H_∞ DYNAMIC OUTPUT-FEEDBACK CONTROL FOR A CLASS OF LINEAR NEUTRAL DELAY SYSTEMS [1]

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Abstract: The H_∞ dynamic output feedback controller design of linear neutral delay systems is studied using Lyapunov-Krasovskii stability theory and linear matrix inequality approach. Based on feasibility positive definite solution to the linear matrix inequalities, we first develop a delay-independent stability criterion and a sufficient condition which makes the system asymptotically stable and guarantees the given H_∞-bound constraint on the disturbance attenuation; Then, we present a scheme of designing a dynamic output feedback H_∞ controller via linear matrix-inequality; Finally, a numerical example is given to demonstrate the validity and effectiveness of the proposed approach. *Copyright© 2005 IFAC.*

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1. INTRODUCTION

It is well known that delay often occurs in many dynamic systems such as communication systems, biological systems, chemical systems and electrical networks. Meanwhile, delay is frequently a source of instability and performance degradation in many dynamic systems, and thus considerable attention has been paid to the research on the stability analysis and controller synthesis of time-delay systems, see (Hale,1977; Els'golts' and Norkin, 1973; Kolmanovskii and Myshkis, 1992).

In addition, there are many control systems having not only delay in the state but also in the state derivative. Such dynamic system are referred to as neutral delay systems(Hu, 1996; Mahmoud, 2000) and (Han, 2002). The theory of neutral delay-differential systems is of both theoretical and practical interest. For example, functional differential equations of neutral type are the natural models of fluctuations of voltage and current in problems arising in transmission lines. Also, the neutral systems often appear in the study of automatic control, population dynamics, and vibrating masses attached to an elastic bar. During the last two decades, many researchers studied the stability analysis and stabilization problem for neutral delay systems. Numerous delay-independent and delay-dependent criterions for stability were formulated by means of matrix measure, algebraic Riccati matrix equations or linear matrix inequalities (*LMI*s), while only a few works on controller design for stabilization of the systems has been explored by some researchers, see(Chukwu, 1992; Ma, *et al.*, 1995; Tarn,*et al.*, 1996; Verriest, 1996;) and (Fiagbedzi, 1994).

Recently, Xu, *et al.*(2001) dealt with the H_∞ and developed positive real control problem for linear neutral delay systems and the corresponding controller design schemes , The robust H_∞ control problem for linear uncertain neutral delay systems

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is considered by Mahmoud(2000), and some sufficient conditions for the solvability were presented, but these results are based on all state variables being available for the feedback. Unfortunately, the states of system are often unknown or only partially known, so the former methods cannot be applied. One of the methods to solve this problem is to design a dynamic output feedback controller. Up to now, at the knowledge of the author, no paper treats the H_∞ dynamic control problem for neutral delay systems.

In this paper, we consider the H_∞ dynamic output feedback controller design problem for linear neutral delay systems. The approach here is based on Lyapunov functionals due to Krasovsii. A sufficient condition is derived in terms of linear matrix inequalities. By means of their solutions, the controller is constructed, which stabilizes the system and achieves a prescribed level of H_∞-norm bound of the closed loop systems. Finally, we give a small example to illustrate the validity of the proposed design procedure.

Notations: The following notations will be used throughout the paper: $\mathbf{R}^n$ denotes the n-dimensional Euclidean space, $\mathbf{R}^{n\times m}$ is the set of $n \times m$ real matrices, I_n is the $n\times n$ identity matrix, $diag\{\cdots\}$ denotes a block-diagonal matrix. The notation $X > 0$(respectively, $X \geq 0$) means that the matrix X is real symmetric positive definite (respectively, positive semi-definite). $\mathscr{L}_2$ is the space of square integrable functions on $[0,\infty)$, $\mathscr{C}^{(1)}_{n,h} = \mathscr{C}^{(1)}([-h,0],\mathbf{R}^n)$ denotes the Banach space of continuous vector functions mapping the interval $[-h,0]$ into $\mathbf{R}^n$ with the topology of uniform convergence. The following norms will be used: $||\cdot||$ refers to the Euclidean vector norm, $||\cdot||_2$ denotes the $\mathscr{L}_2$-norm, $\|x_t\|_{c1} = \sup_{-h\leq\theta\leq 0}\{\|x(t+\theta)\|, \|\dot{x}(t+\theta)\|\}$ stands for the norm of a function $\phi \in \mathscr{C}^{(1)}_{n,h}$. $\lambda_M(A)$ and $\lambda_m(A)$ mean the largest and the smallest eigenvalues of matrix A, respectively.

