t) \in \overline{D(r)}$$

where $X_r(t)$ denotes the time evolution of the state vector when mode r is active.

Hence a new formulation of the same conditions:

$$\forall r \in \{0,\ldots,2^m-1\},$$
$$\exists i \in \{1,\cdots,m\} \left| \left[2\rho_i(r) - 1\right] \lim_{t\to+\infty} S_i(X_r) < 0 \right.$$

Eventually one gets their final expression:

$$\begin{array}{l} \forall r \in \{0,\ldots,2^m-1\}, \exists i \in \{1,\cdots,m\} \big| \\ Q_i^T \left[2\rho_i(r) - 1\right] \lim_{t\to+\infty} \left[D X_r(t) - Y_c\right] < 0 \end{array} \tag{9}$$

Thus, crossing conditions provide a system of 2^m different alternatives between m LMI, each individual inequality concerning one of the unknown variables Q_i. In the most general case, those LMI can depend on the initial conditions.

4. SOME COMMENTS ABOUT THE RESOLUTION PROCEDURE

Collecting the early sufficient condition for local invariance of the sliding manifold recalled in section 2 and the new crossing conditions established in section 3, the refined design problem globally boils down to searching for a point $X_0 \in \mathbb{R}^n$ as well as a set of m independent vectors $\{Q_1 \quad Q_2 \quad \cdots \quad Q_m\}$ of $\mathbb{R}^m$ such that properties (3), (4) and (9) be satisfied all together.

So the proposed step of synthesis primarily consists in choosing a particular point X_0 of the state space satisfying constraint (3), and next in seeking a solution $\{Q_1 \quad Q_2 \quad \cdots \quad Q_m\}$ to the combination of LMI defined by (4) and (9). Considered separately, the LMI problem (4) can be decoupled with respect to each variable Q_i where $i \in \{1,\ldots,m\}$, the LMI sub problem related to Q_i being besides easily interpreted in the m-dimensional space of its components: given X_0, its general solution is a cone-shaped domain of $\mathbb{R}^m$ bounded by 2^m hyperplanes all including the origin. Unfortunately, the introduction of the crossing conditions renders the problem more complex, since the latter cannot be decoupled any longer. So instead of reducing it to m distinct sub problems in $\mathbb{R}^m$, as described previously, it is necessary to solve it globally in $\mathbb{R}^{m\times m}$. A trial-and-error procedure is then used, based on the exploration of the $m\times m$-dimensional parameter space. But such an exploration does not need to be exhaustive. Indeed, let consider a particular solution $Q = [Q_1 \quad Q_2 \quad \cdots \quad Q_m]$ to the LMI problem defined by (4) and (9). As seen in section 2, the control law associated with each of its columns can be written as:

$$\rho_i(X) = \frac{1 + \operatorname{sgn}\left[Q_i^T (D X - Y_c)\right]}{2} \tag{10}$$

It is obvious that such a Boolean state feedback remains unchanged if Q_i is multiplied by a strictly positive constant factor. As a consequence, any other matrix $Q' = [\alpha_1 Q_1 \quad \alpha_2 Q_2 \quad \cdots \quad \alpha_m Q_m]$ also constitutes an admissible solution, which besides exactly gives the same control laws, provided that all the α_i coefficients are strictly positive numbers. Especially, the choice $Q_i / \|Q_i\|$, which is always possible since Q_i cannot be null (matrix Q being necessary non singular), provides an equivalent solution. One can therefore, with no loss of generality, restrict the search for each solution Q_i to the unit radius hyper-sphere of $\mathbb{R}^m$. Now this hyper-sphere can be parameterised using $m-1$ angular variables. Thus, the dimension of the global problem corresponding to the search for a matrix solution Q is reduced from m^2 variables taken in $\mathbb{R}$ to $m(m-1)$ ones taken between the bounds 0 and 2π.

5. EXAMPLE OF A 3-LEVEL DC-DC CONVERTER

In its early version reported in (Morvan *et al.*, 2004), our sliding mode control design methodology has already been successfully applied to classical cases of dc-dc power converters with one and two cells. Let now consider the example of a 3-cell multilevel one such as depicted in Figure 1.

