ROBUST ADAPTIVE NEURAL NETWORK CONTROL FOR NONLINEAR MIMO TIME-DELAY SYSTEMS

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Abstract: This paper proposes an adaptive neural network (NN) controller for a class of multi-input multi-output (MIMO) nonlinear systems with unknown delays, but with known bounds on the delay functionals. The use of a separation technique removes the need to make any assumptions with regard to the structure of the delay functionals, thus making our results applicable to a larger class of systems. Given that the bounds on the delay functionals are known, we can construct Lyapunov-Krasovskii functionals and adaptive NNs to obtain a controller that guarantees all signals to be semi-globally uniformly ultimately bounded (SGUUB) while the outputs track specified desired trajectories.

Keywords: Adaptive control, neural networks, time-delay systems, MIMO nonlinear systems

1. INTRODUCTION

Many phenomena and physical systems in the real world involve time delays. Of great concern is the effect of time delay on stability and asymptotic performance (for an overview, see (Kolmanovskii *et al.*, 1999)).

In establishing robust stability for time-delay systems, some of the most useful tools are based on Lyapunov's second method that include the Lyapunov-Krasovskii theorem and the Lyapunov-Razumikhin theorem. Applications of these theorems have been made to linear time-delay systems (Kolmanovskii and Richard, 1999; Gu *et al.*, 2003), as well as nonlinear ones (Dugard and Verriest, 1998; Jankovic, 2001).

Subsequently, Lyapunov-Krasovskii functionals are also used in control design for time-delay systems. In (Wu, 2000), linear systems with nonlinear functions of state-delays are considered, and it was assumed that the delay functionals are bounded by linear functions of delayed states. Nguang used Lyapunov-Krasovskii functionals with backstepping to derive a robust controller for SISO nonlinear time-delay systems with known bounds (Nguang, 2000), but it was later commented that the results could not be constructively obtained (Zhou *et al.*, 2002). In (Ge *et al.*, 2003), an adaptive NN controller is employed for uncertain nonlinear systems with unknown time delays but known bounds on the delay functionals and known sign of the control coefficients. Subsequently the problem was extended to the case of unknown virtual control coefficients, and solved using Nussbaum-type functions (Ge *et al.*, 2004).

In the above-mentioned works, restrictive assumptions have been made regarding the bounds on the delay functionals, to facilitate the cancellation of delay terms when using Lyapunov-Krasovskii functionals. In this paper, we use the separation

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technique of (Lin and Qian, 2002) to decompose the delay functionals into positive bounding functionals of each delayed state. As such, we do not need to make special assumptions on the structure of the delay functionals. Given that the bounds of the delay functionals are known, we construct appropriate Lyapunov-Krasovskii functionals to eliminate time-delay terms, thus enabling the robust design of memoryless tracking controllers for nonlinear MIMO time-delay systems.

2. PROBLEM FORMULATION AND PRELIMINARIES

2.1 MIMO System Dynamics

Consider the following n inputs n outputs continuous-time MIMO nonlinear system in block-triangular form with unknown constant delays :

$$\Sigma_j \begin{cases} \dot{x}_{j,i_j} = f_{j,i_j}(\bar{x}_{j,i_j}) + g_{j,i_j}(\bar{x}_{j,i_j})x_{j,i_j+1} \\ \qquad + h_{j,i_j}(\bar{x}_{\tau_{j,i_j}}), \\ \qquad\qquad \text{for } \ 1 \le i_j \le m_j - 1 \\ \\ \dot{x}_{j,m_j} = f_{j,m_j}(X, \bar{u}_{j-1}) + g_{j,m_j}(X)u_j \\ \qquad + h_{j,m_j}(X_\tau) \end{cases}$$
$$\vdots$$
$$\Sigma_n \begin{cases} \dot{x}_{n,i_n} = f_{n,i_n}(\bar{x}_{n,i_n}) + g_{n,i_n}(\bar{x}_{n,i_n}(t))x_{n,i_n+1} \\ \qquad + h_{n,i_n}(\bar{x}_{\tau_{n,i_n}}), \\ \qquad\qquad \text{for } \ 1 \le i_n \le m_n - 1 \\ \\ \dot{x}_{n,m_n} = f_{n,m_n}(X, \bar{u}_{n-1}) + g_{n,m_n}(X)u_n \\ \qquad + h_{n,m_n}(X_\tau) \end{cases} \tag{1}$$

$$y_j = x_{j,1}, \quad 1 \le j \le n$$

where $x_j = [x_{j,1},\ x_{j,2},\cdots,\ x_{j,n_j}]^T \in R^{m_j}$ are the state variables of the jth subsystem; $u = [u_1,\cdots,\ u_n]^T \in R^n$ are the system inputs; $y = [y_1,\cdots,y_n]^T \in R^n$ are the outputs; $\bar{u}_{j-1} := [u_1,\cdots,u_{j-1}]^T$ $(j = 2,\cdots,n)$ with $u_0 := 0$; $\bar{x}_{j,i_j} := [x_{j,1},\cdots,x_{j,i_j}]^T \in R^{i_j}$; $f_{j,i_j}(\cdot)$, $g_{j,i_j}(\cdot)$ and $h_{j,i_j}(\cdot)$ are unknown and smooth nonlinear functions; the vector $X = [x_1^T, x_2^T \cdots, x_n^T]^T$ contains all states; $x_{\tau_{j,i_j}} := x_{j,i_j}(t-\tau_{j,i_j})$ denotes the delayed state; and j, i_j and m_j are positive integers.