2. PROBLEM FORMULATION

Consider the following linear neutral delay systems:

$$\dot{x}(t) = Ax(t) + A_1x(t-h) + A_2\dot{x}(t-h) + Bu(t) + B_1\omega(t) \quad (1)$$

$$y(t) = C_2x(t) \quad (2)$$

$$z(t) = C_1x(t) + D_1\omega(t) \quad (3)$$

$$x(t) = \phi(t), \quad t \in [-h,0] \quad (4)$$

where $x(t) \in \mathbf{R}^n$ is the state vector, $u(t) \in \mathbf{R}^m$ is the input vector, $\omega(t) \in \mathbf{R}^p$ is the disturbance input which belongs to $\mathscr{L}_2[0,+\infty)$, $z(t) \in \mathbf{R}^q$ is the controlled output, $y(t) \in \mathbf{R}^l$ is the measurement output. The scalar $h > 0$ denotes the delay time in the state and its derivative, $\phi(t) \in \mathbf{R}^n$ is a continuous vector valued initial function. A, A_1, A_2, B, B_1, C_1, C_2 and D_1 are known constant matrices with appropriate dimensions.

Now we consider the problem of the output-feedback control by using a state observer-based control scheme. Let the state observer be described by

$$\dot{\xi}(t) = A\xi(t) + A_1\xi(t-h) + A_2\dot{\xi}(t-h) + L\left(y(t) - \hat{y}(t)\right) + Bu(t), \quad (5)$$

$$u(t) = K\xi(t). \quad (6)$$

$$\hat{y}(t) = C\xi(t). \quad (7)$$

Introducing the variables

$$e(t) = x(t) - \xi(t),\ \tilde{x}(t) = [x^T(t), e^T(t)]^T,\ \tilde{\omega}(t) = [\omega^T(t), \omega^T(t)]^T \quad (8)$$

then the closed-loop system corresponding to (1)-(3), (5)-(8) is

$$\dot{\tilde{x}}(t) = \overline{A}\tilde{x}(t) + \overline{A}_1\tilde{x}(t-h) + \overline{A}_2\dot{\tilde{x}}(t-h) + \overline{B}_1\tilde{\omega}(t) \quad (9)$$

$$z(t) = \overline{C}_1\tilde{x}(t) + \overline{D}_1\tilde{\omega}(t) \quad (10)$$

where $\overline{A} = \begin{bmatrix} A_c & -BK \\ 0 & A_l \end{bmatrix}$, $\overline{A}_1 = \begin{bmatrix} A_1 & 0 \\ 0 & A_1 \end{bmatrix}$, $\overline{A}_2 = \begin{bmatrix} A_2 & 0 \\ 0 & A_2 \end{bmatrix}$, $\overline{B}_1 = \begin{bmatrix} B_1 & 0 \\ 0 & B_1 \end{bmatrix}$, $\overline{C}_1 = \begin{bmatrix} C_1 & 0 \end{bmatrix}$, $\overline{D}_1 = \begin{bmatrix} D_1 & 0 \end{bmatrix}$, $A_c = A + BK$, $A_l = A - LC_2$.

Our objective is to design of a feedback controller which stabilizes the system and achieves a prescribed level of H_∞-norm bound γ of the closed loop systems (9), i.e. $\|z(t)\|_2 < \gamma\|\omega(t)\|_2$.

In the next section we shall develop *LMI*-based methodologies for solving the above problems. We conclude this section by introducing two facts which will be used in the proof of our results.

Fact 1(Schur complement)(Boyd *et al.*, 1994) Given constant symmetric matrices $\Omega_1, \Omega_2, \Omega_3$, where $\Omega_1 = \Omega_1^T$, $0 < \Omega_2 = \Omega_2^T$, then $\Omega_1 + \Omega_3^T\Omega_2^{-1}\Omega_3 < 0$ if and only if

$$\begin{bmatrix} \Omega_1 & \Omega_3^T \\ \Omega_3 & -\Omega_2 \end{bmatrix} < 0 \quad \begin{bmatrix} -\Omega_2 & \Omega_3 \\ \Omega_3^T & \Omega_1 \end{bmatrix} < 0.$$

