DEPTH CONTROL OF THE INFANTE AUV USING GAIN-SCHEDULED REDUCED-ORDER OUTPUT FEEDBACK [1]

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Abstract. The paper addresses the problem of autonomous underwater vehicle (AUV) control in the absence of full state information. An application is made to the control of a prototype AUV in the vertical plane. The methodology adopted for controller design is nonlinear gain scheduling control, whereby a set of *linear, dynamic reduced order output feedback controllers* are designed and scheduled on the vehicle's forward speed. The paper summarizes the basic controller design steps, describes a technique for practical implementation of the nonlinear control systems derived, and presents experimental results obtained with the INFANTE AUV during tests at sea.

Keywords. Marine Systems, Autonomous Underwater Vehicles, Gain-Scheduled Control, Output Feedback.

1. INTRODUCTION

This paper describes a solution to the problem of autonomous underwater vehicle (AUV) control in the vertical plane, in the absence of full state information. An application is made to the control of the prototype INFANTE AUV, built and operated by the Instituto Superior Técnico of Lisbon, Portugal.

The paper starts by introducing a nonlinear dynamic model of the INFANTE AUV shown in Fig. 1. This is followed by control system design for precise maneuvering in the vertical plane. The technique elected for controller design is gain scheduling (Rugh *et al.*, 2000). Using this approach, a set of linear controllers is first derived for a finite number of linearized models of the plant at selected operating points. The resulting controllers are then interpolated on the vehicle's forward speed.

For linear control systems design, the paper exploits the use of reduced order feedback (ROF) techniques, which lead naturally to dynamic output feedback control laws with a very simple structure. In fact, the resulting controllers exhibit only the dynamics introduced by appended integrators (aimed at reducing steady state tracking errors to zero) as well as extra dynamics that act as shaping filters to limit the actuation bandwidth. The importance of output feedback control strategies cannot be overemphasized: in practice, it is often impossible, difficult, or too expensive to measure the full state vector of a given plant. This motivates the development of controllers that rely on output variables only, effectively increasing the simplicity and thus the reliability of the control laws adopted. In the case of the INFANTE AUV, for example, it is difficult to measure the angle of sideslip and the angle of attack in the horizontal and vertical planes, respectively. However, it is crucial to achieve stabilization and high vehicle perfor-

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mance in both planes. Thus the use of output feedback control to meet tight stability and performance criteria.

From a theoretical point of view, the reduced order output feedback control can be converted into a static output feedback problem for a related augmented system (Mäkilä, 1985). However, in spite of the availability of necessary and sufficient conditions for plant stabilizability by static output feedback,"no algorithm is currently available which guarantees to compute a stabilizing gain or determine if such a gain exists" (Iawasaki *et al.*, 1994). Much of the work in this area is well rooted in the theory of Linear Matrix Inequalities (LMIs), which are steadily becoming the tool par excellence for advanced control system design. In fact, many control problems can be cast as LMI problems that can be solved efficiently using convex programming techniques. However, difficulties arise when designing (sub-optimal) static output feedback controllers because the latter problem can only be cast in terms of an equivalent one that involves Bilinear Matrix Inequalities (BMIs) (Grigoriadis *et al.*, 1996). The resulting problem is no longer convex, and no efficient numerical procedures exist for its solution as in the case of LMIs. However, the bilinear characteristics of the problem can still be exploited to yield an iterative procedure whereby two sets of LMIs are solved sequentially. This is the approach pursued in this paper, where the results described in (El Ghaoui *et al.*, 1997) are used to develop a simple algorithm to iteratively search for the solution to (sub-optimal) static output feedback (SOF) control problems. Indirectly, the algorithm yields also a computational procedure to solve the ROF problem studied in this paper by exploiting the relationship between ROF and SOF control design techniques described in (Mäkilä, 1985). Although the algorithm performs a local search with no guarantees of global convergence, it exhibits excellent performance in the current application. The interested reader is referred to (Leibfritz, 2001) for a lucid discussion of different techniques for computational design of static output feedback controllers.

In the work reported here, a finite number of ROF controllers were developed for linearized plant models obtained at different operating conditions determined by the vehicle's forward speed. The controller parameters were then interpolated and scheduled on speed. The final implementation of the resulting non-linear gain scheduled controller was done using the D-methodology described in (Kaminer *et al.*, 1995) that guarantees a fundamental linearization property and avoids the need to feedforward the values of the state variables and inputs at trimming.