The term h_{j,i_j} is a function of the previous $(i_j - 1)$th delayed states of the jth subsystem, while h_{j,m_j}, which appears in the last equation of each subsystem, is a function of the delayed states of *all* subsystems. The arguments of these functions are defined as follows

$$X_\tau := [x_{1,1}(t-\tau_{1,1}),\cdots,x_{j,i_j}(t-\tau_{j,i_j}),\cdots$$
$$x_{j,m_j}(t-\tau_{j,m_j}),\cdots,x_{n,m_n}(t-\tau_{n,m_n})]^T,$$
$$\bar{x}_{\tau_{j,i_j}} := [x_{j,1}(t-\tau_{j,1}),\cdots,x_{j,i_j}(t-\tau_{j,i_j})]^T.$$

where $\tau_{j,i_j} > 0$ is the constant unknown time delay for the i_j state of the jth subsystem. For $t \in [-\tau_{j,i_j}, 0]$ we have

$$x_{j,i_j}(t) = \phi_{j,i_j}(t), \quad 1 \le j \le n,\ 1 \le i_j \le m_j,$$

where the initial function, $\phi_{j,i_j}(t)$, is smooth and bounded. Throughout this paper, for clarity in presentation, we omit the argument t in $x_{j,i_j}(t)$.

Definition 1. (Lin and Saberi, 1995) The solution of (1) is Semi-Globally Uniformly Ultimately Bounded (SGUUB) if, for any compact set $\Omega_0 \subset R^{m_1+m_2+\cdots+m_n}$, there exist $S > 0$ and $T(S, X(t_0))$ such that $\|X(t)\| \le S$ for all $X(t_0) \in \Omega_0$ and $t \ge t_0 + T$.

Lemma 1. (Dawson *et al.*, 1992) For bounded initial conditions, if there exists a C^1 continuous and positive definite Lyapunov function $V(x)$ satisfying $\gamma_1(\|x\|) \le V(x) \le \gamma_2(\|x\|)$, such that$\dot{V}(x) \le -\rho V(x) + c$, where $\gamma_1, \gamma_2 : R^n \to R$ are class K functions and c is a positive constant, then $x(t)$ is SGUUB.

Lemma 2. Separation Lemma (Lin and Qian, 2002): For any continuous function $h(\bar{x}_n) : R^{m_1} \times \cdots \times R^{m_n} \to R$, where $x_j \in R^{m_j}$ $(1 \le j \le n,\ m_j > 0)$, there exist a constant $a \in R \ge 0$ and positive smooth functions $\varrho_j(x_j) : R^{m_j} \to R$ $(1 \le j \le n)$ satisfying $\varrho_j(0) = 0$ such that

$$|h(x_1,\cdots,x_n)| \le a + \sum_{j=1}^{n} \varrho_j(x_j).$$

Remark 1. The condition that $\varrho_j(0) = 0$ is needed to obtain a suitable Lyapunov-Krasovskii functional later. Throughout this paper, we use the notation $\varrho_c^b(x_{\tau_c})$ to denote the bounding function of the delayed state x_{τ_c} belonging to the bth subsystem.

Assumption 1. The signs of $g_{j,i_j}(\bar{x}_{j,i_j})$ are known, and there exist constants $g_{0_{j,i_j}}$ and known smooth functions $\bar{g}_{j,i_j}(\bar{x}_{j,i_j})$ such that $0 < g_{0_{j,i_j}} \le |g_{j,i_j}(\bar{x}_{j,i_j})| \le \bar{g}_{j,i_j}(\bar{x}_{j,i_j})$. Without loss of generality, we further assume that the signs of $g_{j,i_j}(\bar{x}_{j,i_j})$ are all positive.

Assumption 2. The first-order derivatives of all the states are available.

Assumption 3. The unknown time delays $\tau_{j,k}$ $(1 \le j \le n,\ 1 \le k \le m_j)$ are bounded by a known scalar τ_{max}.

The control objective is to ensure that all signals are bounded while tracking the desired trajectories y_{dj}, $1 \le j \le n$ such that the tracking errors converge to a small neighbourhood of the origin, i.e. $\lim_{t\to\infty} |y_j(t) - y_{dj}(t)| \leq \delta$ for some $\delta > 0$.