Fact 2. The solvability of the following two matrix inequalities

$$\Phi = \begin{bmatrix} \Phi_{11} & \Phi_{12} \\ \Phi_{12}^T & \Phi_{22} \end{bmatrix} < 0 \text{ and } \begin{cases} \Phi_{11} < 0 \\ \Phi_{22} < 0 \end{cases}$$

are equivalent, where Φ_{11}, Φ_{12} are a matrix functions with respect to $X = (X_1, X_2, X_3)$, Φ_{22} is a homogeneous matrix function with respect to $Y = (Y_1, Y_2, Y_3)$, $X_i > 0$, $Y_i > 0$, $i = 1,2,3$ are matrices variables.

Proof. The result is trivial if one of the matrices inequalities has no solutions, so it only suffices to prove the case which the two matrices inequalities have solution. In fact, it is obvious that the solution to $\Phi < 0$ satisfies the inequalities, $\Phi_{11} < 0$ and $\Phi_{22} < 0$. On the other hand, if $X = (X_1, X_2, X_3)$ and $Y = (Y_1, Y_2, Y_3)$ are solutions to $\Phi_{11} < 0$ and $\Phi_{22} < 0$ respectively, then

$$\begin{aligned}&\Phi_{22}(\lambda Y) - \Phi_{12}^T(X)\Phi_{11}^{-1}(X)\Phi_{12}(X)\\&= \lambda\Phi_{22}(Y) - \Phi_{12}^T(X)\Phi_{11}^{-1}(X)\Phi_{12}(X),\end{aligned}$$

So if $\lambda > 0$ is sufficiently large, then $\Phi_{22} - \Phi_{12}^T\Phi_{11}^{-1}\Phi_{12} < 0$ holds. By Fact 1, we get $(X, \lambda Y)$ is a set of solutions to $\Phi < 0$. This completes the proof.

Remark 1. Fact 2 plays a key role in our article, so we present the proof in detail.

Remark 2. Φ_{22} is a homogeneous matrix function with respect to $Y = (Y_1, Y_2, Y_3)$ means that there exists a Positive number λ such that $\Phi_{22}(\lambda Y) = \lambda\Phi_{22}(Y)$.

3. MAIN RESULTS

In the sequel we will present our main results on H_∞ dynamic output control.

3.1 H_∞ Performance Analysis

In this section, we will focus on the H_∞ performance analysis for the system (1)-(3). In order to solve this problem, we first consider the problem of asymptotic stability for the system (1)-(3) with $u(t) = 0$ and $\omega(t) = 0$, i.e.

$$\dot{x}(t) = Ax(t) + A_1x(t-h) + A_2\dot{x}(t-h), \quad (11)$$
$$x(t) = \phi(t), \quad t \in [-h, 0]. \quad (12)$$

Theorem 1. System (11) is asymptotically stable if there exist matrices $X > 0$, $Y > 0$, $Z > 0$ satisfying the following *LMI*:

$$\begin{bmatrix} AX + XA^T & A_1Y & A_2Z & XA^T & X \\ YA_1^T & -Y & 0 & YA_1^T & 0 \\ ZA_2^T & 0 & -Z & ZA_2^T & 0 \\ AX & A_1Y & A_2Z & -Z & 0 \\ X & 0 & 0 & 0 & -Y \end{bmatrix} < 0 \quad (13)$$

where $X = P^{-1}$, $Y = H^{-1}$, $Z = S^{-1}$.

Proof. Define the following Lyapunov functional candidate for the system (11):

$$V(x_t) = x^T(t)Px(t) + \int_{t-h}^{t} x^T(s)Hx(s)ds + \int_{t-h}^{t} \dot{x}^T(s)S\dot{x}(s)ds, \quad (14)$$

where $x_t(\theta) = x(t + \theta)$, $\theta \in [-h, 0]$. Apparently, there exist constants δ_1 and δ_2 such that

$$\delta_1\|x(t)\|^2 \le V(x_t) \le \delta_2\|x_t\|_{c1}^2$$

For example, take $\delta_1 = \lambda_{\min}(P)$, $\delta_2 = \lambda_{\max}(P) + h[\lambda_{\max}(H) + \lambda_{\max}(S)]$.

