INVENTORY MANAGEMENT IN HIGH UNCERTAINTY ENVIRONMENT WITH MODEL REFERENCE CONTROL

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Abstract: In this study, the method of model reference control is used to implement a decision support system for inventory management. The inventory is controlled on the basis of minimizing variable costs while satisfying a certain percent of customer demand. Value-at-Risk analysis is used to determine the required safety stock for the controller, so that the uncertainties are buffered into the inventory. The handling of constraints is also investigated in simulations with four different inventory capacity limitations. Results from the simulations show how a model reference control based decision support system, combined with Value-at-Risk based safety stock determination, can operate within strict constraints while still satisfying a certain percentage of customer demand.

Keywords: Inventory management, model predictive control, model reference control, uncertainty

1. INTRODUCTION

Supply chain and inventory control has been a very popular field of study for the recent decades and especially in the areas of control and systems theory. The reason for this popularity is clear as supply chains and inventories involve several features which make them suitable for a control theoretical approach. (Towill, 1982) Supply chains are often controlled via inventory management as inventories can be seen as buffers, dampening the negative effects of uncertainties and delays. Therefore inventories are involved in a very classical form of control theoretical problem with delays, distortions and uncertainties.

As globalization and integration of companies has led to even larger supply chains, the field of supply chain and inventory management research has also concerned more on studying the effects of highly integrated supply chains. Due to this, the consideration of competition has moved from individual companies to whole supply chains. Therefore the inner competition in a supply chain is often neglected. Competition causes major problems to information sharing in the chain, which in many studies has been forgotten about. In the end, truth is that only very few supply chains have the capability to practice centralized control or high level of information sharing. It is in these supply chains, where the handling of uncertainties becomes a vital task (Lee and Billington, 1992). For example, wholesalers or other actuators, who have a very limited knowledge of their customers' actions, need to manage their inventory so that costs are minimized, yet a certain Customer Demand Satisfaction (CDS) level is guaranteed. In this study, we will focus exactly on this problem, as a Decision Support System (DSS) controlling a single inventory in high uncertainty environment is implemented. For this task we will use a one form of a Model Predictive Control (MPC). For the past years MPC has been a very popular method in supply chain and inventory control related studies. This is due to the fact that MPC has many suitable characteristics of which main features are the following

- Demand forecasts can be used effectively
- Supply chain dynamics are simple to implement in the MPC model
- MPC can handle constraints and delays easily
- Control parameters can be acquired straight from actual unit costs, which makes it easy to comprehend.
- MPC has been found to be a relatively robust control method

MPC, in general, has many variations as it has been used in many different control tasks. Common features in all MPC controllers are the receding (rolling) horizon strategy, in which at each instant, the optimization is done on the basis of a certain prediction horizon, but only the first control signal is used in the process and all the rest of the calculated control signals are ignored. On the next instant, the same actions are done again, since new and more accurate information of the system is available. (Camacho and Bordons, 2002) The optimization is done on the basis of a cost function, which in this study is in the quadratic form. The quadratic cost function is not necessarily the most cost efficient one, but is considered to be a rather robust penalizing method. In future research, other nonlinear and linear cost functions are to be considered.

Naturally, supply chain and inventory control with MPC, has its own special features which are mainly dependent on the line of business. In Chapter 2 we will take a closer view on one of the variations, Model Reference Control (MRC). In Chapter 3 the actual control law, with the definition of target inventory level, is presented. In Chapter 4 the case situation of the simulations is presented with some key issues of handling the results. The actual results from the simulations are shown and analyzed in Chapter 5. We will conclude our study in Chapter 6.

2. MODEL REFERENCE CONTROL

Model reference control is a one form of the set of control methods known as model predictive control. The basic operation method is equal to the one in MPC, but the difference is in the cost function. One problem with the traditional MPC strategy, concerning inventory management, is the fact that the basic form of quadratic MPC cost function consists of penalizing changes made in order level. This has few draw backs of which the most severe ones are listed below.