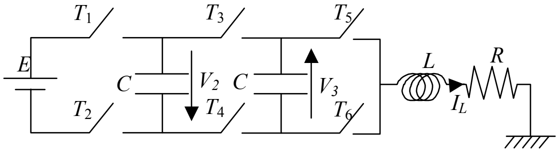

Fig. 1. electrical scheme of a 3-cell dc-dc converter

In normal operating conditions, all its switches commutate by pairs: T_1 together with T_2, T_3 with T_4, and T_5 with T_6. Moreover, each pair is supposed to be controlled by means of a Boolean input. Let $\rho = (\rho_1, \rho_2, \rho_3)^T$ stand for the resulting control vector. Then the overall dynamics of the system can be modelled by a single multimode state equation, which obeys the following form:

$$\dot{X} = \underbrace{\begin{pmatrix} -\frac{R}{L} & -\frac{(1-\rho_1-\rho_2)}{C} & -\frac{1-\rho_2-\rho_3}{C} \\ \frac{1-\rho_1-\rho_2}{L} & 0 & 0 \\ \frac{1-\rho_2-\rho_3}{L} & 0 & 0 \end{pmatrix}}_{A(\rho)} X + \underbrace{\begin{pmatrix} \rho_1 \\ 0 \\ 0 \end{pmatrix}}_{B(\rho)} E \tag{11}$$

where Booleans ρ_1, ρ_2 and ρ_3 respectively control the state of pairs (T_1, T_2), (T_3, T_4) and (T_5, T_6): $\rho_1 = 0$ when T_1 is off and T_2 is on, $\rho_2 = 0$ when T_3 is on and T_4 off, $\rho_3 = 0$ when T_5 is on and T_6 off.

Considering 3 output variables, namely both capacitor voltages V_2, V_3 and the current I_L in the load, the model is made square, with an output equation given by:

$$Y = \begin{pmatrix} I_L \\ V_2 \\ V_3 \end{pmatrix} = \underbrace{\begin{pmatrix} \frac{1}{L} & 0 & 0 \\ 0 & \frac{1}{C} & 0 \\ 0 & 0 & \frac{1}{C} \end{pmatrix}}_{D} X \tag{12}$$

It can be observed that the state equation (11) is affine in the control, since al the terms in matrices $A(\rho)$ and $B(\rho)$ actually are affine functions of the Boolean variables.

In this particular case, each value of the output vector corresponds to one point in the state space. As a consequence, given a prescribed reference Y_c, the sliding manifold is reduced to a single point. So there is no degree of freedom for the choice of X_0:

$$X_0 = \begin{pmatrix} L y_{c1} \\ C y_{c2} \\ C y_{c3} \end{pmatrix} \tag{13}$$

Applying the general methodology described in section 2, the sufficient condition for the invariance of the sliding manifold can be written as:

$$\begin{cases} Q_1^T V_1 < 0 & Q_2^T V_1 > 0 & Q_3^T V_1 > 0 \\ Q_1^T V_2 > 0 & Q_2^T V_2 > 0 & Q_3^T V_2 > 0 \\ Q_1^T V_3 < 0 & Q_2^T V_3 > 0 & Q_3^T V_3 < 0 \\ Q_1^T V_4 > 0 & Q_2^T V_4 > 0 & Q_3^T V_4 < 0 \\ Q_1^T V_5 < 0 & Q_2^T V_5 < 0 & Q_3^T V_5 > 0 \\ Q_1^T V_6 > 0 & Q_2^T V_6 < 0 & Q_3^T V_6 > 0 \\ Q_1^T V_7 < 0 & Q_2^T V_7 < 0 & Q_3^T V_7 < 0 \\ Q_1^T V_8 > 0 & Q_2^T V_8 < 0 & Q_3^T V_8 < 0 \end{cases} \quad (14)$$

where each V_i denotes a constant vector, the value of which is fully determined knowing the prescribed reference and the system constitutive parameters:

$$\begin{cases} V_1 = \begin{pmatrix} \frac{E - Ry_{c1} - y_{c3}}{L} & 0 & \frac{y_{c1}}{C} \end{pmatrix}^T \\ V_2 = \begin{pmatrix} \frac{-Ry_{c1} - y_{c2} - y_{c3}}{L} & \frac{y_{c1}}{C} & \frac{y_{c1}}{C} \end{pmatrix}^T \\ V_3 = \begin{pmatrix} \frac{E - Ry_{c1}}{L} & 0 & 0 \end{pmatrix}^T \\ V_4 = \begin{pmatrix} \frac{-Ry_{c1} - y_{c2}}{L} & \frac{y_{c1}}{C} & 0 \end{pmatrix}^T \\ V_5 = \begin{pmatrix} \frac{E - Ry_{c1} + y_{c2}}{L} & -\frac{y_{c1}}{C} & 0 \end{pmatrix}^T \\ V_6 = \begin{pmatrix} -\frac{Ry_{c1}}{L} & 0 & 0 \end{pmatrix}^T \\ V_7 = \begin{pmatrix} \frac{E - Ry_{c1} + y_{c2} + y_{c3}}{L} & -\frac{y_{c1}}{C} & -\frac{y_{c1}}{C} \end{pmatrix}^T \\ V_8 = \begin{pmatrix} \frac{-Ry_{c1} + y_{c3}}{L} & 0 & -\frac{y_{c1}}{C} \end{pmatrix}^T \end{cases} \quad (15)$$

