A TWO-DEGREE-OF-FREEDOM SMITH CONTROL FOR IMPROVED DISTURBANCE REJECTION

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Abstract: A two-degree-of-freedom Smith control scheme is proposed for improved disturbance rejection for stable delay processes. The resulting set-point and disturbance responses can be tuned by two controllers separately. A novel disturbance controller design is presented with easy tuning and greatly improved performance. The internal and robust stability issues are discussed. Examples are provided for illustration.

Keywords: Process control, delay systems, Smith Predictor, disturbance rejection.

1. INTRODUCTION

In process control, the Smith predictor (Smith, 1959) is a well known and very effective dead-time compensator. One major concern with the normal Smith control is that its disturbance rejection performance is limited due to its one-degree-of-freedom nature. In order to cater to disturbance rejection and robustness as well, a double-controller scheme is presented by Tian and Gao (1998) for stable first order processes with dominant delay. But the improvement of disturbance rejection is not significant, and its performance deteriorates when the process time delay is relatively small. Recently, several 'modified Smith predictor' control schemes have been proposed (Chien *et al.*, 2002; Kaya, 2003; Majhi and Atherton, 2000) to extend applicability of the Smith control to unstable processes. They handle integral or first-order unstable plants by employment of more controllers, and can be applied to stable processes as well through scheme simplification. It is however noted that their characteristic equations are all delay dependent, which is in contrast to delay-free one enjoyed by the normal Smith control and which keeps the stabilization problem as a complicate task. Also, they paid little attention to disturbance rejection. It is undoubtable that disturbance rejection is most important in process control and good solutions have been sought for long time.

In this paper, a two-degree-of-freedom Smith predictor control scheme is proposed for improved disturbance rejection. Its nominal stabilization is of delay free. The resulting set-point response remains the same as in the normal Smith scheme. But the disturbance response can be tuned by one additional controller separately with no effects on the set-point response. Furthermore, a novel method is presented to design this disturbance controller easily and yield substantial control performance improvement.

The rest of the paper is organized as follows. In Section 2, the two-degree-of-freedom control scheme is presented. Stability analysis is given in Section 3. Controller designs are detailed for first-

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order plus dead time (FOPDT) and second-order plus dead time (SOPDT) processes in Section 4 to demonstrate our methods. In Section 5, three examples are provided. And finally, Section 6 concludes this paper.

2. THE PROPOSED SCHEME

In this paper, we consider stable delay processes. Our goal is to devise some new control scheme which can keep nominal delay-free stabilization of the closed-loop system like that in the normal Smith control, yet, provide some additional means to improve disturbance rejection, and hopefully one can tune the set-point and disturbance responses separately and easily. After many trials, we come up with the two-degree-of-freedom Smith control scheme as depicted in Figure 1, which can fulfill all the above requirements. In Figure 1, $G(s) = G_0(s)e^{-Ls}$ and $\hat{G}(s) = \hat{G}_0(s)e^{-\hat{L}s}$ are the stable process and its model respectively. Suppose that the model matches the plant dynamics perfectly, i.e., $\hat{G}_0 = G_0$ and $\hat{L} = L$. It follows that the closed-loop transfer function from the set-point to the output is given by

$$H_r = \frac{G_0 C_1}{1 + G_0 C_1} e^{-Ls}. \quad (1)$$

For the disturbance path, it can be shown that the transfer function is

$$H_d = \frac{1 + G_0 C_1 - G_0 C_1 C_2 e^{-Ls}}{1 + G_0 C_1} G_0 e^{-Ls}, \quad (2)$$

which shares the same delay-free denominator as in H_r. To see the difference and benefits of the new scheme. Letting $C_2 = 1$ reduces the scheme to the normal Smith system which has the same set-point transfer function as in (1) but a different disturbance transfer function as