For the sake of simplicity, we write $x(t) = x$, $x(t-h) = x_h$, $\dot{x}(t-h) = \dot{x}_h$, $\omega(t) = \omega$ in the following proof. The time derivative of $V(x_t)$ along the solution of Eq.(11) is given by

$$\frac{\mathrm{d}}{\mathrm{d}t}V(x_t) = \bar{x}^T\Omega_1\bar{x} \quad (15)$$

where $\Omega_1 = \begin{bmatrix} \Phi & PA_1 + A^TSA_1 & PA_2 + A^TSA_2 \\ * & A_1^TSA_1 - H & A_1^TSA_2 \\ * & * & A_2^TSA_2 - S \end{bmatrix}$,

$\Phi = PA + A^TP + H + A^TSA$, $\bar{x} = [x^T \ x_h^T \ \dot{x}_h^T]^T$.

Under the condition of Theorem 1, we have $\frac{\mathrm{d}}{\mathrm{d}t}V(x_t) < 0$ using the Fact 1. Applying Lyapunov-Krasovskii stability theory, we can obtain the asymptotical stability of System (11).

Remark 3. Some connections betweens the stability results obtained using the norms $\|x_t\|_c = \sup_{-h\le\theta\le0}\{\|x(t+\theta)\}$ and $\|x_t\|_{c1}$ could be found in Els'golts' and Norkin(1973).

We now give the solution to H_∞ performance analysis problem based on Theorem 1. Consider system (1)-(3)with $u(t) = 0$, i.e.

$$\dot{x}(t) = Ax(t) + A_1x(t-h) + A_2\dot{x}(t-h) + B_1\omega(t), \quad (16)$$
$$z(t) = C_1x(t) + D_1\omega(t) \quad (17)$$
$$x(t) = \phi(t), \quad t \in [-h, 0]. \quad (18)$$

Theorem 2. Given scalar $\gamma > 0$, the system (16)-(18) is asymptotically stable with disturbance attenuation γ if there exist symmetric positive-definite matrices X, Y, Z satisfying the following *LMI*:

$$\begin{bmatrix} G(X) & A_1Y & A_2Z & B_1 & XA^T & XC_1^T & X \\ YA_1^T & -Y & 0 & 0 & YA_1^T & 0 & 0 \\ ZA_2^T & 0 & -Z & 0 & ZA_2^T & 0 & 0 \\ B_1^T & 0 & 0 & -\gamma^2I & B_1^T & D_1^T & 0 \\ AX & A_1Y & A_2Z & B_1 & -Z & 0 & 0 \\ C_1X & 0 & 0 & D_1 & 0 & -I & 0 \\ X & 0 & 0 & 0 & 0 & 0 & -Y \end{bmatrix} < 0, \quad (19)$$

or

$$\begin{bmatrix} G(X) & A_1^TY & A_2^TZ & C_1^T & XA & XB_1 & X \\ YA_1 & -Y & 0 & 0 & YA_1 & 0 & 0 \\ ZA_2 & 0 & -Z & 0 & ZA_2 & 0 & 0 \\ C_1 & 0 & 0 & -\gamma^2I & C_1 & D_1 & 0 \\ A^TX & A_1^TY & A_2^TZ & C_1^T & -Z & 0 & 0 \\ B_1^TX & 0 & 0 & D_1^T & 0 & -I & 0 \\ X & 0 & 0 & 0 & 0 & 0 & -Y \end{bmatrix} < 0, \quad (20)$$

where X, Y, Z are same as in Theorem 1, $G(X) \triangleq AX + XA^T$.

Proof. By applying Theorem 1, the asymptotic stability of the system (11) follows from the inequality (19) or (20).

Next, we are only to establish the H_∞ performance for the system (16)-(18), assuming zero initial conditions for Eq.(16). Replacing P, H, S with X, Y, Z in (19), multiplying (19) on both sides by $diag\{P, H, S, I, I, I, I\}$ and then applying Schur complement to the result, we have

$$\Omega_2 = \begin{bmatrix} \Gamma & PA_1 + A^TSA_1 & PA_2 + A^TSA_2 & PB_1 + A^TSB_1 + C_1^TD_1 \\ A_1^TP + A_1^TSA & A_1^TSA_1 - H & A_1^TSA_2 & A_1^TSB_1 \\ A_2^TP + A_2^TSA & A_2^TSA_1 & A_2^TSA_2 - S & A_2^TSB_1 \\ B_1^TP + B_1^TSA + D_1^TC_1 & B_1^TSA_1 & B_1^TSA_2 & B_1^TSB_1 + D_1^TD_1 - \gamma^2 I \end{bmatrix} < 0 \tag{21}$$

where $\Gamma = PA + A^TP + H + A^TSA + C_1^TC_1$.