The paper is organized as follows. Section 2 introduces a nonlinear model for the vertical plane dynamics of the INFANTE AUV. Section 3 details the techniques that were used for depth control system design and implementation. Finally, Section 4 contains experimental results obtained during sea trials of the vehicle in the Azores, Portugal.

2. VEHICLE DYNAMICS

This section describes the dynamic model of the INFANTE AUV in the vertical plane. See (Silvestre, 2000) for a complete study of the AUV dynamics. The vehicle is 4.5 m long, 1.1 m wide and 0.6 m high. It is equipped with two main thrusters (propellers and nozzles) for cruising and fully moving surfaces (rudders, bow planes and stern planes) for vehicle steering and diving in the horizontal and vertical planes, respectively.

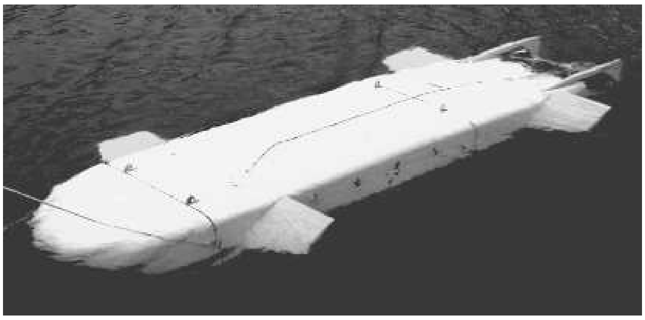

Fig. 1. The INFANTE Vehicle

The notation used and the structure of the vehicle model are standard (Silvestre, 2000; Fossen, 1994). The variables u and w denote surge and heave speeds, while θ, q, and z denote pitch, pitch rate, and depth, respectively. The symbols δ_b, and δ_s represent the bow and stern plane deflections, respectively. With this notation, and neglecting the roll stable motion, the dynamics of the AUV in the vertical plane can be written in compact form as

$$\begin{aligned} m\,(\dot{w} - uq) &= (W - B)\cos(\theta) + \frac{\rho}{2}L^2 Z_w uw + \\ &\quad \frac{\rho}{2}L^3 Z_q uq + \frac{\rho}{2}L^2 u^2\left[Z_{\delta_b}\delta_b + Z_{\delta_s}\delta_s\right] + \\ &\quad \frac{\rho}{2}L^3 Z_{\dot{w}}\dot{w} + \frac{\rho}{2}L^4 Z_{\dot{q}}\dot{q}, \end{aligned} \tag{1}$$

$$\dot{z} = -u\sin(\theta) + w\cos(\theta), \tag{2}$$

$$\begin{aligned} I_y\dot{q} &= z_{CB}B\sin(\theta) + \frac{\rho}{2}L^3 M_w uw + \frac{\rho}{2}L^4 M_q uq + \\ &\quad \frac{\rho}{2}L^3 u^2[M_{\delta_b}\delta_b + M_{\delta_s}\delta_s] + \frac{\rho}{2}L^4 M_{\dot{w}}\dot{w} + \frac{\rho}{2}L^5 M_{\dot{q}}\dot{q}, \end{aligned} \tag{3}$$

$$\dot{\theta} = q, \tag{4}$$

where equations (1) and (3) describe the heave and pitch motion respectively $Z_{(.)}$ and $M_{(.)}$ are hydrodynamic derivative terms and z_{CB} represents the metacentric distance. See (Silvestre, 2000) for numerical values of the hydrodynamic parameters. The variables m, L, W, B, and I_y are the vehicle's mass, length, weight, buoyancy, and moment of inertia about the y axis, respectively and ρ is the density of the water.

3. CONTROL SYSTEM DESIGN AND IMPLEMENTATION

This section focuses on the design of a depth control system for the AUV INFANTE, based on the dynamic model presented in Section 2. The methodology adopted for controller design is nonlinear gain-scheduled control, whereby the design of a controller to achieve stabilization and adequate performance of a given nonlinear plant (system to be controlled) involves the following steps (Rugh *et al.*, 2000):

i) *Linearizing* the plant about a finite number of representative operating points,
ii) Designing *linear controllers* for the plant linearizations at each operating point,
iii) *Interpolating* the parameters of the linear controllers of Step ii) to achieve adequate performance of the linearized closed-loop systems at all points where the plant is expected to operate. The interpolation is performed according to an external scheduling variable (vehicle's forward speed), and the resulting family of linear controllers is referred to as a *gain scheduled controller*,
iv) *Implementing* the gain scheduled controller on the original nonlinear plant.