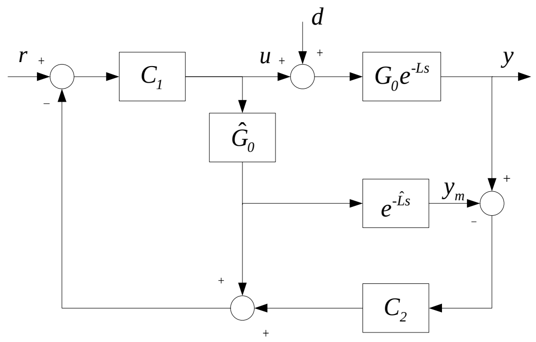


Fig. 1. Block diagram of the proposed scheme

$$H_{d1} = \frac{1 + G_0 C_1 - G_0 C_1 e^{-Ls}}{1 + G_0 C_1} G_0 e^{-Ls}.$$

Obviously, with C_1 designed for closed-loop stability and the set-point response, the normal Smith scheme simply does not have any freedom to manipulate the disturbance response. Owing to great importance of disturbance rejection in process control industry, it is definitely desirable to have a means to improve it. In the new scheme, C_2 appears in the numerator of H_d, and thus can be utilized to reduce or minimize H_d. It is also noted that C_2 is not in the set-point transfer function (1). Hence, C_1 and C_2 can be tuned separately as follows. C_1 is designed to have the desired stability and set-point response. This is a standard task and there are many solutions already. Our focus here is on C_2, that is, design it to achieve best disturbance rejection.

In view of (2), intuitively, one might attempt to determine C_2 by frequency response fitting, i.e., by minimizing

$$|H_d| = \left|1 - \frac{G_0 C_1 e^{-j\omega L}}{1 + G_0 C_1} C_2\right| \left|G_0 e^{-Ls}\right| = |1 - H_r C_2| \, |G_0|$$

for some working frequency range, $\underline{\omega} \le \omega \le \bar{\omega}$, so that the disturbance response is attenuated. Such optimization falls into the model matching category and sounds reasonable. However, it fails to produce ideal performance, as will be demonstrated in Example 2 later. This is because the optimization tends to get C_2 as $C_2 = 1/H_r$ over $[\underline{\omega},\ \bar{\omega}]$. The resulting C_2 would mimic the behavior of $1/H_r$ that contains pure time leading $e^{j\omega L}$ with counter-clockwise Nyquist curve, and would exhibit large magnitude for $\omega > \bar{\omega}$. This increases the corresponding $|H_d|$, and may even make the scheme more susceptible to unmodelled high frequency dynamics or uncertainties.

In order to attain better disturbance rejection in face of the delay term in the numerator of H_d, a novel method is proposed as follows. For a given type disturbance, say $D(s)$, it follows from (2) that the disturbance response is

$$\begin{aligned} Y_d =& \frac{1 + G_0 C_1 - G_0 C_1 C_2 e^{-Ls}}{1 + G_0 C_1} G_0 e^{-Ls} D \\ =& Y_{da} - Y_{db}, \end{aligned} \quad (3)$$

where

$$Y_{da} = G_0 D e^{-Ls} \quad (4)$$

is fixed and

$$Y_{db} = \frac{G_0 G_0 C_1 C_2}{1 + G_0 C_1} D e^{-2Ls} \quad (5)$$

is manipulatable by C_2. Suppose that the disturbance occurs at $t = 0$. Then non-zero responses in $y_{da}(t)$ and $y_{db}(t)$ come in at $t = L$ and $t = 2L$, respectively. Obviously, the disturbance response during $t = L$ to $t = 2L$ is solely from $y_{da}(t)$ and fixed. Any effort to change it during this time period is useless but causes the problem on controller design. The best achievable disturbance rejection is to zero the disturbance response from $t = 2L$ onwards:

$$y_d(t) = y_{da} - y_{db} = \begin{cases} y_{da}(t), & 0 < t < 2L \\ 0, & t \ge 2L \end{cases}$$

which requires the compensating response $y_{db}(t)$ to be

$$y_{db}(t) = \begin{cases} 0, & t < 2L \\ y_{da}(t), & t \geq 2L \end{cases} = y_{da}(t)1(t-2L), \quad (6)$$