In order to be able to express the crossing conditions, the asymptotic behaviour of the system must be studied in each possible operating mode: given the initial state $X(t_0)$, the evolution of the state trajectories have to be known when time tends towards infinity. In the present example, assuming that no commutation occurs, each mode leads to a unique final state which depends on the initial one. The resulting end points in the various modes are defined in a generic way as:

$$X_{0,0,0} = \begin{pmatrix} 0 \\ x_2(t_0) \\ -x_2(t_0) \end{pmatrix} \quad X_{1,0,0} = \begin{pmatrix} 0 \\ x_2(t_0) \\ CE \end{pmatrix}$$
$$X_{0,0,1} = \begin{pmatrix} 0 \\ 0 \\ x_3(t_0) \end{pmatrix} \quad X_{1,0,1} = \begin{pmatrix} LE/R \\ x_2(t_0) \\ x_3(t_0) \end{pmatrix} \quad (16)$$
$$X_{0,1,0} = \begin{pmatrix} 0 \\ x_2(t_0) \\ x_3(t_0) \end{pmatrix} \quad X_{1,1,0} = \begin{pmatrix} 0 \\ -CE \\ x_3(t_0) \end{pmatrix}$$
$$X_{0,1,1} = \begin{pmatrix} 0 \\ x_2(t_0) \\ 0 \end{pmatrix} \quad X_{1,1,1} = \begin{pmatrix} 0 \\ x_2(t_0) \\ -x_2(t_0) - CE \end{pmatrix}$$

At this stage, new constants vectors are introduced, which depend on the previous ones:

$$\begin{cases} V_9 = DX_{0,0,0} - Y_c & V_{13} = DX_{1,0,0} - Y_c \\ V_{10} = DX_{0,0,1} - Y_c & V_{14} = DX_{1,0,1} - Y_c \\ V_{11} = DX_{0,1,0} - Y_c & V_{15} = DX_{1,1,0} - Y_c \\ V_{12} = DX_{0,1,1} - Y_c & V_{16} = DX_{1,1,1} - Y_c \end{cases} \quad (17)$$

The crossing conditions can be straightforwardly expressed using the latter constant vectors:

$$\begin{cases} Q_1^T V_9 > 0 \\ \text{or} \\ Q_2^T V_9 > 0 \\ \text{or} \\ Q_3^T V_9 > 0 \end{cases} \& \begin{cases} Q_1^T V_{10} > 0 \\ \text{or} \\ Q_2^T V_{10} > 0 \\ \text{or} \\ Q_3^T V_{10} < 0 \end{cases} \& \begin{cases} Q_1^T V_{11} > 0 \\ \text{or} \\ Q_2^T V_{11} < 0 \\ \text{or} \\ Q_3^T V_{11} > 0 \end{cases} \& \begin{cases} Q_1^T V_{12} > 0 \\ \text{or} \\ Q_2^T V_{12} < 0 \\ \text{or} \\ Q_3^T V_{12} < 0 \end{cases}$$
$$\&$$
$$\begin{cases} Q_1^T V_{13} < 0 \\ \text{or} \\ Q_2^T V_{13} > 0 \\ \text{or} \\ Q_3^T V_{13} > 0 \end{cases} \& \begin{cases} Q_1^T V_{14} < 0 \\ \text{or} \\ Q_2^T V_{14} > 0 \\ \text{or} \\ Q_3^T V_{14} < 0 \end{cases} \& \begin{cases} Q_1^T V_{15} < 0 \\ \text{or} \\ Q_2^T V_{15} < 0 \\ \text{or} \\ Q_3^T V_{15} > 0 \end{cases} \& \begin{cases} Q_1^T V_{16} < 0 \\ \text{or} \\ Q_2^T V_{16} < 0 \\ \text{or} \\ Q_3^T V_{16} < 0 \end{cases} \quad (18)$$

LMI combinatorial systems (14) and (18) fully formalize the design problem. Indeed, admissible solutions for the set of switching hyperplanes that will determine the sliding mode control laws are defined by regular square matrices Q of $\mathbb{R}^{3\times3}$ whose columns satisfy both systems.