In what follows a brief summary is given of the work carried out at each of the design steps, leading to the development of a controller for the vehicle that is scheduled on forward speed. For the sake of brevity, the linear design methodology is illustrated for the case of a single operating condition that corresponds to a forward speed of 2 m/s.

Linearization. Open-Loop System Analysis The model for the vertical plane was linearized about the equilibrium point determined by $(w_0, q_0, z_0, \theta_0)^T = (0,0,0,0)^T$ and $\mathbf{u}_0 = (\delta_b, \delta_s)^T = (0,0)^T$.

The resulting linearized model eigenvalues are presented in Fig. 2. The model exhibits an eigenvalue at zero (corresponding to a pure integrator in the depth coordinate z) and three *stable* eigenvalues that link together the variables w, q, and θ. Notice the overall trend in the plot, where the two complex eigenvalues at low speed

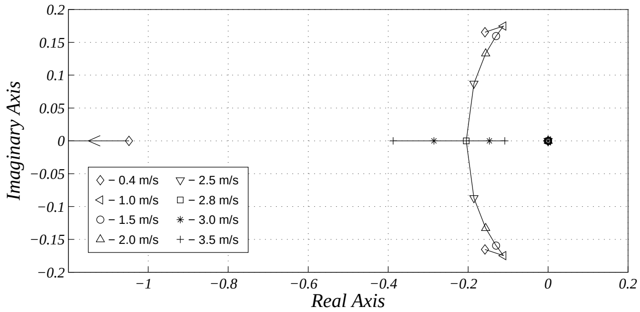

Fig. 2. Linearized model eigenvalues as functions of the forward speed u_0

degenerate into two real eigenvalues at higher speed. In the input matrix, the bow and stern plane deflections δ_b and δ_s affect directly the state variables w and q.

3.1 *Design Specifications*

The linear depth controllers were required to meet the following design specifications:

Zero Steady State Error. Achieve zero steady state values for the error variable in response to the input commands z_{cmd}.

Bandwidth Requirements. The input-output command response bandwidth for the depth command channel should be on the order of 0.1 rad/s; the control loop bandwidth for the bow and stern planes channels should not exceed 5 rad/s; these figures were selected to ensure that the actuators would not be driven beyond their normal actuation bandwidth.

Closed Loop Damping and Stability Margins. The closed loop eigenvalues should have a damping ratio of a least 0.7. It was also required that the steady state deflection of the bow planes in response to a step input command in depth be $\delta_b = 0$.

3.2 *Linear Control System Design*

The methodology selected for linear control system design was reduced order output feedback with an $\mathcal{H}_\infty$ criterion (Grigoriadis *et al.*, 1996). This method rests on a firm theoretical basis and leads naturally to an interpretation of control design specifications in the frequency domain. Furthermore, it provides clear guidelines for the design of controllers so as to achieve robust performance in the presence of plant uncertainty.

The Reduced Order Output Feedback (ROF) control problem can solved by converting it into a Static Output Feedback (SOF) control problem, using a well-known system augmentation technique. To that effect, consider the original plant dynamics $\Sigma_m = \{A_m, B_m, C_m\}$ and the appended dynamics $\Sigma_k = \{A_k = 0_k, B_k = I_k, C_k = I_k\}$ of order k. Let $\mathbf{u_k} \in \mathbb{R}^k$, and $\mathbf{x_k} \in \mathbb{R}^k$, be the control input and state of the appended dynamics. It can be shown (Mäkilä, 1985) that the ROF stabilization problem has a solution of order k if and only if the augmented system

$$A = \begin{bmatrix} A_k & 0 \\ 0 & A_m \end{bmatrix},\ B = \begin{bmatrix} B_k & 0 \\ 0 & B_m \end{bmatrix},\ C = \begin{bmatrix} C_k & 0 \\ 0 & C_m \end{bmatrix}$$

admits a static output-feedback stabilizing solution. The remainder of this section focuses on the SOF problem.