as displayed in Figure 2. We now derive an

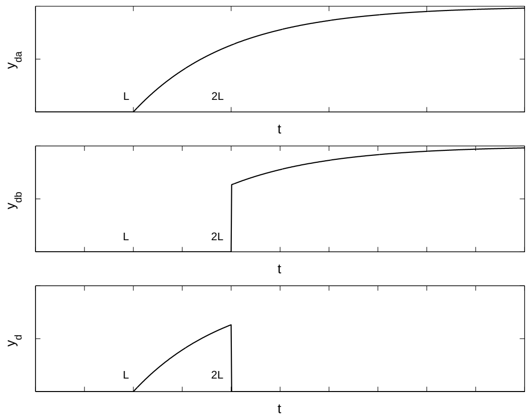


Fig. 2. Illustration of desired disturbance rejection

analytical solution for $C_2(s)$ to meet (6). In view of (4), Y_{da} can be expressed using the partial fraction expansion as, say,

$$Y_{da}(s) = G_0 D e^{-Ls} = \left(\frac{\alpha_0}{s} + \sum_i \frac{\alpha_i}{s+\beta_i} \right) e^{-Ls},$$

and its time domain form is

$$y_{da}(t) = \left[\alpha_0 + \sum_i \alpha_i e^{-\beta_i (t-L)} \right] 1(t-L).$$

It follows that

$$\begin{aligned} y_{da}(t)1(t-2L) &= \left[\alpha_0 + \sum_i \alpha_i e^{-\beta_i (t-L)} \right] 1(t-2L) \\ &= \left[\alpha_0 + \sum_i \alpha_i e^{-\beta_i L} e^{-\beta_i (t-2L)} \right] 1(t-2L) \\ &\triangleq \hat{y}_{da}(t-2L)1(t-2L), \end{aligned} \quad (7)$$

where

$$\hat{y}_{da}(t) = \alpha_0 + \sum_i \alpha_i e^{-\beta_i L} e^{-\beta_i t},$$

with

$$\hat{Y}_{da}(s) = \frac{\alpha_0}{s} + \sum_i \frac{\alpha_i e^{-\beta_i L}}{s+\beta_i}.$$

Laplace transform of (6) with help of (5) and (7) gives

$$\frac{G_0 G_0 C_1 C_2}{1+G_0 C_1} D e^{-2Ls} = \hat{Y}_{da}(s) e^{-2Ls},$$

and its solution is

$$C_2^* = \frac{\hat{Y}_{da}(s)(1+G_0 C_1)}{G_0 G_0 C_1 D}. \quad (8)$$

Since C_2^* is improper in general, a low-pass filter should be added for practical implementation so that the actual C_2 is given by

$$C_2 = \frac{1}{(\tau s+1)^n} \frac{\hat{Y}_{d1}(s)(1+G_0 C_1)}{G_0 G_0 C_1 D}. \quad (9)$$

Detailed controller design will be provided for several typical industrial processes in Section 4 after the stability analysis section.

Before concluding this section, we would highlight novelty and advantage of our new scheme over the standard two-degree-of-freedom control scheme (either single-loop based or Smith predictor based) where a prefilter is added between the reference input and the negative feedback. In the standard two-degree-of-freedom control scheme, obviously, the prefilter does not affect the disturbance response and could only be utilized to tune the set-point response. Then, this leaves its primary controller responsible for both closed-loop stabilization and disturbance response, and thus limits disturbance rejection performance. On the other hand, in our scheme, C_2 deals solely with the disturbance. It is easier to design and superior in disturbance rejection performance. In the extreme case where the process is bi-proper, C_2 may eliminate the disturbance response completely from $t = 2L$, which is impossible for the standard two-degree-of-freedom control scheme and any other schemes where the controller taking care of disturbance rejection also needs to cope with closed-loop stability and/or pole placement.

As for the system stability, it could be shown (Wang *et al.*, 1999) that the system is internally stable if and only if C_1 stabilizes G_0 and C_2 is stable. As for robust stability analysis, let the total uncertainty be given by

$$\Delta_G(s) = \left| \frac{G(s) - \hat{G}(s)}{\hat{G}(s)} \right|. \quad (10)$$

Assume nominal stability. Then by the small gain theorem, the closed-loop system is robustly stable if and only if

$$\left| \frac{C_1 C_2}{1+C_1 \hat{G}_0} \hat{G} \Delta_G \right|_\infty < 1. \quad (11)$$

By invoking (1) and (9), (11) reduces to

$$\left| \frac{1}{H_r(j\omega) C_2^*(j\omega)} \right| (\tau^2 \omega^2 + 1)^{n/2} > |\Delta_G|, \ \forall \omega > 0. \quad (12)$$

It can been seen from (9) and (12) that a trade-off is to be made by C_2, or tuning of the parameter τ: a decrease in τ will improve the disturbance rejection performance but reduce the robust stability, and vise versa.