With the zero initial conditions($x(t) = 0,\ t \in [-\tau, 0]$), we consider

$$\begin{aligned} J_T &= \int_0^T (z^T(t)z(t) - \gamma^2\omega^T(t)\omega(t) + \dot{V}(x_t) \\ &\quad - \dot{V}(x_t))dt \\ &\le \int_0^T (z^T(t)z(t) - \gamma^2\omega^T(t)\omega(t) + \dot{V}(x_t))dt \end{aligned}$$

Since the time derivative of $V(x_t)$ along the trajectory of Eq.(16) is given by

$$\begin{aligned} \dot{V}(x_t) &= \dot{x}^T(t)Px(t) + x^T(t)P\dot{x}(t) + x^T(t)Hx(t) \\ &\quad + \dot{x}^T(t)S\dot{x}(t) - x^T(t-h)Hx(t-h) \\ &\quad - \dot{x}^T(t-h)S\dot{x}(t-h) \\ &= \eta^T\Omega_3\eta \end{aligned}$$

where

$$\Omega_3 = \begin{bmatrix} PA + A^TP + H + A^TSA & PA_1 + A^TSA_1 & PA_2 + A^TSA_2 & PB_1 + A^TSB_1^T \\ * & A_1^TSA_1 - H & A_1^TSA_2 & A_1^TSB_1 \\ * & * & A_2^TSA_2 - S & A_2^TSB_1 \\ * & * & * & B_1^TSB_1 \end{bmatrix},$$

$\eta = [x^T, x_h^T, \dot{x}_h^T, \omega^T]^T$.

$$\begin{aligned} & z^T(t)z(t) - \gamma^2\omega^T(t)\omega(t) \\ &= \begin{bmatrix} x \\ x_h \\ \dot{x}_h \\ \omega \end{bmatrix}^T \begin{bmatrix} C_1^TC_1 & 0 & 0 & C_1^TD_1 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ D_1^TC_1 & 0 & 0 & D_1^TD_1 - \gamma^2 I \end{bmatrix} \begin{bmatrix} x \\ x_h \\ \dot{x}_h \\ \omega \end{bmatrix}, \end{aligned}$$

hence

$$z^T(t)z(t) - \gamma^2\omega^T(t)\omega(t) + \dot{V} = \eta^T\Omega_2\eta < 0,$$

where Ω_2 is defined as the former, this means

$$\begin{aligned} \int_0^T z^T(t)z(t)dt &< \gamma^2 \int_0^T \omega^T(t)\omega(t))dt \\ &\le \int_0^\infty \omega^T(t)\omega(t))dt \end{aligned}$$

holds for all $T > 0$. Hence controlled output $z(t) \in \mathscr{L}_2[0, \infty)$ and satisfies $\|z\|_2 \le \gamma||\omega||_2$, which concludes the proof.

3.2 H_∞ Dynamic Output Control

In this subsection, we'll present a solution to the H_∞ dynamic output control problem for the system (9)-(10) based on the H_∞ performance analysis in Theorem 2.

By considering Theorem 2, the system (9)-(10) is asymptotically stable with disturbance attenuation γ if the following matrix inequality holds :

$$\begin{bmatrix} F(X) & \overline{A}_1^TY & \overline{A}_2^TZ & \overline{C}_1^T & X\overline{A} & X\overline{B}_1 & X \\ Y\overline{A}_1 & -Y & 0 & 0 & Y\overline{A}_1 & 0 & 0 \\ Z\overline{A}_2 & 0 & -Z & 0 & Z\overline{A}_2 & 0 & 0 \\ \overline{C}_1 & 0 & 0 & -\gamma^2 I & \overline{C}_1 & \overline{D}_1 & 0 \\ \overline{A}^TX & \overline{A}_1^TY & \overline{A}_2^TZ & \overline{C}_1^T & -Z & 0 & 0 \\ \overline{B}_1^TX & 0 & 0 & \overline{D}_1^T & 0 & -I & 0 \\ X & 0 & 0 & 0 & 0 & 0 & -Y \end{bmatrix} < 0, \tag{22}$$

where $F(X) = \overline{A}^TX + X\overline{A}$, $X = diag\{X_1\ X_2\}$, $Y = diag\{Y_1\ Y_2\}$, $Z = diag\{Z_1\ Z_2\}$ are positive definite matrices. $\overline{A}$, $\overline{A}_1$, $\overline{A}_2$, $\overline{C}_1$, $\overline{D}_1$, $\overline{X}$, $\overline{Y}$, $\overline{Z}$ are same as the former.