In this paper, we shall use Radial Basis Function (RBF) neural network (NN), which are linearly parametrized, to approximate the continuous function $p(Z) : R^q \to R$ as

$$p(Z) = W^T S(Z) \tag{2}$$

where the input vector $Z \in \Omega_Z \subset R^q$, weight vector $W \in R^l$, and basis function vector $S(Z) = [s_1(Z), s_2(Z), ..., s_l(Z)]^T \in R^l$, with l being the NN node number and $s_i(Z)$ chosen as the commonly used Gaussian functions, which have the form $s_i(Z) = \exp[-(Z-\mu_i)^T(Z-\mu_i)/\eta_i^2]$, $i = 1, ..., l$ where $\mu_i = [\mu_{i1}, \mu_{i2}, ..., \mu_{iq}]^T$ is the center of the receptive field and η_i is the width of the Gaussian function. Universal approximation results in (Sanner and Slotine, 1992) indicate that, if l is chosen sufficiently large, $W^T S(Z)$ can approximate any continuous function to any desired accuracy over a compact set $\Omega_Z \subset R^q$ to arbitrary any accuracy as

$$p(Z) = W^{*T} S(Z) + \varepsilon(Z), \forall Z \in \Omega_Z \subset R^q \tag{3}$$

where W^* is the ideal constant weight vector, and $\varepsilon(Z)$ is the approximation error which is bounded over the compact set, i.e., $|\varepsilon(Z)| \leq \varepsilon^*$, $\forall Z \in \Omega_Z$ where $\varepsilon^* > 0$ is an unknown constant. The ideal weight vector W^* is an "artificial" quantity required for analytical purposes. W^* is defined as the value of W that minimizes $|\varepsilon|$ for all $Z \in \Omega_Z \subset R^q$, i.e., $W^* := \arg\min_{W\in R^l}\{\sup_{Z\in\Omega_Z} |p(Z) - W^T S(Z)|\}$.

3. ADAPTIVE NN CONTROL DESIGN

The main idea of the control design is essentially a robust control approach based on the use of *memoryless* affine controls to dominate the delayed effects so that the overall closed loop system stably achieves a desired level of tracking performance.

Noting that each subsystem is in strict-feedback form, our control design adopts embedded backstepping. Within the jth ($1 \leq j \leq n$) subsystem in strict feedback form, virtual controls are designed via backstepping up to the $(m_j - 1)$th step. For the m_jth ($1 \leq j \leq n$) equation of each subsystem, the interconnections with the states, delayed states, and inputs of all other subsystems are present, but the block triangular structure allows backstepping to be used across the subsystems, thereby guaranteeing stablility of the entire interconnected MIMO system.

Step j, i_j Consider the i_jth equation of the jth subsystem. Let $z_{j,i_j+1} = x_{j,i_j+1} - \alpha_{j,i_j}$, and $\alpha_{j,0} := y_{dj}$. To avoid controller singularity, we employ integral Lyapunov functions (Ge *et al.*, 2000):

$$V_{z_{j,i_j}} = z_{j,i_j}^2 \int_0^1 \theta g_{\lambda_{j,i_j}}^{-1}(\bar{x}_{j,i_j-1}, \theta z_{j,i_j} + \alpha_{j,i_j-1}) d\theta,$$

where $g_{\lambda_{j,i_j}}^{-1} := \bar{g}_{j,i_j}(\cdot)/g_{j,i_j}(\cdot)$. Differentiating $V_{z_{j,i_j}}$ along the desired and plant trajectories, and using Young's Inequality on $z_{j,i_j} g_{\lambda_{j,i_j}}^{-1} h_{j,i_j}$ yields

$$\begin{aligned}\dot{V}_{z_{j,i_j}} \leq z_{j,i_j}\big[& g_{\lambda_{j,i_j}}^{-1}(f_{j,i_j} + a_{j,i_j}) + z_{j,i_j+1} + \alpha_{j,i_j} \\ &- \dot{\bar{x}}_{j,i_j-1}^T \int_0^1 \theta \frac{\partial g_{\lambda_{j,i_j}}^{-1}}{\partial \bar{x}_{j,i_j-1}} d\theta - \dot{\alpha}_{j,i_j-1} \int_0^1 g_{\lambda_{j,i_j}}^{-1} d\theta \\ &+ \frac{1}{2} z_{j,i_j} g_{\lambda_{j,i_j}}^{-2}\big] + \frac{1}{2}\sum_{k=1}^{i_j} \left(\varrho_{j,k}^{j,i_j}(x_{\tau_{j,k}})\right)^2\end{aligned}$$

where $a_{j,i_j} \geq 0$ and $\varrho_{j,k}^{j,i_j}(0) = 0$.