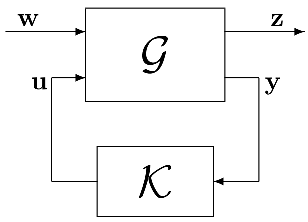

Fig. 3. Feedback interconnection

In what follows, the standard set-up and nomenclature in (Zhou *et al*, 1995) is adopted, leading to the feedback system represented in Fig. 3 with realization

$$\begin{cases} \dot{\mathbf{x}}(t) = \mathcal{A}\mathbf{x}(t) + \mathcal{B}_w\mathbf{w}(t) + \mathcal{B}\mathbf{u}(t) \\ \mathbf{z}(t) = \mathcal{C}_z\mathbf{x}(t) + \mathcal{D}\mathbf{w}(t) + \mathcal{E}\mathbf{u}(t), \\ \mathbf{y}(t) = \mathcal{C}\mathbf{x}(t) + \mathcal{F}\mathbf{w}(t) \end{cases} \quad \mathbf{u}(t) = \mathcal{K}\mathbf{y}(t), \quad (5)$$

where $\mathbf{x}$ is the state vector. The symbol $\mathbf{w}$ denotes the input vector of exogenous signals (including commands and disturbances), $\mathbf{z}$ is the output vector of errors to be reduced, $\mathbf{y}$ is the vector of measurements that are available for feedback, and $\mathbf{u}$ is the vector of actuator signals. The generalized plant $\mathcal{G}$ consists of the augmented system described before together with weights that shape the exogenous and internal signals, see Section 3.3. Suppose that the feedback system is well-posed, and let $\mathcal{T}_{\mathbf{zw}}$ denote the closed loop operator from $\mathbf{w}$ to $\mathbf{z}$. The (sub-optimal) $\mathcal{H}_\infty$ SOF synthesis problem consists of finding (if it exists) a static controller $\mathcal{K}$ that stabilizes the closed loop system and makes the infinity norm $\|\mathcal{T}_{\mathbf{zw}}\|_\infty$ of the operator $\mathcal{T}_{\mathbf{zw}}$ smaller than a desired bound $\gamma > 0$. The technique used for controller design is based on two following standard results.

Result 1: The closed loop system with realization (4) has all the eigenvalues in the semi-plane $\lambda \in C : \mathrm{Re}(\lambda) < \alpha$ if a real symmetric matrix $X > 0$ and a real matrix $\mathcal{K}$ exist such that the closed loop Lyapunov inequality

$$X(\mathcal{A}' - \alpha I) + X\mathcal{C}'\mathcal{K}'\mathcal{B}' + (\mathcal{A} - \alpha I)X + \mathcal{B}\mathcal{K}\mathcal{C}X < 0 \quad (6)$$

is satisfied.

Result 2: The $\mathcal{H}_\infty$ norm of the operator $\mathcal{T}_{\mathbf{zw}}$ is less than a positive number γ, that is, $\|\mathcal{T}_{\mathbf{zw}}\|_\infty < \gamma$, if a real symmetric matrix $X > 0$ and a real matrix $\mathcal{K}$ exist such that the LMI

$$\begin{bmatrix} X(\mathcal{A}' + \mathcal{C}'\mathcal{K}'\mathcal{B}') + (\mathcal{A} + \mathcal{B}\mathcal{K}\mathcal{C})X & * & * \\ \mathcal{B}'_w & -\gamma I & * \\ \mathcal{C}_z X + \mathcal{E}\mathcal{K}\mathcal{C} & \mathcal{D} & -\gamma I \end{bmatrix} < 0 \quad (7)$$

holds.

In the case of a square full rank matrix $\mathcal{C}$ the standard transformation $W = \mathcal{K}\mathcal{C}X$ converts the above nonlinear LMIs into convex LMIs. However, in the case of a noninvertible $\mathcal{C}$ matrix, the problem of determining a sub-optimal SOF controller involves Bilinear Matrix Inequalities (BLMIs). In this situation, the problem at hand is no longer convex, thus making the task of finding numerical solutions hard. It is important to point out that given an arbitrary dynamic system, there are no guarantees that a SOF controller exists that will stabilize the system. Furthermore, even if the existence of a stabilizing controller can be established, the nonconvex characteristics of the optimization problem are such that no assurances can be given as to whether a numerical procedure will converge to a solution. Therefore, the following algorithms for the computation of a sub-optimal $\mathcal{H}_\infty$ SOF controller should only be adopted if sound judging is applied to establish if a solution to the (sub-optimal) $\mathcal{H}_\infty$ SOF synthesis problem can indeed be found.