3. CONTROLLER DESIGN

It follows from the preceding sections that in our scheme, C_1 is designed to have stable closed-loop and good set-point response, and C_2 has to be stable and meet (9). It is noted that most typical industrial processes of interests could be approximated by FOPDT or SOPDT ones. Detailed controller design will be carried out for each case and closed-form formulas for controller parameters are given as follows for easy reference.

FOPDT Processes Consider the following stable FOPDT process:

$$G(s) = G_0(s)e^{-Ls} = \frac{k_0}{T_0 s + 1} e^{-Ls},$$

where all coefficients are positive. The closed-loop transfer function for set-point tracking is chosen to be

$$H_r = \frac{k_0 C_1}{T_0 s + 1 + k_0 C_1} e^{-Ls} = \frac{1}{T_r s + 1} e^{-Ls},$$

where T_r is the desired closed-loop time constant and $T_r \geq T_0$ is recommended. This gives rise to

$$C_1 = \frac{T_0 s + 1}{k_0 T_r s}. \qquad (13)$$

Then the corresponding closed-loop transfer function from disturbance is

$$H_d = \frac{k_0}{T_0 s + 1} e^{-Ls} - \frac{k_0 C_2}{(T_r s + 1)(T_0 s + 1)} e^{-2Ls}. \qquad (14)$$

Consider the most typical case of step disturbance: $D(s) = k_D/s$. It follows from definitions in Section 2 that

$$y_{da}(t) = k_D k_0 [1 - e^{-(t-L)/T_0}] 1(t - L),$$
$$\hat{y}_{da}(t) = k_D k_0 [1 - e^{-L/T_0} e^{-t/T_0}],$$
$$\hat{Y}_{da}(s) = k_D k_0 \left(\frac{1}{s} - \frac{e^{-L/T_0}}{s + 1/T_0} \right),$$

and

$$C_2^* = (T_r s + 1)[T_0(1 - e^{-L/T_0})s + 1].$$

Obviously, $n = 2$ is needed to implement C_2^* as

$$C_2 = \frac{(T_r s + 1)[T_0(1 - e^{-L/T_0})s + 1]}{(\tau s + 1)^2}. \qquad (15)$$

A large τ will increase the system robustness, and a small one will yield better disturbance rejection. The recommended range for τ is $\tau = 0.2T_r \sim T_r$.

SOPDT Processes with real poles Consider the following stable SOPDT process:

$$G(s) = G_0(s)e^{-Ls} = \frac{k_0}{(T_1 s + 1)(T_2 s + 1)} e^{-Ls},$$

where all coefficients are positive. Choose the desired set-point transfer function as

$$H_r = \frac{\omega_n^2}{s^2 + 2\xi_n \omega_n s + \omega_n^2} e^{-Ls},$$

and C_1 is given by

$$C_1 = \frac{\omega_n^2}{k_0} \frac{(T_1 s + 1)(T_2 s + 1)}{(s + 2\xi_n \omega_n)s}. \qquad (16)$$

Still for step type disturbance $D(s) = k_D/s$, it follows that

$$Y_{da}(s) = k_D k_0 \left(\frac{1}{s} - \frac{T_1/(T_1 - T_2)}{s + 1/T_1} + \frac{T_2/(T_1 - T_2)}{s + 1/T_2} \right),$$
$$\hat{y}_{da}(t) = k_D k_0 \left(1 - \frac{T_1 e^{-L/T_1} e^{-t/T_1}}{T_1 - T_2} + \frac{T_2 e^{-L/T_2} e^{-t/T_2}}{T_1 - T_2} \right),$$
$$\hat{Y}_{da}(s) = k_D k_0 \left(\frac{1}{s} - \frac{a_1 T_1}{T_1 s + 1} + \frac{a_2 T_2}{T_2 s + 1} \right),$$

where

$$a_1 = \frac{T_1}{T_1 - T_2} e^{-L/T_1}, \; a_2 = \frac{T_2}{T_1 - T_2} e^{-L/T_2}.$$

