EXTENDED MOVING BOUNDARY MODEL FOR TWO-PHASE FLOWS

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Abstract: Moving boundary models developed to date are based on several hypotheses that cannot be maintained in several applications, such as solar evaporators. In this work, the main modification in the hypotheses is the difference in pressure along the evaporator, due to the length of the real installations to model (500 m and 1400 m). Modelling of dynamics associated to momentum conservation is necessary in this case, following the scheme in 'staggered-grid' and the Finite Volume Method. This paper presents the development of an extended moving boundary model containing information on the momentum conservation in two additional control volumes.

Keywords: dynamic modelling, two-phase flows, moving boundary models.

1. INTRODUCTION

Control of distributed evaporators with two-phase flows requires dynamic modelling to predict the transient behaviour of the mentioned systems against determined disturbances. For control purposes, these models should not require excessive computing effort because for model-based controllers, they should be solved in each control action. Also, the error shall be bounded and acceptable for the control system purposes. Generalized Moving Boundary Models (GMBM) meet these general specifications, on condition of being more inaccurate than models based on spatial discretization techniques. This paper shows an extension to Generalized Moving Boundary Models, in those cases where the evaporator to be modelled does not meet some of the hypotheses where these models are based on.

2. GENERALIZED MOVING BOUNDARY MODELS

In the scope of dynamic modelling for evaporators control, moving boundary models (MBM) fulfill a compromise between simplicity and numerical robustness allowing, under certain simplifications, to obtain low order dynamic models that can be used in model based controllers. They are based on the idea of discretizing the spatial domain of the PDE set solution obtained from the formulation of principles of mass and energy conservation, on a flow subjected to phase changes due to the exchange of energy with its environment. This is the case of Parabolic-Trough Collectors (PTC) (Valenzuela *et al.*, 2004) (Zarza, 2003), where the flow is water-steam. MBMs have the advantage of avoiding modelling of phase transitions in thermodynamic equilibrium, which could give numerical problems in 'chattering' form, what is frequent in discretized models. On the other hand, it entails too aggressive approximations that may lead to high deviations in the models predictions as well as more limited utilization ranges with respect to an operation point. This document is based on the generalized MBM (GMBM) presented in (Jensen and Tummescheit, 2002). In that paper the authors show the development of a generalized model of an evaporator with the definition of three control volumes (CV) for the three regions corresponding to the subcooled liquid, two-phase and superheated. In one CV the equations of mass and energy conservation in the fluid, as well as energy conservation for the corresponding section of the evaporator pipe are stated. The developments take into account that CV's boundaries vary dynamically.

2.1 Hypothesis base of the GMBM

The hypotheses of GMBMs (Jensen, 2003) are:

- Negligible pressure drop in the evaporator.
- Average properties in each region.
- Negligible gravitational forces.
- Negligible changes in the kinetic energy.
- Constant cross section in the pipe.
- Negligible axial thermal conduction in the fluid and pipe.
- In two-phase flow region the thermodynamic equilibrium is considered in both phases.

Some of these hypotheses cannot be maintained in evaporators with certain length, as in the water-steam distributed evaporators in 'one-through' mode, with dimensions of 500m and 1400m respectively. These installations are part of CIEMAT's Solar Thermal Energy Generation Plants (Spanish Ministry of Education and Science) DISS and Solar Thermal Central Receiver CESA-1, located at the Plataforma Solar de Almeria (South-East Spain). In both cases, due to the length and/or the evaporator's tilt, neither the pressure drop nor the gravitational forces in the evaporator can be neglected. It is necessary to extend the GMBM with the dynamics not considered in those models.

3. EXTENDED GENERALIZED MOVING BOUNDARY MODELS

In those cases where the hypothesis of pressure independent from space cannot be maintained, it may be necessary the inclusion of the modelling of the fluid momentum conservation. Extending the GMBM models with equations of momentum conservation allows knowing the different pressures established in the subcooling, two-phase and superheated sections. Depending on the number of CV's where the momentum conservation is considered, the inlet and outlet pressure of the evaporator should be obtained with higher accuracy. This paper deals with the development of a model that considers two CV's where the fluid momentum is conserved. A more accurate model can be obtained with four CV's, although it is not depicted due to space restrictions.