The algorithms proposed can be briefly explained as follows: i) compute (if it exists) a SOF stabilizing controller for the generalized plant $\mathcal{G}$, and ii) use this controller as a starting point to find an $\mathcal{H}_\infty$ sub-optimal controller. The first algorithm finds a stabilizing controller using Result 1, starting with an arbitrary $\mathcal{K}$ of appropriate dimensions.

Algorithm 1: *SOF stabilizing controller*

(1) For an arbitrary $\mathcal{K}$ find α such that $Re(\lambda_i(A - \alpha I)) < 0$, $i = 1, ..., n$.
(2) Fix $\mathcal{K}$ and solve LMI (6) (feasibility problem) with respect to variable X.
(3) Fix X and solve the optimization problem of minimizing α subject to the LMI constraint (6) in the variables α, and $\mathcal{K}$.
(4) If $\alpha \geq 0$ go to step 1, else end.

The second algorithm computes a SOF $\mathcal{H}_\infty$ sub-optimal controller using Result 2, adopting the stabilizing static controller $\mathcal{K}$ obtained before as a starting point.

Algorithm 2: *SOF $\mathcal{H}_\infty$ sub-optimal controller*

(1) Fix $\mathcal{K}$ and solve the optimization problem of minimizing γ subject to the LMI constraint (7) in the variables γ and X. Set $\gamma_1 = \gamma$ found.

(2) Fix X and solve the optimization problem of minimizing γ subject to the LMI constraint (7) in the variables γ and $\mathcal{K}$. Set $\gamma_2 = \gamma$ found.
(3) If $|\gamma_1 - \gamma_2| > \zeta$ go to step 1, else end.

In this design exercise ζ was set to 0.001.

Finally, the ROF controller is computed from the augmented system and the gain $\mathcal{K}$.

3.3 *Synthesis Model and Controller Design*

The first step in the controller design procedure is the development of a synthesis model that can serve as an interface between the designer and the $\mathcal{H}_\infty$ controller synthesis algorithm. Consider the feedback system shown

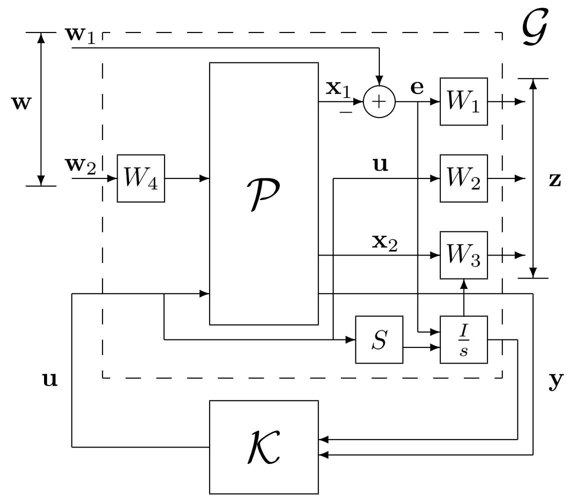

Fig. 4. Synthesis model

in Fig. 4, where $\mathcal{P}$ is the augmented linearized model of the AUV in the vertical plane, and $\mathcal{K}$ is a SOF controller to be designed. The block $\mathcal{G}$ within the dashed line is the synthesis model, which is derived from the linear augmented model of the plant by appending the depicted weights. In practice, the weights serve as tuning "knobs" which the designer can adjust to meet the desired performance specifications.

In the figure, $\mathbf{w}_1$ represents the depth command z_{cmd} that must be tracked. The vector $\mathbf{w}_2$ includes the input noise to each of the sensors that provide measurements of depth, pitch, and pitch rate as well as disturbance inputs to the states w and q of the plant. The signal $\mathbf{u}$ represents the augmented system control inputs that consist of $\mathbf{u}_k$ and the bow and stern plane deflections δ_b, and δ_s, respectively, whereas $\mathbf{e} = \mathbf{w}_1 - \mathbf{x}_1$ is the respective depth tracking error. The signal $\mathbf{x}_2$ contains the remaining state variables that must be penalized in the design process, that is, w, q, and θ. The matrices W_i; $i = 1, \ldots, 4$ correspond to dynamic weights that penalize input, state, and tracking variables. Finally, the signal $\mathbf{y}$ consists of the variables $\mathbf{x}_k$, q, θ, z, $\mathbf{e}/s$ and δ_b/s that are available for feedback. To meet the depth step command response requirement the weighting function W_1 was chosen as $W_1 = 0.1$. The bow and stern planes deflection bandwidth constraint was achieved with $W_2 = \text{diag}(0.5, 0.5, 2(s/6+1)/(s/30+1), 2(s/6+1)/(s/30+1))]$. The weight W_3 was set to $\text{diag}(22.0, 0.1, 0.01, 0.05, 0.05)$ to meet the command bandwidth requirements, and $W_4 = 0.001 I_4$. Notice the existence of a block of integrators I/s that operates on the tracking errors $\mathbf{e}$ and on the entries of the control input vector $\mathbf{u}$ that are selected by the matrix S. Integral action on the errors is required to ensure zero steady state in response to step commands in $\mathbf{w}_1$. Integral action on the entries of $\mathbf{u}$ introduces a "washout" on the particular control inputs selected. In the present case the "washout" ensures zero bow plane deflection at trimming conditions. After several iterations the controller order was set to $k = 2$ to accommodate the required actuators bandwidth constraints.