Exchanging some rows and corresponding columns of the matrix in (22), we have

$$\begin{bmatrix} \Phi_{11} & \Phi_{12} \\ \Phi_{12}^T & \Phi_{22} \end{bmatrix} < 0 \tag{23}$$

where

$$\Phi_{11} = \begin{bmatrix} G_1(X_1, K) & X_1A_c & A_1^TY_1 & X_1 & A_2^TZ_1 & X_1B_1 & C_1^T \\ A_c^TX_1 & -Z_1 & A_1^TY_1 & 0 & A_2^TZ_1 & 0 & C_1^T \\ Y_1A_1 & Y_1A_1 & -Y_1 & 0 & 0 & 0 & 0 \\ X_1 & 0 & 0 & -Y_1 & 0 & 0 & 0 \\ Z_1A_2 & Z_1A_2 & 0 & 0 & -Z_1 & 0 & 0 \\ B_1^TX_1 & 0 & 0 & 0 & 0 & -I & D_1^T \\ C_1 & C_1 & 0 & 0 & 0 & D_1 & -\gamma^2 I \end{bmatrix},$$

$$\Phi_{12} = \begin{bmatrix} -X_1BK & -X_1BK & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix},$$

$$\Phi_{22} = \begin{bmatrix} G_2(X_2) & X_2A_l & A_2^TZ_2 & X_2B_1 & A_1^TY_2 & X_2 \\ A_l^TX_2 & -Z_2 & A_2^TZ_2 & 0 & A_1^TY_2 & 0 \\ Z_2A_2 & Z_2A_2 & -Z_2 & 0 & 0 & 0 \\ B_1^TX_2 & 0 & 0 & -I & 0 & 0 \\ Y_2A_1 & Y_2A_1 & 0 & 0 & -Y_2 & 0 \\ X_2 & 0 & 0 & 0 & 0 & -Y_2 \end{bmatrix},$$

$G_1(X_1, K) = A_c^T X_1 + X_1 A_c$, $G_2(X_2) = A_l^T X_2 + X_2 A_l$.

Note that Φ_{11}, Φ_{12}, Φ_{22} satisfy the conditions of Fact 2. Applying Fact 2, we obtain that the condition of (23) has a solution is equivalent to that $\Phi_{11} < 0$ and $\Phi_{22} < 0$ have solutions.

Taking $K = -B^T X_1$ and substituting it into Φ_{11}, we have

$$\Phi_{11} = \begin{bmatrix} A^T X_1 + X_1 A - 2\Xi & X_1 A - \Xi & A_1^T Y_1 & X_1 & A_2^T Z_1 & X_1 B_1 & C_1^T \\ A^T X_1 - \Xi & -Z_1 & A_1^T Y_1 & 0 & A_2^T Z_1 & 0 & C_1^T \\ Y_1 A_1 & Y_1 A_1 & -Y_1 & 0 & 0 & 0 & 0 \\ X_1 & 0 & 0 & -Y_1 & 0 & 0 & 0 \\ Z_1 A_2 & Z_1 A_2 & 0 & 0 & -Z_1 & 0 & 0 \\ B_1^T X_1 & 0 & 0 & 0 & 0 & -I & D_1^T \\ C_1 & C_1 & 0 & 0 & 0 & D_1 & -\gamma^2 I \end{bmatrix}$$

$$\leq \begin{bmatrix} A^T X_1 + X_1 A - \Xi & X_1 A & A_1^T Y_1 & X_1 & A_2^T Z_1 & X_1 B_1 & C_1^T \\ A^T X_1 & -Z_1 + \Xi & A_1^T Y_1 & 0 & A_2^T Z_1 & 0 & C_1^T \\ Y_1 A_1 & Y_1 A_1 & -Y_1 & 0 & 0 & 0 & 0 \\ X_1 & 0 & 0 & -Y_1 & 0 & 0 & 0 \\ Z_1 A_2 & Z_1 A_2 & 0 & 0 & -Z_1 & 0 & 0 \\ B_1^T X_1 & 0 & 0 & 0 & 0 & -I & D_1^T \\ C_1 & C_1 & 0 & 0 & 0 & D_1 & -\gamma^2 I \end{bmatrix}$$