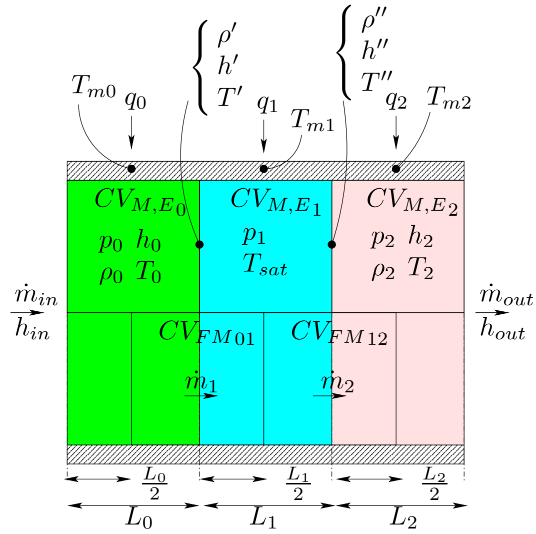

Fig. 1. Control Volumes in a staggered-grid.

3.1 Development Fundamentals

Figure 1 shows the spatial distribution of the three regions of the fluid. The total volume occupied by the fluid is discretized following the Finite Volume Method (FVM) with the discretization scheme in 'staggered-grid', commonly applied in Computational Fluid Dynamics (CFD), (Patankar, 1980) (Versteeg and Malalasekera, 1995). This scheme avoid errors due to coupling between fluids and pressures when evaluated in the same spatial positions, as in the Method of Lines (MOL) (Schiesser and Silebi, 1997), or discretizations based on finite differences technique. This method has been successfully applied in thermofluid systems modelling (Eborn and Nilsson, 1994), (Eborn, 1998), (Tummescheit and Eborn, 1998), (Eborn, 2001), (Tummescheit, 2002), and (Jensen, 2003). It is necessary to take into account that boundaries between CV's become dynamic variables that evolve during time. All system variables will be the same than those defined in the original model (Jensen and Tummescheit, 2002), therefore, the reader is recommended to read this paper for more details.

To calculate the equations of the model mass, energy and momentum conservation, it will be necessary to establish the following hypotheses:

- There will be one averaged and different pressure for each of the subcooling, two-phase and superheating sections of the fluid, each of which will be modelled with a CV with time-varying boundaries, in which the mass and energy is conserved. These pressures will be: $p_0(t)$ for the subcooled region, $p_1(t)$ for two-phase region, and $p_2(t)$ for superheated region. These CVs are referenced in figure 1 as CV_{M,E_i}, with $i = 0, 1, 2$.
- There will be two CV's in which the momentum will be conserved, located between the subcooled and two-phase region, and the two-phase and overheated region respectively ('staggered-grid' idea). These CV dimensions will be variable and associated to those of CV_{M,E_i}. They are referenced in figure 1 as $CV_{FM\,i-j}$, with $i = 0, 1$ and $j = 1, 2$.
- The evaporator may have a tilt α over the horizontal plane.

The method for the derivation of the model consists in the application on the CV's the equations of conservation corresponding to each CV. In CV_{M,E_i} mass and energy are conserved, while in $CV_{FM\,i}$ is the momentum. In the application of conservation equations, it will be necessary to take into account that the boundaries of CV_{M,E_i} and $CV_{FM\,i-j}$ will evolve along the time. In a CV with variable dimensions, the conservation of a specific magnitude c for which there is a source term ϕ on that magnitude, and a superficial flow $\boldsymbol{J}$ through the surface containing the CV, can be expressed as (Jensen, 2003), (Todreas and Kazimi, 1993):

$$\frac{d}{dt}\int_V \rho c dV + \oint_S \rho c(\boldsymbol{w} - \boldsymbol{w}_s)\hat{\boldsymbol{n}} dS = \int_V \rho\phi dV + \oint_S \boldsymbol{J}\hat{\boldsymbol{n}} dS \quad (1)$$