- The cost for making a change in order level is impossible to determine analytically
- Since no such cost actually exists, there is no cost based explanation for using such penalizing
- As shown in (Rasku *et al.*, 2004), the results from such penalizing are not ideal for inventory management

Even though the flaws of this kind of penalizing are known, it is still used in supply chain related MPC controllers, for example in (M. W. Braun and Kempf, 2003), since it is necessary to have some kind of limitations for the control signal behavior. This move suppression term is needed to manage the notorious bullwhip effect which is the amplification of oscillations in order rates in the upstream supply chain. The basic form of quadratic MRC cost function can be written as

$$J = \sum_{i=N_1}^{N_2} (y^*(i) - P(q^{-1})\hat{y}(x_i))^2, \tag{1}$$

where $y^*(\cdot)$ is the target output level, $\hat{y}(\cdot)$ is predicted output level and x_k is the optimal control signal at instant k. The inverted discrete filter $P(q^{-1})$, which usually is a first or second order transfer function, can be written as

$$P(q^{-1}) = \frac{1 - p_1 q^{-1} - p_2 q^{-2} - \cdots}{1 - p_1 - p_2 - \cdots}. \tag{2}$$

As can be seen from the Equation (1), the target output level $y^*(\cdot)$ is needed in the control. The quadratic cost function presented is very simple itself, but the target output level holds great influence to the resulting control policy as will be seen in the following chapter.

3. CONTROL LAW

When the MRC control strategy is used to inventory management, the cost function presented in Equation (1) can be written as

$$J = \sum_{i=N_1}^{N_2} (I_d(i) - P(q^{-1})\hat{I}(x_i))^2, \tag{3}$$

where $I_d(\cdot)$ is the target inventory level, $\hat{I}(\cdot)$ is predicted inventory level and x_i is the optimal order level at instant i. The inverted discrete filter $P(q^{-1})$ is equal to the one presented in Equation (2). In this study, we used the most basic form of the filter, that is, a first order transfer function. As mentioned in the previous chapter, the target inventory level has great influence on the control, and therefore the actual cost function does not need to be more complex. This is due to the fact, that when an inventory is controlled on the basis of minimizing costs while satisfying customer demand, these two goals can be managed separately. The cost function itself can handle the minimizing of costs, and the guaranteeing of a certain CDS-level is handled with the definition of the target inventory level $I_d(\cdot)$. One should also bare in mind, that the control strategy pursued in this study is a DSS-based one. This means, that the most detailed control features are not taken into consideration, but left as the decision maker's responsibility.

3.1 Defining the Target Inventory Level

In several studies concerning inventory management, the task of defining a target inventory level for a company is often neglected. During recent years this problem has been tackled especially by using the method of Value-at-Risk (VaR) analysis. This kind of approaches can be seen, for example, in (Luciano *et al.*, 2003) and (Tapiero, 2003). The VaR analysis is very suitable as inventories are often managed on the basis of a certain Customer Demand Satisfaction (CDS) level which is often presented as a certain percent of customer demand satisfied, usually 95% or 99%. (Luciano *et al.*, 2003). This is demonstrated graphically in Figure 1 via an imaginary example situation, where the uncertainty in demand forecast has been approximated. By determining the required CDS level, the required safety stock level $I_d(\cdot)$ can be calculated from the cumulative distribution by solving Equation (4).

$$\Phi_{CDS}(i) = \int_{-\infty}^{I_d(i)} F_{\hat{D}(i)}(x)dx, \qquad (4)$$

where $I_d(\cdot)$ is the desired inventory level, $\Phi_{CDS}(\cdot)$ is the required customer demand satisfaction level and $F_{\hat{D}(\cdot)}(x)$ is the cumulative distribution function of forecasted demand $\hat{D}(\cdot)$ as a function of x, which in this case stands for units in the demand forecast distribution. As also illustrated in Figure 1, the distribution presenting the total uncertainty does not necessarily need to be normal, but in this case we will consider all distributions normal in order to keep the results understandable. When this method of determining the target inventory level is implemented into an MRC controller, the feature of receding horizon needs to be taken into consideration. Due to the receding horizon, the determination of desired inventory level needs to be done separately for each instant in the horizon. This is demonstrated in Figure 2, where the distributions of the demand forecast from the whole prediction horizon are used to determine the required safety stock levels for each instant

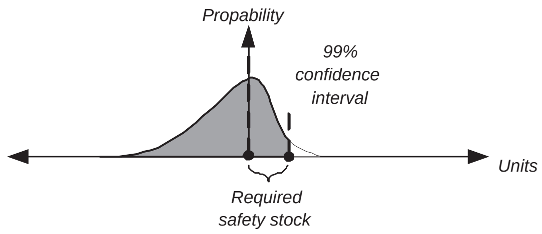

Fig. 1. Safety stock determination for a single period from the distribution of uncertainty.