$$\leq \begin{bmatrix} \Sigma_1 & X_1 A & A_1^T Y_1 & X_1 & A_2^T Z_1 & X_1 B_1 & C_1^T & 0 \\ A^T X_1 & -Z_1 & A_1^T Y_1 & 0 & A_2^T Z_1 & 0 & C_1^T & X_1 B \\ Y_1 A_1 & Y_1 A_1 & -Y_1 & 0 & 0 & 0 & 0 & 0 \\ X_1 & 0 & 0 & -Y_1 & 0 & 0 & 0 & 0 \\ Z_1 A_2 & Z_1 A_2 & 0 & 0 & -Z_1 & 0 & 0 & 0 \\ B_1^T X_1 & 0 & 0 & 0 & 0 & -I & D_1^T & 0 \\ C_1 & C_1 & 0 & 0 & 0 & D_1 & -\gamma^2 I & 0 \\ 0 & B_1^T X_1 & 0 & 0 & 0 & 0 & 0 & -I \end{bmatrix} = \Delta \quad (24)$$

where $\Sigma_1 = A^T X_1 + X_1 A + QBB^T Q^T - QBB^T X_1 - \Xi$, $\Xi = X_1 BB^T X_1$. Thus for a given matrix Q, $\Delta < 0$ is a LMI, here we use the inequality $-X_1 BB^T X_1 \leq QBB^T Q^T - QBB^T X_1 - X_1 BB^T Q^T$. In fact, for any matrix Q with appropriate dimension, $(Q - X_1)BB^T(Q - X_1)^T \geq 0$ holds.

From Schur complement, $\Delta < 0$ is equivalent to

$$\begin{bmatrix} \Sigma_1 & X_1 A & A_1^T Y_1 & X_1 & A_2^T Z_1 & X_1 B_1 & C_1^T & 0 \\ A^T X_1 & -Z_1 & A_1^T Y_1 & 0 & A_2^T Z_1 & 0 & C_1^T & X_1 B \\ Y_1 A_1 & Y_1 A_1 & -Y_1 & 0 & 0 & 0 & 0 & 0 \\ X_1 & 0 & 0 & -Y_1 & 0 & 0 & 0 & 0 \\ Z_1 A_2 & Z_1 A_2 & 0 & 0 & -Z_1 & 0 & 0 & 0 \\ B_1^T X_1 & 0 & 0 & 0 & 0 & -I & D_1^T & 0 \\ C_1 & C_1 & 0 & 0 & 0 & D_1 & -\gamma^2 I & 0 \\ 0 & B_1^T X_1 & 0 & 0 & 0 & 0 & 0 & -I \end{bmatrix} < 0, \quad (25)$$

By the similar way, take $L = X_2^{-1} N$, then Φ_{22} becomes the following LMI:

$$\begin{bmatrix} \Sigma_2 & \Sigma_3 & A_2^T Z_2 & X_2 B_1 & A_1^T Y_2 & X_2 \\ \Sigma_3^T & -Z_2 & A_2^T Z_2 & 0 & A_1^T Y_2 & 0 \\ Z_2 A_2 & Z_2 A_2 & -Z_2 & 0 & 0 & 0 \\ B_1^T X_2 & 0 & 0 & -I & 0 & 0 \\ Y_2 A_1 & Y_2 A_1 & 0 & 0 & -Y_2 & 0 \\ X_2 & 0 & 0 & 0 & 0 & -Y_2 \end{bmatrix} < 0 \quad (26)$$

where $\Sigma_2 = A^T X_2 + X_2 A - NC_2 - C_2^T N^T$, $\Sigma_3 = X_2 A - NC_2$.

If the inequalities (25), (26) hold, then (23) holds. Moreover, if $(X_i, Y_i, Z_i), i = 1, 2$ are solutions to (25) and (26), respectively, we can get the control gain matrix $K = -B^T X_1$, and observer gain matrix $L = \lambda X_2^{-1} N$, where the positive number λ is determined by

$$\lambda \Phi_{22}(X_2, Y_2, Z_2) - \Phi_{12}^T(X_1, Y_1, Z_1) \times \Phi_{11}^{-1}(X_1, Y_1, Z_1)\Phi_{12}(X_1, Y_1, Z_1) < 0 \quad (27)$$

Summarizing the above analysis, about the H_∞ dynamic output control problem of the system (9)-(10), we have the following result.