(1) is called General Balance Equation (GBE), where: $\boldsymbol{w}$ is the fluid velocity, $\boldsymbol{w}_s$ is the CV surface velocity and $\hat{\boldsymbol{n}}$ is the unitary vector normal to CV surface. Depending on the magnitude conserved in the CV, the variables $\{c,\ \phi,\ \boldsymbol{J}\}$ will have the meaning set on table 1, where $\boldsymbol{g}$ is the gravity acceleration, $\overline{\tau}$ is the surface friction tensor, $\overline{I}$ is the identity matrix, $\boldsymbol{q}$ is the energy flux per unit of area on the surface S, p is the pressure and u is the specific internal energy.

Assuming properties spatially averaged in each CV, and applying Reynolds Transport Theorem (B.1), in the appendix B, for the first term of the left hand side of (1), it results in (2):

Table 1. For each magnitude in the first column, the conservation law for that magnitude shall be expressed replacing the corresponding expressions: $\{c,\ \phi,\ \boldsymbol{J}\}$

	c	ϕ	$\boldsymbol{J}$
Mass	1	0	0
Momentum	$\boldsymbol{w}$	$\boldsymbol{g}$	$\overline{\tau} - p\overline{I}$
Energy	$u + \frac{w^2}{2}$	$\boldsymbol{g}\boldsymbol{w}$	$-\boldsymbol{q} + (\overline{\tau} - p\overline{I})\boldsymbol{w}$

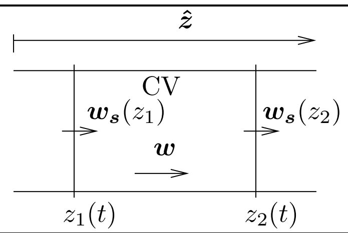

Fig. 2. CV's with variable boundaries

$$\frac{d}{dt}\int_V \overline{\rho c} dV = \frac{\mathrm{d}\overline{\rho c}}{dt}\int_{V(t)} dV + \overline{\rho c}\oint_{S(t)} \boldsymbol{w_s}\hat{\boldsymbol{n}} dS \quad (2)$$

where $\overline{\rho c} = \frac{1}{V(t)}\int_{V(t)} \rho c dV$. So, GBE could be stated according to (3):

$$\frac{\mathrm{d}\overline{\rho c}}{dt}\int_V dV + \overline{\rho c}\oint_S \boldsymbol{w_s}\hat{\boldsymbol{n}} dS + \oint_S \rho c(\boldsymbol{w} - \boldsymbol{w}_s)\hat{\boldsymbol{n}} dS = \int_V \rho\phi dV + \oint_S \boldsymbol{J}\hat{\boldsymbol{n}} dS \quad (3)$$

The application of (3) over CV's CV_{M,E_i} in figure 1, is developed in (Tummescheit, 2002), (Jensen and Tummescheit, 2002) and (Jensen, 2003) for mass and energy, therefore it will not be repeated here. In these works the authors use the differential formulation of conservation equation instead of the integral formulation used in this paper. The contribution of this article consists in the development of the momentum conservation, replacing $\{c,\ \phi,\ \boldsymbol{J}\}$ for the corresponding expressions according to table 1 for that physical magnitude, in the CV's $CV_{FM\,i-j}$ in figure 1.

4. MOMENTUM CONSERVATION IN CV_{FM}

Figure 2 represents a CV where the boundaries only vary in one dimension, that of vector $\hat{\boldsymbol{z}}$ (flow direction). This is the case for the modelled evaporator. In this figure the boundaries are defined by variables $z_1(t)$ y $z_2(t)$. The one-dimensional projection of equation (3) is (4), where A is the cross section of the evaporator:

$$A\frac{\mathrm{d}\overline{\rho c}}{dt}(z_2 - z_1) + A\overline{\rho c}\left(\frac{dz_2}{dt} - \frac{dz_1}{dt}\right) \quad (4)$$
$$+A\left((\rho c(\boldsymbol{w} - \boldsymbol{w}_s))\,|_{z_2} - (\rho c(\boldsymbol{w} - \boldsymbol{w}_s))\,|_{z_1}\right) =$$
$$A\int_{z_1}^{z_2} \rho\phi dz + \oint_S \boldsymbol{J}\hat{\boldsymbol{n}} dS$$