Using the results of section 4, candidate solutions can be parameterised as:

$$Q = \begin{pmatrix} \cos\varphi_1 \sin\alpha_1 & \cos\varphi_2 \sin\alpha_2 & \cos\varphi_3 \sin\alpha_3 \\ \sin\varphi_1 \sin\alpha_1 & \sin\varphi_2 \sin\alpha_2 & \sin\varphi_3 \sin\alpha_3 \\ \cos\alpha_1 & \cos\alpha_2 & \cos\alpha_3 \end{pmatrix} \quad (19)$$

Based upon a common sampling of interval $[0, 2\pi[$ for all the angular variables φ_i and α_i where $i \in \{1,3\}$, the discrete set of matrices Q fitting (19) is scanned, testing for LMI conditions (14) and (18), until a solution is found.

A set of typical values of the parameters is assumed for the numerical resolution:
C=1.0e-3 F; L=75.0e-3 H;R=20.0 Ω;E=90.0 V;

Then, starting from initial conditions located at the origin of the state space (system at rest), an output reference is arbitrarily defined by $Y_c = (2A \quad -30V \quad 60V)^T$. In the state space, it corresponds to the desired equilibrium point X_0 of coordinates $(0.15 \quad -0.03 \quad 0.06)$. In such conditions, a particular solution is given by:

$$Q = \begin{pmatrix} -0.0955 & 0.25 & -0.6545 \\ 0.2939 & 0.1816 & -0.4755 \\ -0.309 & 0.309 & 0.809 \end{pmatrix} \quad (20)$$

The related expressions of the switching functions can be written as:

$$\begin{cases} S_1(X) = -1.27x_1 + 293.89x_2 - 309.02x_3 + 27.55 \\ S_2(X) = 3.33x_1 + 181.64x_2 + 309.02x_3 - 13.59 \\ S_3(X) = -8.73x_1 - 475.53x_2 + 809.02x_3 - 61.50 \end{cases} \quad (21)$$

Figure 2 shows the state trajectory obtained when simulating the resulting Boolean control.

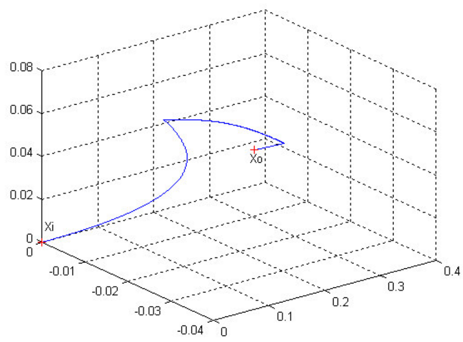

Fig. 2. state trajectory in sliding mode

It can be verified that starting from the initial point, the state actually converges towards the desired equilibrium point. In addition, looking separately at the time evolutions of the individual state variables, as depicted in Figure 3, it appears that a time response of 30 ms is sufficient for the regulation objective to be achieved by applying the synthesized control, when the system is initially taken at rest.

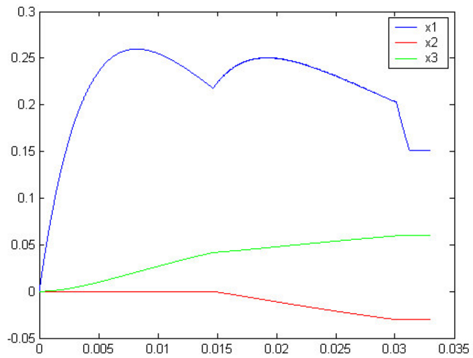

Fig. 3. time evolution of the state variables

6. CONCLUSION

This paper deals with a generic step to design sliding mode controllers for a particular class of switching systems, namely multilevel power converters, the commutations of such systems being controlled by the values of Boolean inputs. Recapitulating a method initially proposed in a previous paper, whose specific feature is to directly provide Boolean control actions, it refines this method in several respects.

First, introducing so-called crossing conditions, which restrict the field of possible control solutions, it increases the chances to induce a sliding motion on a prescribed manifold when the initial point is not chosen inside the domain where the later manifold is attractive with respect to state trajectories. Thus, it allows starting a regulation process from any point in the state space, and it does not require that the attractivity region of the sliding manifold be precisely known. From a formal point of view, the crossing conditions are a combination of LMI, which complete the former LMI expressing the design problem such as it has been set in the original approach. The solutions of the resulting LMI problem are regular matrices whose columns define the parameters of hyperplanes that delimit the switching laws and that all intersect on the sliding manifold.

As a second point, the present paper investigates some aspects related to the resolution of the LMI problem. It establishes some kind of invariance property concerning the Boolean control laws with respect to changes of the matrix solution. Thus it allows reducing the dimension of the domain where this solution must be looked for.

Eventually it is shown in detail how the improved methodology can be applied to regulate a multilevel dc-dc converter with three cells. Further work will consist in searching for criteria to optimize the solution of the LMI problem, and if possible to investigate the relation between its choice and the performances of the control. Another interesting perspective is the extension of this approach to tracking, in the case of a time varying reference.

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