To eliminate the terms with time-delay, we augment the Lyapunov function with a Lyapunov-Krasovskii functional:

$$V_{U_{j,i_j}} = \frac{1}{2}\sum_{k=1}^{i_j} \int_{t-\tau_{j,k}}^{t} \left(\varrho_{j,k}^{j,i_j}(x_{j,k}(\tau))\right)^2 d\tau, \tag{4}$$

which has time-derivative:

$$\dot{V}_{U_{j,i_j}} = \frac{1}{2}\sum_{k=1}^{i_j}\left[\left(\varrho_{j,k}^{j,i_j}(x_{j,k})\right)^2 - \left(\varrho_{j,k}^{j,i_j}(x_{\tau_{j,k}})\right)^2\right].$$

Define $U_{j,i_j} = V_{z_{j,i_j}} + V_{U_{j,i_j}}$. It can be seen that $\dot{U}_{j,i_j}$ no longer contains time delay terms as they are cancelled when summing $\dot{V}_{z_{j,i_j}}$ and $\dot{V}_{U_{j,i_j}}$:

$$\begin{aligned}\dot{U}_{j,i_j} \leq z_{j,i_j}\big[& g_{\lambda_{j,i_j}}^{-1}(f_{j,i_j} + a_{j,i_j}) + z_{j,i_j+1} + \alpha_{j,i_j} \\ &- \dot{\bar{x}}_{j,i_j-1}^T \int_0^1 \theta \frac{\partial g_{\lambda_{j,i_j}}^{-1}}{\partial \bar{x}_{j,i_j-1}} d\theta - \dot{\alpha}_{j,i_j-1} \int_0^1 g_{\lambda_{j,i_j}}^{-1} d\theta \\ &+ \frac{1}{2} z_{j,i_j} g_{\lambda_{j,i_j}}^{-2}\big] + \frac{1}{2}\sum_{k=1}^{i_j} \left(\varrho_{j,k}^{j,i_j}(x_{j,k})\right)^2\end{aligned} \tag{5}$$

The virtual control is chosen as

$$\begin{aligned}\alpha_{j,i_j} = p_{j,i_j}(z_{j,i_j})\big(& -z_{j,i_j-1} - \kappa_{j,i_j} z_{j,i_j} \\ &- \frac{1}{2z_{j,i_j}}\sum_{k=1}^{i_j} \varrho_{j,k}^2(x_{j,k}) + \hat{W}_{j,i_j}^T S(Z_{j,i_j})\big),\end{aligned} \tag{6}$$

where the discontinuous function

$$p_{j,i_j}(z_{j,i_j}) := \begin{cases} 1, & |z_{j,i_j}| \geq \epsilon_{j,i_j} \\ 0, & |z_{j,i_j}| < \epsilon_{j,i_j} \end{cases} \tag{7}$$

for all $1 \leq j \leq n$ and $1 \leq i_j \leq m_j$ is used to ensure a realizable controller by 'switching off' the control and adaptation laws whenever z_{j,i_j} is below a specified tolerance level ϵ_{j,i_j}.

The term $\hat{W}_{j,i_j}^T S(Z_{j,i_j})$ approximately cancels the terms within the square brackets in (5). The neural network inputs are given by

$$Z_{j,i_j} = [\bar{x}_{j,i_j}, \alpha_{j,i_j-1}, \dot{\alpha}_{j,i_j-1}]^T, \tag{8}$$

where $\dot{\alpha}_{j,i_j-1}$ is computable as

$$\begin{aligned}\dot{\alpha}_{j,i_j-1} = &\sum_{k=1}^{i_j-1}\left(\frac{\partial \alpha_{j,i_j-1}}{\partial x_{1,k}}\dot{x}_{j,k} + \frac{\partial \alpha_{j,i_j-1}}{\partial \hat{W}_{j,k}}\dot{\hat{W}}_{j,k}\right)\\ &+\sum_{k=0}^{i_j-1}\frac{\partial \alpha_{j,i_j-1}}{\partial y_{dj}^{(k)}} y_{dj}^{(k+1)},\end{aligned} \tag{9}$$

with $y_{dj}^{(k)}$ denoting $\frac{d^k}{dt^k}[y_{dj}]$.

Remark 2. In the present construction, we only consider the case when $p_{j,i_j}(z_{j,i_j}) = 1$, for all $1 \le j \le n$, $1 \le i_j \le m_j$, such that α_{j,i_j-1} is continuous. Subsequently, in Theorem 1, we will analyze the closed loop stability by considering the different cases when $p_{j,i_j}(z_{j,i_j})$ can be 0 or 1.