in this particular case, $\boldsymbol{w_s}|_{z_i} = \frac{dz_i}{dt}$. Developing (4) for magnitudes $\{c,\ \phi,\ \boldsymbol{J}\} = \{\boldsymbol{w},\ \boldsymbol{g},\ \overline{\tau} - p\overline{I}\}$ corresponding to momentum conservation, equation (4) reads:

$$(z_2 - z_1)\frac{d\overline{\dot{m}}}{dt} + \left(\dot{m}(z_1) - \overline{\dot{m}}\right)\frac{dz_1}{dt} \quad (5)$$
$$+\left(\overline{\dot{m}} - \dot{m}(z_2)\right)\frac{dz_2}{dt} =$$
$$\dot{I}_{z_1} - \dot{I}_{z_2} - A\rho g(z_2 - z_1)sin(\alpha) -$$
$$AP_{loss} + A\left(p(z_2) - p(z1)\right)$$

where:

- $\overline{\dot{m}} = \frac{1}{z_2 - z_1}\int_{z_1}^{z_2} \dot{m}(z)dz$ is the averaged mass flow in the interval $[z_1, z_2]$.
- $\dot{m}(z_i)$ is the mass flow evaluated at z_i.
- $\dot{I}(z_i) = \dot{m}(z_i)|\dot{w}(z_i)|$ is the momentum flux evaluated in z_i.
- α is the tilt angle over the evaporator horizontal.
- $P_{loss} = \xi\frac{(z_2 - z_1)\overline{\dot{m}}|\overline{\dot{m}}|}{2A^2\rho(\overline{\dot{m}})D_{in}}$, is the equivalent pressure of the friction forces in CV assuming turbulent flow [1], (Johnson, 1998). Where: $\rho\left(\overline{\dot{m}}\right)$ is the density 'linked' to CV, and its definition varies depending of the approximation used ('centered differences' or 'upwind') (Patankar, 1980); ξ is the friction factor; and D_{in} is the hydraulic diameter.
- $p(z_i)$ is the pressure at z_i.

To obtain the expression of the momentum conservation equation for $CV_{FM_{0-1}}$ and $CV_{FM_{1-2}}$ in figure 1, it is necessary to replace the variables in equation (5) for each CV as shown in table 2: As shown in figure 1, $\rho(\dot{m}_1)$ and $\rho(\dot{m}_2)$ might be defined as ρ' and ρ'' respectively. Due to the averaged condition of the thermodynamic magnitudes, the arithmetic mean used may be consider valid. The resulting equations for the momentum conservation for $CV_{FM_{0-1}}$ and $CV_{FM_{1-2}}$ are shown in figure A.1 of appendix A, and they are A.1 and A.2, respectively.

5. COMPLETE MODEL

Repeating the same procedure for mass and energy conservation in CV's CV_{M,E_i} in figure 1, the complete model is given by the equations

Table 2. Equations and aproximations for the variables in $CV_{FM_{i-j}}$

Variable	CV_{FM0-1}	CV_{FM1-2}
z_1	$\frac{L_0}{2}$	$L_0 + \frac{L_1}{2}$
z_2	$L_0 + \frac{L_1}{2}$	$L_0 + L_1 + \frac{L_2}{2}$
$\overline{\dot{m}}$	$\dot{m}_1$	$\dot{m}_2$
$\dot{I}_1$	$\frac{\dot{m}(z_1)\lvert\dot{m}(z_1)\rvert}{A\rho_0}$	$\frac{\dot{m}(z_1)\lvert\dot{m}(z_1)\rvert}{A\rho_1}$
$\dot{I}_2$	$\frac{\dot{m}(z_2)\lvert\dot{m}(z_2)\rvert}{A\rho_1}$	$\frac{\dot{m}(z_2)\lvert\dot{m}(z_2)\rvert}{A\rho_2}$
$p(z_1)$	p_0	p_1
$p(z_2)$	p_1	p_2
P_{loss}	$\xi\frac{(Lo+L1)\dot{m}_1\lvert\dot{m}_1\rvert}{4A^2\rho(\dot{m}_1)\ D_{in}}$	$\xi\frac{(L1+L2)\dot{m}_2\lvert\dot{m}_2\rvert}{4A^2\rho(\dot{m}_2)D_{in}}$
$\rho(\dot{m}_1)$	$\frac{\rho_0+\rho_1}{2}$	
$\rho(\dot{m}_2)$		$\frac{\rho_1+\rho_2}{2}$
$\dot{m}(z_1)$	$\frac{\dot{m}_{in}+\dot{m}_1}{2}$	$\frac{\dot{m}_1+\dot{m}_2}{2}$
$\dot{m}(z_2)$	$\frac{\dot{m}_1+\dot{m}_2}{2}$	$\frac{\dot{m}_2+\dot{m}_{out}}{2}$