Theorem 3. Consider system (9)-(10) with a given matrix Q. If there exist matrices $X_i > 0$, $Y_i > 0$, $Z_i > 0, i = 1, 2$ and N satisfying (25) and (26), respectively, then system (9)-(10) is asymptotically stable with disturbance attenuation γ. Moreover, control gain matrix and observer gain matrix are given by $K = -B^T X_1$, $L = \lambda X_2^{-1} N$, respectively, where the positive number λ is determined by (27).

Remark 3. The positive number λ determined by (27) is not unique. We can obtain the minimum of it by using the function *mincx* in Matlab software.

4. NUMERICAL EXAMPLE

In this section, a numerical example is presented to illustrate the effectiveness of the proposed method in this paper.

Example Consider the following neutral delay system with $A = \begin{bmatrix} -1 & 1 \\ 1 & -1 \end{bmatrix}$, $A_1 = \begin{bmatrix} 0.15 & 0.05 \\ 0 & 0.1 \end{bmatrix}$, $A_2 = \begin{bmatrix} -0.05 & 0.02 \\ 0.01 & 0 \end{bmatrix}$, $B = \begin{bmatrix} 0.5 \\ 1 \end{bmatrix}$, $B_1 = \begin{bmatrix} -0.01 \\ 0.03 \end{bmatrix}$,

$C_1 = \begin{bmatrix} 1 & 1 \\ -1 & 0 \end{bmatrix}$, $C_2 = \begin{bmatrix} 1 & 1 \end{bmatrix}$, $D_1 = \begin{bmatrix} 0.5 & 0.01 \end{bmatrix}$,

Let $\gamma = 1$, $Q = \begin{bmatrix} 1 & 2 \\ 0 & 1 \end{bmatrix}$. Based on the Theorem 3, using the function *mincx* in Matlab software, we obtain $\lambda_{min} = 2.4310$ by solving $LMIs$ (25), (26) and (27), the corresponding solutions to the LMIs are

$X_1 = \begin{bmatrix} 4.8485 & 0.5449 \\ 0.5449 & 3.4759 \end{bmatrix}$, $Y_1 = \begin{bmatrix} 18.2579 & -0.8855 \\ -0.8855 & 18.1880 \end{bmatrix}$,

$Z_1 = \begin{bmatrix} 20.0207 & -0.6683 \\ -0.6683 & 19.6801 \end{bmatrix}$, $X_2 = \begin{bmatrix} 6.7098 & 1.2049 \\ 1.2049 & 6.6086 \end{bmatrix}$,

$Y_2 = \begin{bmatrix} 19.3482 & 0.7075 \\ 0.7075 & 19.7011 \end{bmatrix}$, $Z_2 = \begin{bmatrix} 20.7215 & 0.9112 \\ 0.9112 & 20.2727 \end{bmatrix}$,

$K = \begin{bmatrix} -2.9692 & -3.7484 \end{bmatrix}$, $L = \begin{bmatrix} 1.4326 \\ 1.3457 \end{bmatrix}$,

Consequently, we can get the H_∞ dynamic output controller:

$$\begin{aligned} \dot{\xi}(t) &= \begin{bmatrix} -3.9171 & -2.3067 \\ -3.3149 & -6.0941 \end{bmatrix} \xi(t) + \begin{bmatrix} 0.15 & 0.05 \\ 0 & 0.1 \end{bmatrix} \times \\ &\quad \xi(t-h) + \begin{bmatrix} -0.05 & 0.02 \\ 0.01 & 0 \end{bmatrix} \dot{\xi}(t-h) \\ &\quad + \begin{bmatrix} 1.4326 \\ 1.3457 \end{bmatrix} y(t) \\ u(t) &= \begin{bmatrix} -2.9692 & -3.7484 \end{bmatrix} \xi(t) \end{aligned}$$

5. CONCLUSIONS

In this paper, the H_∞ dynamic output feedback controller design problem for linear neutral delay systems is considered. A sufficient condition is derived in terms of linear matrix inequalities. By means of their solutions, the controller is constructed, which stabilizes the system and achieves a prescribed level of H_∞-norm bound of the closed loop systems. Finally, we give a small example to illustrate the validity of the proposed design procedure.

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