Choosing the Lyapunov function as

$$V_{j,i_j} = V_{j,i_j-1} + U_{j,i_j} + \frac{1}{2}\tilde{W}_{j,i_j}^T \Gamma_{j,i_j}^{-1}\tilde{W}_{j,i_j},$$

with adaptation law

$$\begin{aligned}\dot{\hat{W}}_{j,i_j} = &-p_{j,i_j}(z_{j,i_j})\Gamma_{j,i_j}[S(Z_{j,i_j})z_{j,i_j}\\ &+\sigma_{j,i_j}(\hat{W}_{j,i_j} - W_{j,i_j}^0)],\end{aligned}$$

and noting that $z_{j,i_j}\varepsilon_{j,i_j} \le \frac{1}{4\lambda}z_{j,i_j}^2 + \lambda\varepsilon_{j,i_j}^2$, $\lambda > 0$, we have

$$\begin{aligned}\dot{V}_{j,i_j} \le &\dot{V}_{j,i_j-1} - \kappa_{j,i_j}z_{j,i_j}^2 + z_{j,i_j}z_{j,i_j+1}\\ &-z_{j,i_j-1}z_{j,i_j} + \frac{1}{4\lambda}z_{j,i_j}^2 + \lambda\varepsilon_{j,i_j}^2\\ &-\frac{\sigma_{j,i_j}}{2}\|\tilde{W}_{j,i_j}\|^2 + \frac{\sigma_{j,i_j}}{2}\|W_{j,i_j}^* - W_{j,i_j}^0\|^2\\ \le &-\sum_{k=1}^{i_j}\left(\kappa_{j,k} - \frac{1}{4\lambda}\right)z_{j,k}^2 + z_{j,i_j}z_{j,i_j+1}\\ &-\sum_{k=1}^{i_j}\frac{\sigma_{j,k}}{2}\|\tilde{W}_{j,k}\|^2 + \sum_{k=1}^{i_j}(\lambda\varepsilon_{j,k}^2 + \frac{\sigma_{j,k}}{2}\\ &\times\|W_{j,k}^* - W_{j,k}^0\|^2)\end{aligned} \tag{10}$$

where $\kappa_{j,k}$ will be specified later. The $z_{j,i_j}z_{j,i_j+1}$ term will be cancelled in the $(i_j+1)th$ step.

Step j,m_j Consider the last equation of subsystem Σ_j, where the control input u_j appears, and will be designed to stabilize the jth subsystem. Let $x_{j,m_j}^c \subset X$ such that $x_{j,m_j}^c \bigcup x_{j,m_j} = X$ and $x_{j,m_j}^c \bigcap x_{j,m_j} = 0$. Define integral Lyapunov function

$$V_{z_{j,m_j}} = z_{j,m_j}^2\int_0^1 \theta g_{\lambda_{j,m_j}}^{-1}(x_{j,m_j}^c, \theta z_{j,m_j} + \alpha_{j,m_j-1})d\theta.$$

The time-derivative along the desired and plant trajectories is

$$\begin{aligned}\dot{V}_{z_{j,m_j}} \le &z_{j,m_j}[g_{\lambda_{j,m_j}}^{-1}(f_{j,m_j} + a_{j,m_j}) + u_j\\ &-\int_0^1 \theta\frac{\partial g_{\lambda_{j,m_j}}^{-1}}{\partial x_{j,m_j}^c}d\theta\ \dot{x}_{j,m_j}^c\\ &-\dot{\alpha}_{j,m_j-1}\int_0^1 g_{\lambda_{j,m_j}}^{-1}d\theta + \frac{1}{2}z_{j,m_j}g_{\lambda_{j,m_j}}^{-2}]\\ &+\frac{1}{2}\sum_{i=1}^{n}\sum_{k=1}^{m_i}\left(\varrho_{i,k}^{j,m_j}(x_{\tau_{i,k}})\right)^2,\end{aligned} \tag{11}$$

where $x_{\tau_{j,k}} := x_{j,k}(t - \tau_{j,k})$.

In view of the interconnections between the different subsystems in the last equation, and according to Lemma 2, we consider the following Lyapunov-Krasovskii functional, which has a form slightly different from that of the previous $m_j - 1$ equations.

$$V_{U_{j,m_j}} = \frac{1}{2}\sum_{i=1}^{n}\sum_{k=1}^{m_i}\int_{t-\tau_{i,k}}^{t}\left(\varrho_{i,k}^{j,m_j}(x_{i,k}(\tau))\right)^2 d\tau,$$

Denoting $U_{j,m_j} = V_{z_{j,m_j}} + V_{U_{j,m_j}}$, it can be shown that

$$\begin{aligned}\dot{U}_{j,m_j} \le &z_{j,m_j}[g_{\lambda_{j,m_j}}^{-1}(f_{1,m_1} + a_{j,m_j}) + u_j\\ &-\int_0^1 \theta\frac{\partial g_{\lambda_{j,m_j}}^{-1}}{\partial x_{j,m_j}^c}d\theta\dot{x}_{j,m_j}^c\\ &-\dot{\alpha}_{j,m_j-1}\int_0^1 g_{\lambda_{j,m_j}}^{-1}d\theta + \frac{1}{2}z_{j,m_j}g_{\lambda_{j,m_j}}^{-2}]\\ &+\frac{1}{2}\sum_{i=1}^{n}\sum_{k=1}^{m_i}\left(\varrho_{i,k}^{j,m_j}(x_{i,k})\right)^2.\end{aligned}$$