Then the ideal C_2^* is derived from (8) as

$$C_2^* = \frac{1}{\omega_n^2}[(1 - a_1 + a_2)T_1 T_2 s^2 + (T_1 + T_2 - a_1 T_1 + a_2 T_2)s + 1][s^2 + 2\omega_n \xi_n s + \omega_n^2],$$

and implemented by

$$C_2 = \frac{C_2^*}{(\tau s + 1)^4}, \qquad (17)$$

with $\tau = 0.1/\omega_n \sim 1/\omega_n$ recommended.

SOPDT Processes with complex poles Consider the following stable SOPDT process:

$$G(s) = G_0(s)e^{-Ls} = \frac{k_0}{s^2 + 2\xi_0 \omega_0 s + \omega_0^2} e^{-Ls},$$

where all coefficients are positive and $0 < \xi_0 < 1$. Choose the desired set-point transfer function as

$$H_r = \frac{\omega_n^2}{s^2 + 2\xi_n \omega_n s + \omega_n^2} e^{-Ls},$$

C_1 is given by

$$C_1 = \frac{\omega_n^2}{k_0} \frac{s^2 + 2\xi_0 \omega_0 s + \omega_0^2}{(s + 2\xi_n \omega_n)s}. \qquad (18)$$

For step disturbance $D(s) = k_D/s$, it follows that

$$\hat{y}_{da}(t) = \frac{k_D k_0}{\omega_0^2} \left[1 - e^{-\alpha(t+L)} \right] \left\{ \cos[\beta(t + L)] + \frac{\alpha}{\beta} \sin[\beta(t + L)] \right\},$$
$$\hat{Y}_{da}(s) = \frac{k_D k_0}{\omega_0^2} \left(\frac{1}{s} - \frac{b_1 s + b_2}{s^2 + 2\xi_0 \omega_0 s + \omega_0^2} \right),$$

with

$$\alpha = \omega_0 \xi_0, \; \beta = \omega_0 \sqrt{1 - \xi_0^2},$$
$$b_1 = e^{-\alpha L}[\cos(\beta L) + \frac{\alpha}{\beta} \sin(\beta L)],$$
$$b_2 = \beta e^{-\alpha L}[\frac{\alpha}{\beta} \cos(\beta L) - \sin(\beta L)].$$

The C_2^* is then derived from (8) as

$$C_2^* = \frac{1}{\omega_0^2 \omega_n^2}[s^2 + 2\omega_n \xi_n s + \omega_n^2] [(1 - b_1)s^2 + (2\alpha - b_1 \alpha - b_2)s + \omega_0^2],$$

and implemented by

$$C_2 = \frac{C_2^*}{(\tau s + 1)^4}, \qquad (19)$$

with $\tau = 0.1/\omega_n \sim 1/\omega_n$ recommended.

4. EXAMPLES

In this section, we demonstrate our designs in Section 4 by three examples, one for each case. The set-point input and load disturbance are both step signals with magnitude of 1 and 0.5, respectively, throughout the Examples.

Example 1: Consider an stable FOPDT process with dominant delay:

$$G(s) = \frac{1}{s+1} e^{-3s}.$$

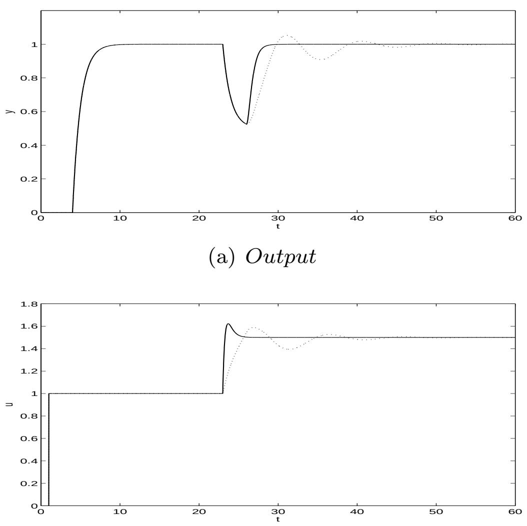
The controller parameters of the double-controller Smith scheme from Tian and Gao (1998) are $G_{c1} = 1 + 1/s$ and $G_{c2} = 0.667 + 0.222/s$. Choose $T_r = T_0 = 1$ to achieve the same set-point response as Tian's. For $\tau = 0.4T_0 = 0.4$, it follows from (13) and (15) that