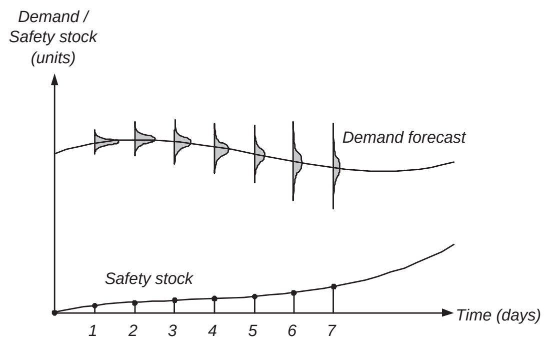

Fig. 2. Safety stock determination for a whole period of forecast

in the horizon. This curve, consisting of the required safety stock levels, is the target inventory level used as the reference trajectory in the MRC controller. Here needs to be stressed, that the assumption of losing unmet demand is made. This assumption is approximated by considering the demand forecast uncertainty distributions as independent distributions which may lead to occasional excessive inventory levels. In future research this approximation is replaced by skewing the distributions.

By using this strategy in the definition of target inventory level, the role of demand forecasting is reduced. The accuracy of the forecast is no longer vital, as long as the level of accuracy is known. That is, if one knows the form of the distribution, the inventory can be used to handle this uncertainty. The actual accuracy of the forecaster is now relevant only from the inventory capacity's point of view. The more inaccurate forecaster, the more inventory capacity is needed to handle the uncertainty. This does not concern only demand forecasts, as all other aspects concerning uncertainty can be handled this way, as long as the amount of uncertainty can be approximated.

4. CASE SITUATION OF THE SIMULATIONS

As mentioned in the introduction, the case of a company operating in a supply chain with low information sharing level is often neglected

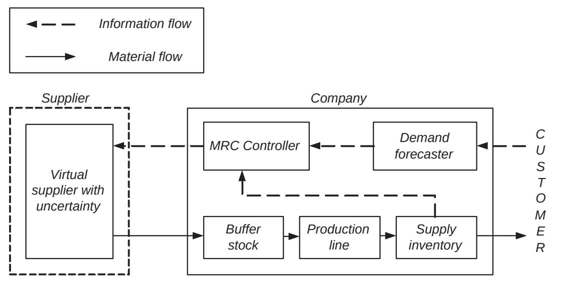

Fig. 3. Block diagram presentation of the simulator used in this study

these days. Therefore, in these simulations we will present a case situation just like this. That is, a sole company operating on the basis of inventory management with very little information of the rest of the chain. Basically all the information company receives, is the customer demand data. The simulator used is presented in Figure 3. The simulator includes a total delay of 3 time units so that the virtual supplier supplies the material ordered with total delay of 2 time units: One time unit of production delay and one time unit of transportation delay. Also the production line of the company simulated has a delay of one time unit. In the simulations we will investigate how such company can handle the uncertainties with its inventory management and also investigate the influence of constrained inventory capacity to the handling of uncertainty.

Uncertainties are present in both customer demand and supplier CDS-level. The customer demand is presented in Figure 4 and consists of both seasonal components and normally distributed disturbance components. The variance of the normally distributed distortion in customer demand is set to be $\sigma^2_{Dem} = 400$. The distortions in the virtual supplier's CDS-level are also normally distributed, for the sake of convenience, with variance $\sigma^2_{Sup} = 300$. This can be taken as a feature of back ordering policy of the supplier, as in the end of the simulation, the total amount ordered is always supplied. As explained earlier, for the company simulated, the unmet demand is always lost, so no back ordering policy is used. The time scale in demand data can be taken as weeks, so that each year consists of 52 weeks. Then the data consists of three years, of which the first one is used as learning data for the demand forecaster, and the rest five years are used in the actual simulations. The demand is forecasted with a simple multiple regression method, which can be used to generate both the expectation value and the required safety stock level. An example of a such demand forecaster can be found in, for example, (Pindyck and Rubinfeld, 1998). Naturally the forecaster continues to improve as the simulations run, but the improving is rather minor after the first year.