shown in table 3, referenced with regard to figure A.1 of appendix A. To finish the complete model definition, it is necessary to express equations of pressure and metal temperature approximations in the boundaries of CV_{M,E_i}:

$$p(L_0) = \frac{p_0 + p_1}{2}; p(L_0 + L_1) = \frac{p_1 + p_2}{2} \quad (6)$$

$$T_w(L_0) = \begin{cases} T_{w1} & \text{if } \frac{dL_0}{dt} > 0 \\ T_{w0} & \text{if } \frac{dL_0}{dt} \leqslant 0 \end{cases} \quad (7)$$

$$T_w(L_0 + L_1) = \begin{cases} T_{w2} & \text{if } \frac{dL_1}{dt} > 0 \\ T_{w1} & \text{if } \frac{dL_1}{dt} \leqslant 0 \end{cases} \quad (8)$$

If $p = p_0 = p_1 = p_2$ are assumed, the equations of mass and energy conservation match with those of the original GMBM model (Jensen and Tummescheit, 2002) and the statement of momentum conservation equations would be meaningless.

Table 3. Equations for mass and energy conservation for each region.

Region	Mass	Fluid Energy	Tube Energy
Subcooled	A.3	A.4	A.5
Two-Phase	A.6	A.7	A.8
Superheated	A.9	A.11	A.10

6. CONCLUDING REMARKS

This paper presents an extension to MBM, due to the necessity to apply this type of models to long length water-steam distributed evaporators. In these evaporators it is not possible to assume some of the initial hypotheses under which the MBM were developed, and it is necessary to modify these hypotheses and reformulate the model in accordance. The main contribution is the inclusion of the momentum dynamic balance in two CV's with variable dimensions, located following a distribution in 'staggered grid', according to the structure recommended by FVM.

[1] In those cases in which the model has validity in laminar regime, it must be used the corresponding expression (Elmqvist *et al.*, 2003).

7. ACKNOWLEDGEMENTS

This work has been financed by CICYT-FEDER funds (projects DPI2002-04375-C03, DPI2004-01804 and DPI2004-07444-C04-04) and by the Consejería de Innovación, Ciencia y Empresa de la Junta de Andalucía. This work has been also performed within the scope of the specific collaboration agreement between the Plataforma Solar de Almería and the Automatic Control, Electronics and Robotics (TEP197) research group of the Universidad de Almería titled "Development of control systems and tools for thermosolar plants".

Appendix A. MODEL EQUATIONS

The figure A.1 shows the complete model equations, containing the equations for the balance of mass, energy in fluid, and energy in tube in CV_{M,E_i}; and momentum balances in CV_{FM0-1} y CV_{FM1-2}.

Appendix B. REYNOLDS TRANSPORT THEOREM

For a magnitud $\phi_{\boldsymbol{r}}(t)$, this theorem is stated as (Egeland and Gravdahl, 2002):

$$\frac{d}{dt}\int_{V(t)} \phi(\boldsymbol{r},t)dV = \int_{V(t)} \frac{\partial \phi(\boldsymbol{r},t)}{\partial t}dV + \oint_{S(t)} \phi \boldsymbol{v_s}\hat{\boldsymbol{n}}dS \tag{B.1}$$