3.4 *Non-linear Controller Implementation*

A set of controllers was designed for a finite number of operating points, and their parameters interpolated according to the vehicle's forward speed (scheduling variable). The implementation of the resulting non-linear gain scheduled controller was done using the D-methodology described in (Kaminer *et al.*, 1995). This leads to the general structure for the implementation of discrete-time gain scheduled controllers depicted in Fig. 5, where $F(u)$ denotes the block that interpolates the reduced order output feedback controllers obtained from the discretization of the linear controller designs in Section 3.2. In the present case a sampling frequency of 10 Hz was selected.

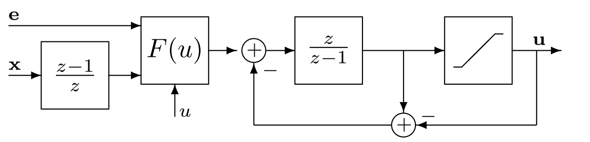

Fig. 5. D Controller implementation with anti-windup mechanism

4. TESTS AT SEA

To assess the performance of the controller developed, a series of tests were carried out at sea. The vehicle was operated at constant heading under the influence of strong wave action. Figs. 6 through 10 show some of the practical results obtained during depth changing maneuvers, together with the results of simulations obtained with a full nonlinear model of the vehicle. At the beginning of this maneuver INFANTE was at surface;

20 seconds into the maneuver the depth controller was switched on and a command to dive to 8 meters depth was applied; this was followed by a command to dive to 10 meters at $t = 150$ seconds. In the figures, the dashed and solid lines represent the experimental and the simulation results, respectively. The vehicle's forward speed was kept approximately constant at 1.8 m/s.