$$C_1 = 1 + \frac{1}{s},$$

and

$$C_2 = \frac{0.9502s^2 + 1.95s + 1}{(0.4s+1)^2}.$$

The responses from both schemes are compared in Figure 3, and the performance improvement of the proposed design is clear. For better comprehension, the curves for y_{da}, y_{db} and the error $y_d = y_{da} - y_{db}$ are given in Figure 4. It verifies that the proposed C_2 results in a y_{db} which approaches y_{da} after $t \geq 2L$, and improves the disturbance response significantly.



(a) *Output*

(b) *Input*

Fig. 3. Step responses of Example 1 (—— *Proposed*; $\cdots\cdot$ *Tian's*)

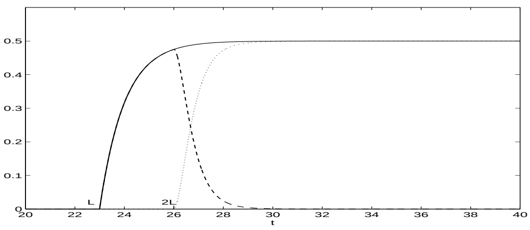


Fig. 4. Illustration of y_{da}, y_{db} and y_d for Example 1 (—— y_{da}; $\cdots\cdot$ y_{db}; - - - y_d)

Example 2: Consider an stable SOPDT process with distinct real poles:

$$G(s) = \frac{2}{(10s+1)(2s+1)} e^{-3s}.$$

By choosing $\omega_n = 0.2$, $\xi_n = 1$ and $\tau = 0.15/\omega_n = 0.75$, it follows from (16) and (17) that

$$C_1 = \frac{s^2 + 0.6s + 0.05}{2.5s^2 + s},$$

and

$$C_2 = \frac{64.88s^4 + 97.24s^3 + 56.11s^2 + 12.85s + 1}{(0.75s+1)^4}.$$

The PI-PD Smith scheme from Kaya (2003) is adopted for comparison, whose controller parameters are calculated as $G_{c1} = 0.4 + 0.04/s$ and $G_{c2} = -0.1 - s$ to provide the same set-point response. The model match design as described in Section 2 is also investigated, and the second order controller C_{2mm} is derived (Wang *et al.*, 2003) as

$$C_{2mm} = \frac{50.11s^2 + 4.044s + 1}{(0.5s+1)^2}.$$

The responses from all three schemes are plotted in Figure 5, the proposed scheme provides best disturbance rejection, and the model match design's performance is also better than Kaya's design. As for the robust performance, suppose $\pm 20\%$ gain change or $\pm 10\%$ time delay change. We plot (12) in Figure 6, which indicates robust stability. And the corresponding time responses are provided in Figures 7 and 8.

Example 3: Consider an oscillating stable SOPDT process with $\xi_0 = 0.6$ and $\omega_0 = 0.5$:

$$G(s) = \frac{0.3}{s^2 + 0.6s + 0.25} e^{-3s}.$$

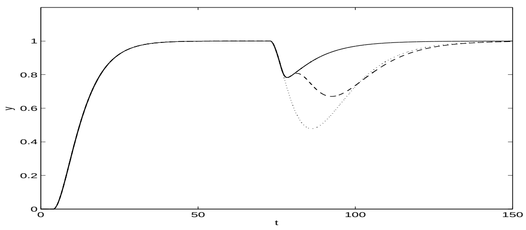
By choosing $\omega_n = \omega_0 = 0.5$, $\xi_n = 1$ and $\tau = 0.2/\omega_0 = 0.4$, it follows from (18) and (19) that