Making the aproximation $\phi_{\boldsymbol{r}}(t) = \frac{1}{V(t)}\int_{V(t)} \phi_{\boldsymbol{r}}(t)dV$:

$$\frac{d}{dt}\int_{V(t)} \phi_{\boldsymbol{r}}(t)dV = \frac{d\phi_{\boldsymbol{r}}(t)}{dt}\int_{V(t)} dV + \phi_{\boldsymbol{r}} \oint_{S(t)} \boldsymbol{v_s}\hat{\boldsymbol{n}}dS \tag{B.2}$$

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$$\left(\frac{L_0+L_1}{2}\right)\frac{d\dot{m}_1}{dt}+0.25(\dot{m}_{in}-\dot{m}_1)\frac{dL_0}{dt}+0.5(\dot{m}_1-\dot{m}_2)\left(\frac{dL_0}{dt}+0.5\frac{dL_1}{dt}\right)=\frac{\dot{m}\left(\frac{L_0}{2}\right)|\dot{m}\left(\frac{L_0}{2}\right)|}{\rho_0 A} \tag{A.1}$$

$$-\frac{\dot{m}\left(L_0+\frac{L_1}{2}\right)|\dot{m}\left(L_0+\frac{L_1}{2}\right)|}{\rho_1 A}-A\rho(\dot{m}_1)\frac{L_0+L_1}{2}gsin(\alpha)-\xi\frac{(L_0+L_1)\,\dot{m}_1|\dot{m}_1|}{4A\rho(\dot{m}_1)\;D_{in}}+A\,(p_0-p_1)$$

$$\left(\frac{L_1+L_2}{2}\right)\frac{d\dot{m}_2}{dt}+0.5(\dot{m}_1-\dot{m}_2)\left(\frac{dL_0}{dt}+0.5\frac{dL_1}{dt}\right)+0.5(\dot{m}_2-\dot{m}_{out})\left(\frac{dL_0}{dt}+\frac{dL_1}{dt}+0.5\frac{dL_2}{dt}\right) \tag{A.2}$$

$$=\frac{\dot{m}\left(L_0+\frac{L_1}{2}\right)|\dot{m}\left(L_0+\frac{L_1}{2}\right)|}{\rho_1 A}-\frac{\dot{m}\left(L_0+L_1+\frac{L_2}{2}\right)|\dot{m}\left(L_0+L_1+\frac{L_2}{2}\right)|}{\rho_2 A}-A\rho(\dot{m}_2)\frac{L_1+L_2}{2}gsin(\alpha)$$

$$-\xi\frac{(L_1+L_2)\,\dot{m}_2|\dot{m}_2|}{4A\rho(\dot{m}_2)\;D_{in}}+A\,(p_1-p_2)$$

$$A(\rho_0-\rho')\frac{dL_0}{dt}+AL_0\left(\frac{\partial\rho_0}{\partial p}|_h+\frac{1}{2}\frac{\partial\rho_0}{\partial h}|_p\frac{dh'}{dp}\right)\frac{dp_0}{dt}+\frac{1}{2}AL_0\frac{\partial\rho_0}{\partial h}|_p\frac{dh_{in}}{dt}=\dot{m}_{in}-\dot{m}_1 \tag{A.3}$$

$$\frac{1}{2}A\left(\rho_0(h_{in}+h')-2\rho'h'-2p_0+2p(L_0)\right)\frac{dL_0}{dt} \tag{A.4}$$

$$+\frac{1}{2}AL_0\left(\rho_0+\frac{1}{2}(h_{in}+h')\frac{\partial\rho_0}{\partial h}|_p\right)\frac{dh_{in}}{dt}$$

$$+\frac{1}{2}AL_0\left(\rho_0\frac{dh'}{dp}+(h_{in}+h')\left(\frac{\partial\rho_0}{\partial p}|_h+\frac{1}{2}\frac{\partial\rho_0}{\partial h}|_p\frac{dh'}{dp}-2\right)\right)\frac{dp_0}{dt}$$