The practical control law for subsystem Σ_j is

$$\begin{aligned}u_j = &p_{j,m_j}(z_{j,m_j})\big(-z_{j,m_j-1} - \kappa_{j,m_j}z_{j,m_j}\\ &-\frac{1}{2z_{j,m_j}}\sum_{i=1}^{n}\sum_{k=1}^{m_i}\left(\varrho_{i,k}^{j,m_j}(x_{i,k})\right)^2\\ &+\hat{W}_{j,m_j}^T S(Z_{j,m_j})\big)\end{aligned} \tag{12}$$

where $p_{j,m_j}(\cdot)$ is defined in (7); κ_{j,m_j} will be defined later and $\hat{W}_{j,m_j}^T S(Z_{j,m_j})$ approximately cancels the terms within the square bracket in (11). The neural network inputs Z_{j,m_j} are:

$$\begin{aligned}Z_{j,m_j} = &[X, \dot{x}_{1,m_1}, \dot{x}_{2,m_2}, \cdots, \dot{x}_{j,m_j-1}, \dot{x}_{j,m_j+1}, \cdots,\\ &\dot{x}_{n,m_n}, \alpha_{j,m_j-1}, \dot{\alpha}_{j,m_j-1}, \bar{u}_{j-1}]^T\end{aligned} \tag{13}$$

wherein $\dot{\alpha}_{j,m-1}$ is a computable function of $\bar{x}_{j,m_j}, \hat{W}_{j,1}, \cdots, \hat{W}_{j,m_j}, y_{dj}, \cdots, y_{dj}^{(m_j)}$.

We consider the Lyapunov function as

$$V_{j,m_j} = V_{j-1,m_{(j-1)}} + V_{j,m_j-1} + U_{j,m_j} + \frac{1}{2}\tilde{W}^T_{j,m_j}\Gamma^{-1}_{j,m_j}\tilde{W}_{j,m_j}, \quad (14)$$

and choose adaptation law to be

$$\dot{\hat{W}}_{j,m_j} = -p_{j,m_j}(z_{j,m_j})\Gamma_{j,m_j}[S(Z_{j,m_j})z_{j,m_j} + \sigma_{j,m_j}(\hat{W}_{j,m_j} - W^0_{j,m_j})]. \quad (15)$$

Then, taking the derivative of V_{j,m_j} along (1), (12), (15), and noting that $z_{j,m_j}\varepsilon_{j,m_j} \leq \frac{1}{4\lambda}z^2_{j,m_j} + \lambda\varepsilon^2_{j,m_j}$, $\lambda > 0$, we obtain

$$\dot{V}_{j,m_j} \leq -\sum_{i=1}^{j}\sum_{k=1}^{m_i}\left[\left(\kappa_{i,k} - \frac{1}{4\lambda}\right)z^2_{i,k} + \frac{\sigma_{i,k}}{2}\|\tilde{W}_{i,k}\|^2\right] + \sum_{i=1}^{j}\sum_{k=1}^{m_i}\left(\lambda\varepsilon^2_{i,k} + \frac{\sigma_{i,k}}{2}\|W^*_{i,k} - W^0_{i,k}\|^2\right) \quad (16)$$

where $\kappa_{i,k}$ will be specified later.

<u>*Step (n, m_n)*</u> This is the final step, where the nth input will be designed to ensure the stability of the entire plant. Let $z_{n,m_n} = x_{n,m_n} - \alpha_{n,m_n-1}$. Consider the following Lyapunov function

$$V_{n,m_n} = V_{n-1,m_{(n-1)}} + V_{n,m_n-1} + V_{z_{n,m_n}} + V_{U_{n,m_n}} + \frac{1}{2}\tilde{W}^T_{n,m_n}\Gamma^{-1}_{n,m_n}\tilde{W}_{n,m_n}, \quad (17)$$

with control law:

$$u_n = p_{n,m_n}(z_{n,m_n})\Big[- z_{n,m_n-1} - \kappa_{n,m_n}z_{n,m_n} - \frac{1}{2z_{n,m_n}}\sum_{j=1}^{n}\sum_{k=1}^{m_j}\left(\varrho^{n,m_n}_{j,k}(x_{j,k}(\tau))\right)^2 + \hat{W}^T_{n,m_n}S(Z_{n,m_n})\Big] \quad (18)$$

and adaptation law:

$$\dot{\hat{W}}_{n,m_n} = -p_{n,m_n}(z_{n,m_n})\Gamma_{n,m_n}[S(Z_{n,m_n})z_{n,m_n} + \sigma_{n,m_n}(W^*_{n,m_n} - W^0_{n,m_n})]. \quad (19)$$