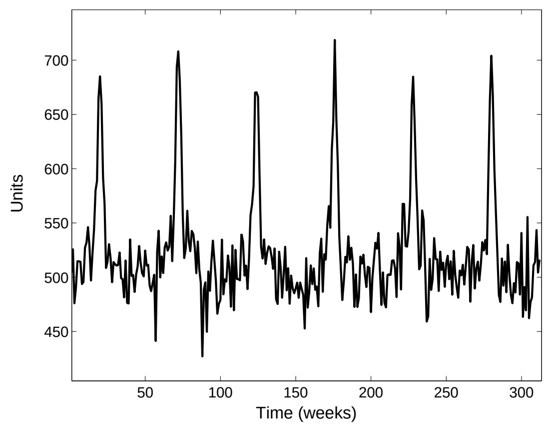

Fig. 4. Customer demand data used in the simulations

The controller in the simulations was tuned to be a rather calm one, with the first order transfer function $P(q^{-1})$ set to operate with the parameter $p_1 = 0.9$. The receding horizon length was set to be 7 time units, i.e. weeks in this case. The selection of the horizon length is mainly based on the length of the delay in the system, as the horizon always needs to be longer than the maximum delay in the system. The controller was also set to operate on the CDS-level of 99%, but still, on the basis of minimizing costs.

4.1 Analysis of the Results

The results from the simulations were evaluated on the basis of costs and CDS-levels, which has been the main stream method in recent studies. The costs were calculated cumulatively with Equation (5)

$$J_{tot}(k) = \sum_{i=1}^{k} (\alpha I(i) + \beta I_{SO}(i) + \gamma I_E(i)), \qquad (5)$$

where $J_{tot}(\cdot)$ is the total cumulative cost, $I(\cdot)$ is the inventory level, $I_{SO}(\cdot)$ is the inventory stock out level and $I_E(\cdot)$ is the excessive inventory, i. e. the amount of supply which cannot be stored in the inventory due to inventory capacity limitations. All the coefficients, α, β and γ are approximations of the true costs of stock holding, stock out and excessive material supply, respectively. Especially the cost of abandoning material due to limited inventory capacity, is difficult to determine accurately, since it depends on production costs and lost sales profits and is, in general, very case dependent. All that can be stated is, that this cost is usually higher than the unit cost of stock holding and lower than the cost of stock out. This cost has no effect in the situation of unlimited inventory capacity, naturally. In the evaluation of simulation results in this study, these coefficients are set to be $\alpha = 1, \beta = 100$ and $\gamma = 50$ in order to make the effects clear. The other measure of efficiency, the CDS-level, was determined as the relation of satisfied demand and total demand with Equation (6)

$$CDS = 100\% \cdot \frac{\sum_{i=1}^{T_{sim}} S(i)}{\sum_{i=1}^{T_{sim}} D(i)}, \tag{6}$$

where CDS is the customer demand satisfaction level, $S(\cdot)$ is the supplied material of the Company, $D(\cdot)$ is the customer demand and T_{sim} is the simulation time, i. e. the length of the simulated period. This methods gives an easy-to-understand percentage of the CDS-level achieved.

5. RESULTS FROM THE SIMULATIONS

The simulations were made in the Matlab Simulink environment with the set of universal supply chain blocks presented in (Rasku *et al.*, 2004), so that the company controlled is simulated with the universal production block, and the supplier is simulated with the virtual supplier block. A set of 4 simulation runs were conducted with 4 different inventory capacities: The unconstrained, 200-unit, 150-unit and 100-unit storage capacities were tested. The respective CDS-levels can be seen in Table 1. As can be seen, the three largest inventory capacities were large enough to satisfy the CDS-level target of 99%, but the storage capacity of 100 units was found to be too strict. Though the target CDS-level was achieved with the larger storage capacity limitations, the CDS-level percentage declines as the limitation gets stricter. This is understandable and also shows, that MRC can operate under rather strict limitations, as the inventory level rised up to 200 units when unlimited.

Table 1. CDS-levels of the company simulated with four different inventory capacity levels.

Storage capacity	Infinite	200	150	100
CDS-level	99,5%	99,4%	99,2%	98,7%

An illustrative example of how exactly the constraints change the behavior of the controller can be seen in Figure 5, where inventory levels from a period of one year are presented. The constraint levels are presented with the thin horizontal lines and respective inventory levels with the thicker oscillating lines. As can be seen, the constraints have major effect on inventory levels even if the constraints are not eventually reached, especially with the two most strict constraints. On the other hand, the inventory levels from the controller handling the 200-unit inventory capacity constraint follows almost exactly the inventory levels of the unconstrained controller. Only during the seasonal peak (time span $20 - 30$), the lines can be separated. This is due to the high uncertainty during the seasonal peak, which causes the constrained controllers to order more carefully to avoid excessive ordering.