$$=\dot{m}_{in}h_{in}-\dot{m}_1h'+\pi D_iL_0\alpha_{i0}(T_{w0}-T_0)$$

$$A_w\rho_wc_{p,w}\left(L_0\frac{dT_{w0}}{dt}+(T_{w0}-T_w(L_0))\frac{dL_0}{dt}\right)=\pi D_iL_0\alpha_{i0}(T_0-T_{w0})+q_0L_0 \tag{A.5}$$

$$A(\rho'-\rho'')\frac{dL_0}{dt}+A(1-\overline{\gamma})(\rho'-\rho'')\frac{dL_1}{dt}+AL_1\left(\overline{\gamma}\frac{d\rho''}{dp}+(1-\overline{\gamma})\frac{d\rho'}{dp}\right)\frac{dp_1}{dt}=\dot{m}_1-\dot{m}_2 \tag{A.6}$$

$$A(\rho'h'-\rho''h''-p(L_0)+p(L_0+L_1))\frac{dL_0}{dt}+A\left((1-\overline{\gamma})(\rho'h'-\rho''h'')-p_1+p(L_0+L_1)\right)\frac{dL_1}{dt} \tag{A.7}$$

$$+AL_1\left(\overline{\gamma}\frac{d(\rho''h'')}{dp}+(1-\overline{\gamma})\frac{d(\rho'h')}{dp}-1\right)\frac{dp_1}{dt}=\dot{m}_1h'-\dot{m}_2h''+\pi D_iL_1\alpha_{i1}(T_{w1}-T_1)$$

$$A_w\rho_wc_{p,w}\left(L_1\frac{dT_{w1}}{dt}+(T_w(L_0)-T_{w1})\frac{dL_0}{dt}+(T_{w1}-T_w(L_0+L_1))\frac{dL_1}{dt}\right)= \tag{A.8}$$

$$\pi D_iL_1\alpha_{i1}(T_1-T_{w1})+q_1L_1$$

$$A(\rho''-\rho_2)\frac{dL_0}{dt}+A(\rho''-\rho_2)\frac{dL_1}{dt}+AL_2\left(\frac{1}{2}\frac{\partial\rho_2}{\partial h}|_p\frac{dh''}{dp}+\frac{\partial\rho_2}{\partial p}|_h\right)\frac{dp_2}{dt} \tag{A.9}$$

$$+\frac{1}{2}AL_2\frac{\partial\rho_2}{\partial h}|_p\frac{dh_{out}}{dt}=\dot{m}_2-\dot{m}_{out}$$

$$A\left(\rho''h''-\frac{1}{2}\rho_2(h''+h_{out})+p_2-p(L_0+L_1)\right)\left(\frac{dL_0}{dt}+\frac{dL_1}{dt}\right) \tag{A.10}$$

$$+\frac{1}{2}AL_2\left((h''+h_{out})\left(\frac{1}{2}\frac{\partial\rho_2}{\partial h}|_p\frac{dh''}{dp}+\frac{\partial\rho_2}{\partial p}|_h\right)+\rho_2\frac{dh''}{dp}-2\right)\frac{dp_2}{dt}$$

$$+\frac{1}{2}AL_2\left(\rho_2+\frac{1}{2}\frac{\partial\rho_2}{\partial h}|_p(h''+h_{out})\right)\frac{dh_{out}}{dt}=\dot{m}_2h''-\dot{m}_{out}h_{out}+\pi D_iL_2\alpha_{i2}(T_{w2}-T_2)$$

$$A_w\rho_wc_{p,w}\left(L_2\frac{dT_{w2}}{dt}+(T_w(L_0)-T_{w1})\frac{dL_0}{dt}+(T_w(L_0+L_1)-T_{w2})\left(\frac{dL_0}{dt}+\frac{dL_1}{dt}\right)\right)= \tag{A.11}$$

$$\pi D_iL_2\alpha_{i2}(T_2-T_{w2})+q_2L_2$$

Fig. A.1. Momentum conservation equations in $CV_{FM_{0-1}}$ (A.1) and $CV_{FM_{1-2}}$ (A.2). Conservation equations for mass, energy in fluid and energy in tube. For regions of subcooled liquid, two-phase and superheated, respectively: (A.3), (A.4), (A.5), (A.6), (A.7), (A.8), (A.9), (A.10) y (A.11)