It can be shown that the derivative of V_{n,m_n} along (1), (18), (19), and noting that $z_{n,m_n}\varepsilon_{n,m_n} \leq \frac{1}{4\lambda}z^2_{n,m_n} + \lambda\varepsilon^2_{n,m_n}$, $\lambda > 0$, we obtain

$$\dot{V}_{n,m_n} \leq -\sum_{j=1}^{n}\sum_{k=1}^{m_j}\left[\left(\kappa_{j,k} - \frac{1}{4\lambda}\right)z^2_{j,k} + \frac{\sigma_{j,k}}{2}\|\tilde{W}_{j,k}\|^2\right] + \sum_{j=1}^{n}\sum_{k=1}^{m_j}\left(\lambda\varepsilon^2_{j,k} + \frac{\sigma_{j,k}}{2}\|W^*_{j,k} - W^0_{j,k}\|^2\right). \quad (20)$$

In order to modify (20) to the form in Lemma 1, we choose $\kappa_{j,k}$ as follows:

$$\kappa_{j,i_j}(t) = \frac{1}{4\lambda} + \kappa_{0_{j,i_j}}\left(\int_0^1 \theta\bar{g}_{j,i_j}d\theta + \frac{1}{z^2_{j,i_j}} \times \sum_{k=1}^{i_j}\int_{t-\tau_{max}}^{t}\frac{1}{2}\left(\varrho^{j,i_j}_{j,k}(x_{j,k}(\tau))\right)^2 d\tau\right),$$

for $j = 1, \cdots, n$, $i_j = 1, \cdots, m_j - 1$, and

$$\kappa_{j,m_j}(t) = \frac{1}{4\lambda} + \kappa_{0_{j,m_j}}\left(\int_0^1 \theta\bar{g}_{j,m_j}d\theta + \frac{1}{z^2_{j,m_j}} \times \sum_{i=1}^{n}\sum_{k=1}^{m_i}\int_{t-\tau_{max}}^{t}\frac{1}{2}\left(\varrho^{j,m_j}_{i,k}(x_{i,k}(\tau))\right)^2 d\tau\right),$$

for $j = 1, \cdots, n$. Noting Assumption 3 and the property

$$\frac{z^2_{j,i_j}}{2} \leq V_{z_{j,i_j}} \leq \frac{z^2_{j,i_j}}{g_{0_{j,i_j}}}\int_0^1 \theta\bar{g}_{j,i_j}d\theta, \quad (21)$$

for $j = 1, \cdots, n$ and $i_j = 1, \cdots, m_j$, it can be shown that the time derivative of V_{n,m_n} along the solutions of (1), (18), and (19) satisfies the following:

$$\dot{V}_{n,m_n} \leq -\rho V_{n,m_n} + C, \quad (22)$$

$$\rho = \min_{j,i_j}\left(\kappa_{0_{j,i_j}}g_{0_{j,i_j}},\ \kappa_{0_{j,i_j}},\ \frac{\sigma_{j,i_j}}{\lambda_{max}(\Gamma^{-1}_{j,i_j})}\right),$$

$$C = \sum_{j=1}^{n}\sum_{k=1}^{m_j}(\lambda\varepsilon^2_{j,k} + \frac{\sigma_{j,k}}{2}\|W^*_{j,k} - W^0_{j,k}\|^2).$$

Remark 3. The parameters λ, $\sigma_{j,k}$, and $W^0_{j,k}$ can be designed to make the constant C arbitrarily small. At the same time, $\Gamma_{j,k}$ and $\kappa_{0_{j,k}}$ can be chosen to make ρ large, such that the steady state compact set of V_{n,m_n} given by C/ρ can be made as small as desired.

Now, we are ready to present the results of this paper under the following theorem.

Theorem 1. The closed loop system consisting of nonlinear MIMO time-delay plant (1) under Assumptions 1–3, with control law u_j given by (12) and adaptation law given by (15) for $j = 1, 2, ...n$, is SGUUB, and the error signals $z = [z_{1,1}, \cdots, z_{j,i_j}, \cdots, z_{n,m_n}]^T$ eventually converge to the compact set:

$$\Omega_z := \{z \in R^{m_1+\cdots+m_n} |\ \|z\|^2 \leq \mu\},$$

where $\mu := \max(2C/\rho,\ 2C_K/\rho_K,\ E)$, with

$$C_K := \sum_{k,i_k}(\lambda\varepsilon^2_{k,i_k} + \frac{\sigma_{k,i_k}}{2}\|W^*_{k,i_k} - W^0_{k,i_k}\|^2),$$

$$\rho_K := \min_{k,i_k}\left(\kappa_{0_{k,i_k}}g_{0_{k,i_k}},\ \kappa_{0_{k,i_k}},\ \frac{\sigma_{k,i_k}}{\lambda_{max}(\Gamma^{-1}_{k,i_k})}\right),$$

$$E := \sum_{j=1}^{n}\sum_{k=1}^{m_j}\epsilon^2_{j,k}\ ,$$

and $\{k, i_k\} \subset \{j, i_j\}$ for $1 \leq j \leq n$ and $1 \leq i_j \leq m_j$, such that $z_{k,i_k} < \epsilon_{k,i_k}$.