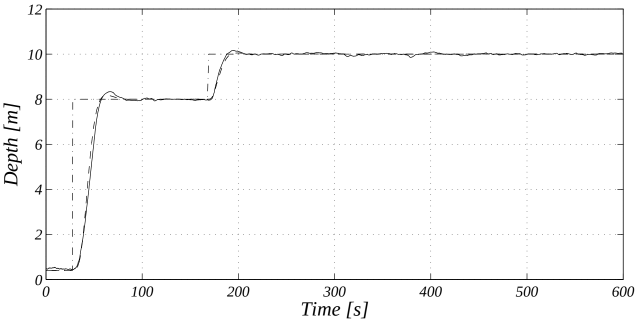

Fig. 6. Commanded and measured depth - simulated and measured values

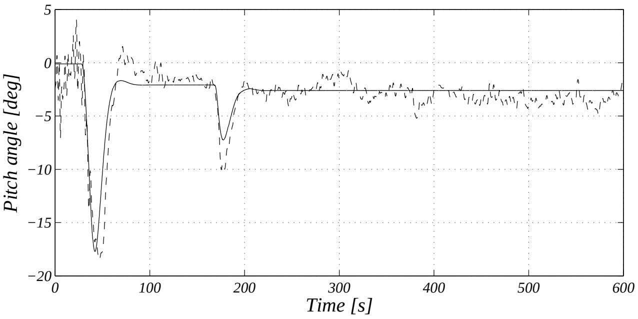

Fig. 7. Pitch angle

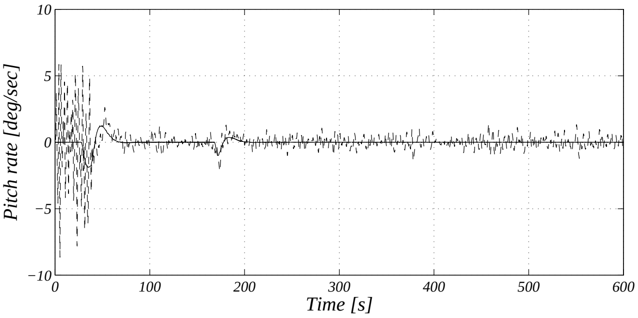

Fig. 8. Pitch rate

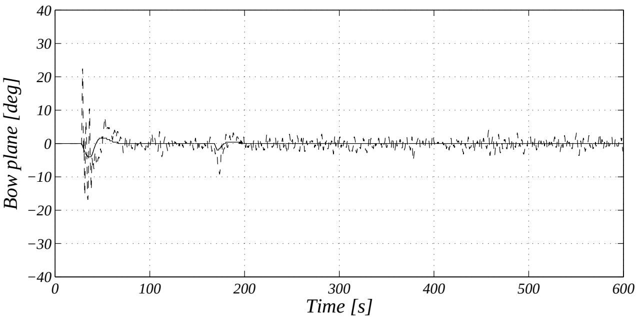

Fig. 9. Bow plane deflection under strong wave action

Figs. 6, 7 and 8 show commanded and measured depth, pitch, and pitch rate activity, respectively. Figs. 9 and 10

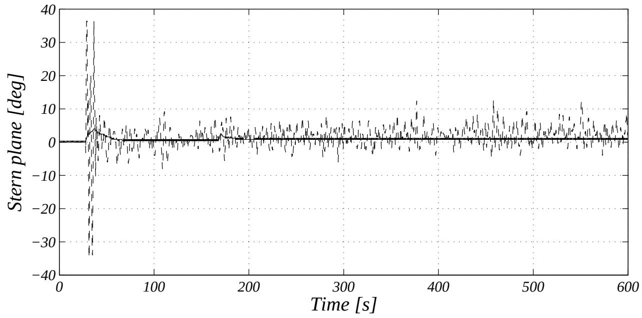

Fig. 10. Stern plane deflection under strong wave action

display the activity of the bow and stern planes respectively. Notice the strong coupling between wave action and control planes deflection near the surface, mainly induced by pitch and pitch rate. Leaving aside the influence of the waves (which was not addressed explicitly in the controller design phase), the figures reveal close agreement between predicted and actual maneuvers.

5. ACKNOWLEDGEMENTS

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6. REFERENCES

El Ghaoui, L., Oustry, F., AitRami, M. (1997). A Cone Complementary Linearization Algorithm for Static Output-Feedback and Related Problems, *IEEE Trans. Automatic Control*, Vol. 42, No. 8, pp 1171-1176.

Fossen, T. (1994). *Guidance and Control of Ocean Vehicles*, John Wiley and Sons.

Grigoriadis, K. M. and Skelton, R. E. (1996). Low Order Control Design for LMI problems Using Alternating Projection Methods, *Automatica*, Vol. 32, pp. 1117–1125.

Iwasaki, T., Skelton, R., Geromel, J. (1994). Linear Quadratic Suboptimal Control with Static Output Feedback, *Systems and Control Letters*, Vol. 32, No. 6, pp 421-430.

Kaminer, I., Pascoal, A., Khargonekar, P. and Coleman, E. (1995). A Velocity Algorithm for the Implementation of Gain-Scheduled Controllers, *Automatica*, Vol. 31, pp. 1185–1191.

Leibfritz, F. (2001). An LMI-based Algorithm for designing Suboptimal Static Output Feedback Controllers, *SIAM Journal on Control and Optimization*, Vol. 39, No. 6, pp. 1711–1735.

Mäkilä, P. M.(1985). Parametric LQ control, *International Journal of Control*, Vol. 41, No. 6, pp. 1413–1428.

Rugh, W. and Shamma, Jeff S. (2000). A Survey of Research on Gain-Scheduling, *Automatica*, pp 261–268, September.

Silvestre, C. (2000). *Multi-Objective Optimization Theory with Applications to the Integrated Design of Controllers / Plants for Autonomous Vehicles*, PhD thesis, Department of Electrical Engineering and Computer Science, Instituto Superior Técnico, Lisbon, Portugal.

Zhou, Kemin, Doyle, John C., and Glover, Keith (1995). *Robust and Optimal Control*, Prentice Hall, New Jersey, USA.