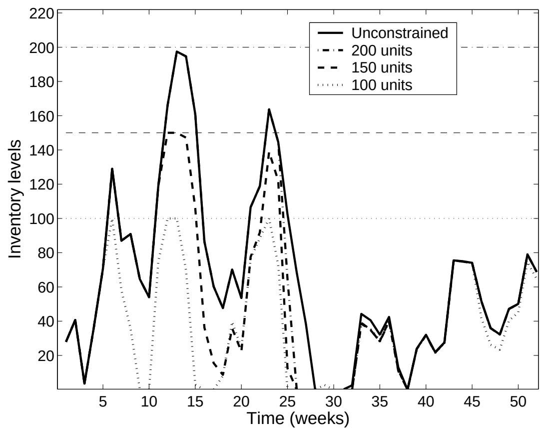

Fig. 5. An example of inventory level behavior with different inventory capacity constraints.

The cumulative costs confirm the results got from CDS-levels: Unlimited inventory can operate with the lowest costs, and as the limitations get stricter, the costs grow higher. This is understandable since the cost function used to analyze the results, Equation (5), penalized the stock out and excessive inventory levels a lot more heavier than the sole stock holding cost. Therefore the occasional stock out and excessive inventory level situations caused the inventory capacity limited situations to operate on higher costs. Especially with the inventory capacity constraint of 100 units, the most rapid rise in costs is due to the penalizing of excessive inventory levels. At this point, it is important to realize, that the costs taken into consideration here are all variable costs, and therefore do not include, for example, the investment made to build the storage building and other major fixed costs. This is the trade-off between a large storage capacity with higher fixed costs and higher CDS-level and a small storage capacity with lower fixed costs and lower CDS-level. From a DSS's point of view, the simulations presented here can give some guide lines for determining the needed inventory capacity and investments made. This issue is not discussed further more in this study, but left for future research.

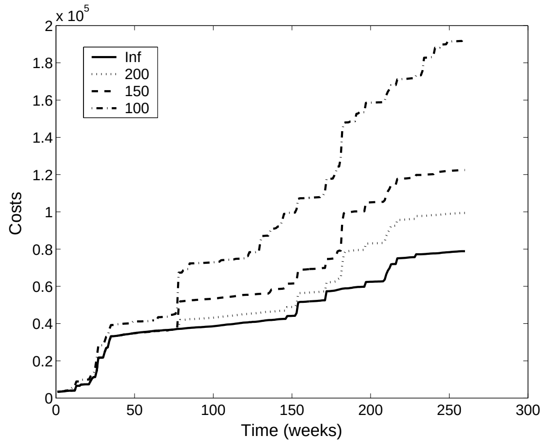

Fig. 6. Cumulative costs with different inventory capacities.

6. CONCLUSIONS

The area of inventory management in a supply chain with poor information sharing is very rarely discussed. Therefore, the objectives in this study were to implement and simulate a Decision Support System (DSS) for inventory management which could operate in high uncertainty environment and to introduce a constrained Model Predictive Control (MPC) based controller with Value-at-Risk-analysis (VaR-analysis) based reference trajectory determination. The method of Model Reference Control was briefly introduced and eventually used in the DSS in order to achieve a robust and efficient controller. The VaR-analysis based safety stock level definition was also presented and implemented within the MPC strategy. This is a major improvement combining the best features of both methods as uncertainty has so far been difficult to handle with traditional MPC strategy and on the other hand, VaR-analysis alone does not consider variable cost minimization. Even the issue of determining a safety stock level itself is very rarely studied. Simulations with the single company simulator and with the MRC controller were presented in this study. In these simulations we studied the performance of the controller as a DSS in a high uncertainty environment with four different inventory capacity limitations: Unconstrained, 200-unit, 150-unit and 100-unit inventory capacities. The results were investigated on the basis of costs and Customer Demand Satisfaction-levels (CDS-levels), which are the main measures of supply chain and inventory performance these days. Controller's ability to handle rather strict constraints and high uncertainties, while satisfying the target CDS-levels, was found very effective. The results show how the DSS implemented in this study can be used as an actual everyday tool for inventory management, inventory capacity determination and can give guide lines for inventory capacity related investment decisions.

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