Proof: The proof is similar to that in (Ge *et al.*, 2003) and will only be outlined briefly here. Noting the discontinuous function $p(\cdot)$ in the control and adaptiation laws, the following three cases are considered.

Case 1): $|z_{j,i_j}| < \epsilon_{j,i_j}, \forall\, j = 1, ..., n,\ i_j = 1, ..., m_j$. The control and adaptation laws are 'switched off' i.e. $\alpha_{j,i_j} = 0$, $u_j = 0$ and $\dot{\hat{W}}_{j,i_j} = 0$. Since y_{dj} and z_{j,i_j} are bounded, we know that x_{j,i_j} are bounded. At the same time, $\hat{W}_i$ remains constant and bounded. For bounded x_{j,i_j}, z_{j,i_j} and $\hat{W}_{j,i_j}$, it can be deduced that V_{n,m_n} is bounded, i.e., there exists a finite C_B such that $V_{n,m_n}(t) \le C_B$. Note that for this case, $\|z\|^2 < E$.

Case 2): $|z_{j,i_j}| \ge \epsilon_{j,i_j}, \forall\, j = 1, ..., n,\ i_j = 1, ..., m_j$. This case has been addressed in the foregoing derivation, and yields $\dot{V}_{n,m_m} \le -\rho V_{n,m_m} + C$. According to Lemma 1, the signals $z_{1,1}, \cdots, z_{j,m_j}$, and $\tilde{W}_{1,1}, \cdots, \tilde{W}_{j,m_j}$ are SGUUB. In addition, it can be shown that $\lim_{t\to\infty} V_{n,m_n}(t) \le C/\rho$. From (21), we thus obtain $\lim_{t\to\infty} \|z(t)\|^2 \le 2C/\rho$.

Case 3): Some $|z_{j,i_j}| < \epsilon_{j,i_j}$ and some $|z_{k,i_k}| \ge \epsilon_{k,i_k}$ for $j \ne k$. For $|z_{j,i_j}| < \epsilon_{j,i_j}$, we define:

$$V_J(t) = \textstyle\sum_{j,i_j} (V_{z_{j,i_j}} + V_{U_{j,i_j}} + \frac{1}{2}\tilde{W}^T_{j,i_j}\Gamma^{-1}_{j,i_j}\tilde{W}_{j,i_j})$$

and conclude that $V_J(t) \le C_J < \infty$. For $|z_{k,i_k}| \ge \epsilon_{k,i_k}$, we define:

$$V_K(t) = \textstyle\sum_{k,i_k} (V_{z_{k,i_k}} + V_{U_{k,i_k}} + \frac{1}{2}\tilde{W}^T_{k,i_k}\Gamma^{-1}_{k,i_k}\tilde{W}_{k,i_k}).$$

From $\dot{V}_K(t) \le -\rho_K V_K(t) + C_K$, we obtain $V_K(t) \le V_K(0) + C_K/\rho_K$. This leads to:

$$V_{n,m_n}(t) = V_K(t) + V_J(t) \le V_K(0) + \frac{C_K}{\rho_K} + C_J,$$

from which it is clear that V_{n,m_n} is bounded. It can be shown that $\lim_{t\to\infty} \sum_{k,i_k} z^2_{k,i_k} \le 2C_K/\rho_K$ and $\sum_{j,i_j} z^2_{j,i_j} \le \sum_{j,i_j} \epsilon^2_{j,i_j}$.

Therefore, the closed loop signals are SGUUB for all 3 cases, and the vector z satisfies $\lim_{t\to\infty} \|z\|^2 \le \max(2C/\rho,\ 2C_K/\rho_K,\ E)$. ∎

4. CONCLUSION

This paper proposed an adaptive neural network controller for a class of block-triangular MIMO nonlinear time-delay systems. With the use of a separation technique, more general forms of delay dynamics (including complex interconnections of state-delays found in MIMO systems) can be handled, such that no assumptions regarding the bounds of the delay dynamics are required. Using Lyapunov-Krasovskii functionals, a stable adaptive NN controller is designed, which guarantees that the tracking error remains bounded within a neighbourhood of the origin that can be made arbitrarily small by design of parameters.

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