$$C_1 = \frac{0.83s^2 + 0.5s + 0.21}{s^2 + s},$$

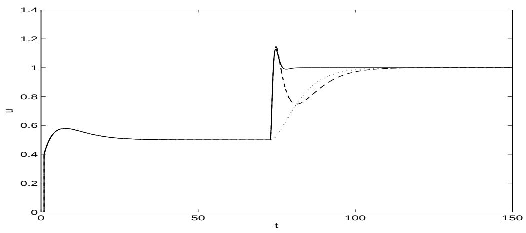
and

$$C_2 = \frac{5.158s^4 + 12.19s^3 + 12.32s^2 + 5.758s + 1}{(0.4s+1)^4}.$$

The proposed design and the normal Smith predictor are compared in Figure 9, and the disturbance response improvement is obvious. In view of



(a) *Output*



(b) *Input*

Fig. 5. Step responses of Example 2
(—— *Proposed*; ···· *Kaya's*; - - - *Model match design*)

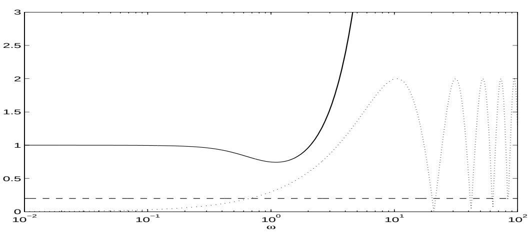


Fig. 6. Robust stability check of Example 2
(—— *Left half part of* (12) for $\tau = 0.75$;
- - - *Right half part of* (12) *for* $|\Delta k| = 20\%$;
···· *Right half part of* (12) *for* $|\Delta L| = 10\%$)

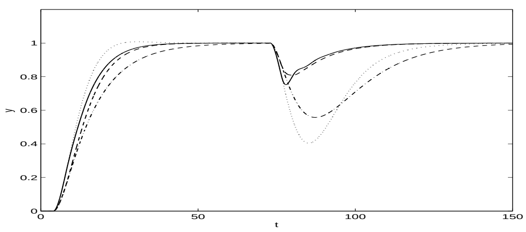


Fig. 7. Responses in case of ±20% gain changes of Example 2
($\Delta k = 20\%$: —— *Proposed*, ···· *Kaya's*;
$\Delta k = -20\%$: - - - *Proposed*, - · - *Kaya's*)

these three examples, the proposed method yield much better disturbance rejection, owing to the additional one more degree-of-freedom provided by C_2.

5. CONCLUSION

Due to great importance of disturbance rejection, a new control scheme, two-degree-of-freedom Smith control, is proposed for better disturbance rejection for stable delay processes. This scheme has an additional degree-of-freedom to manipulate disturbance response. It keeps nominal characteristic equation delay-free, and allows separate and easy design of disturbance controller with superior disturbance rejection, while the set-point response remains the same as in the normal Smith system.

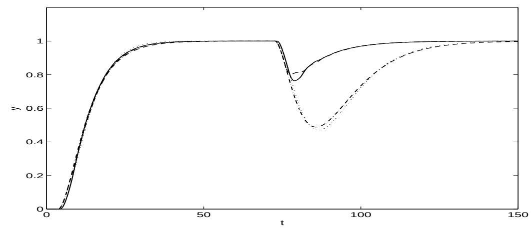
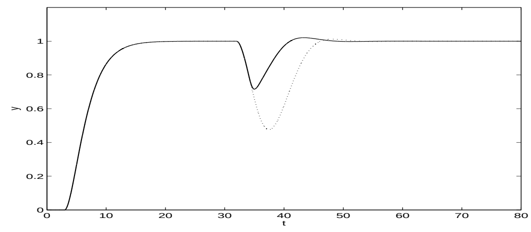


Fig. 8. Responses in case of ±10% time delay changes of Example 2
($\Delta L = 10\%$: —— *Proposed*, ···· *Kaya's*;
$\Delta L = -10\%$: - - - *Proposed*, - · - *Kaya's*)



(a) *Output*

Fig. 9. Step responses of Example 3
(—— *Proposed*; ···· *Normal Smith*)

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