A RISK ADJUSTED APPROACH TO ROBUST SIMULTANEOUS FAULT DETECTION AND ISOLATION

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Abstract: In this paper we address the problem of detecting and isolating faults from noisy input/output measurements of a MIMO uncertain–system, subject to structured dynamic uncertainty. The main result of the paper shows that this problem can be solved in a computationally efficient way by using a combination of sampling and LMI optimization tools. These results are illustrated using a simplified model of a flight control system.

Keywords: Fault Detection, Fault Isolation, Model (In)Validation

1. INTRODUCTION

The problem of Fault Detection and Isolation (FDI) in control systems has been the subject of considerable attention during the past two decades. This research has resulted in a variety of methods and a vast amount of papers in the literature (see for instance (Frank and Ding 1997, Gertler 1998, Patton 1994) and references therein). Many of these methods are based on a *model–based* approach, also known as analytical or functional redundancy. In contrast to approaches based on *physical or hardware redundancy*, the former exploit the mathematical model of the system under consideration, leading to a two stage procedure: (i) residual generation and, (ii) decision making.

While appealing, since it does not require additional hardware, a potential problem with the analytical approach is its *fragility*: a mismatch between the actual plant and the model used in the FDI algorithm can result in false alarms. To avoid this difficulty, the algorithm must be robust both against modelling errors and exogenous disturbances. Robust FDI methods have been well studied (see for instance (Collins and Song 2000, Emaimi-Naeimi *et al.* 1998, Frank and Ding 1997, Henry *et al.* 2001, Jiang *et al.* 2002, Patton 1994, Saberi *et al.* 2000, Stoustrup and Niemann 2003, Zhong *et al.* 2003) and references therein). A potential disadvantage of these methods is the difficulty in isolating the exact location of the fault and in detecting simultaneous faults. Moreover, in the case of dynamic uncertainty, this problem is generically non–convex in all variables involved(Shim and Sznaier 2003) and thus computationally hard to solve.

In this paper we propose to solve these difficulties by pursuing a risk–adjusted approach, based on sampling the uncertainty set. This removes one of the interpolation constraints that renders the problem non-convex, allowing for efficient solutions. The proposed new FDI framework has the following advantages over currently existing methods:

(a) It allows for handling *arbitrary* dynamic uncertainty structures (as opposed to parametric uncertainty)
(b) It allows for arbitrary fault dynamics, rather than having the fault and nominal operation sharing the same dynamic matrix A. In addition, it also provides an estimate of which fault has occurred.
(c) Its computational complexity grows only polynomially with the dimension of the plant.

(d) It avoids being *over–optimistic*, in the sense of assuming that a fault has not taken place when the probability of the experimental data having been generated by the nominal dynamics is low.

Finally, we also present a deterministic, convex relaxation for the special case of multiplicative uncertainty and benchmark both methods using a simple example.

2. PRELIMINARIES

2.1 Notation

Below we summarize the notation used in this paper:

$\lfloor x \rfloor$	largest integer smaller than or equal to $x \in \Re$.
$\mathbf{x}$	real–valued column vector.
$\|\mathbf{x}\|_p$	p-norm of a vector: $\|\mathbf{x}\|_p \doteq (\sum_{k=1}^m \|x_k\|^p)^{\frac{1}{p}}$, $p \in [1,\infty)$, $\|\mathbf{x}\|_\infty \doteq \max_{k=1,\ldots,m} \|x_k\|$.
$\mathbf{A}^T$	conjugate transpose of matrix A.
$\mathbf{A} > 0$	$\mathbf{A} = \mathbf{A}^T$ is positive definite.
$\mathbf{A} <\ (\leq)\mathbf{B}$	$(\mathbf{A} - \mathbf{B}) <\ (\leq) 0$
$\mathbf{I}, \mathbf{0}$	the identity and null matrices of compatible dimensions (when omitted).
$\overline{\sigma}(\mathbf{A})$	maximum singular value of $\mathbf{A}$.
ℓ_2^m	Banach space of vector valued real sequences equipped with the norm: $\|x\|_2 \doteq \left(\sum_{i=0}^\infty \|\mathbf{x}_i\|_2^2\right)^{\frac{1}{2}}$
$\ell_{2[0,n]}^m$	subspace of ℓ_2^m formed by finite sequences of length $n+1$.
$X(\lambda)$	λ–transform of a single–sided real sequence $\{x\}$: $X(\lambda) = \sum_0^\infty x_i \lambda^i$.
$\mathcal{BX}(\gamma)$	closed γ-ball in a normed space $\mathcal{X}$: $\mathcal{BX}(\gamma) = \{x \in \mathcal{X} \colon \|x\|_{\mathcal{X}} \leq \gamma\}$
$\mathcal{H}_\infty$	Space of transfer matrices with bounded analytic continuation inside the unit disk, equipped with the norm: $\|G\|_\infty \doteq ess\sup_{\|\lambda\|<1} \overline{\sigma}(G(\lambda))$.
$\mathcal{RX}$	subspace of $\mathcal{X} \subseteq \mathcal{H}_\infty$ composed of real rational transfer matrices.
$\mathcal{BH}_\infty^n$	set of $(n-1)^{\text{th}}$ order FIR transfer matrices that can be completed to belong to $\mathcal{BH}_\infty$, i.e. $\mathcal{BH}_\infty^n \doteq \{H(\lambda) = \mathbf{H}_0 + \mathbf{H}_1\lambda + \ldots + \mathbf{H}_{n-1}\lambda^{n-1} \colon H(\lambda) + \lambda^n G(\lambda) \in \mathcal{BH}_\infty,$ for some $G(\lambda) \in \mathcal{H}_\infty\}$.

In the sequel, to any finite sequence $\{\mathbf{x}_k\}$, we will associate the following finite lower Toeplitz matrix:

$$\mathbf{T}_x^n = \begin{bmatrix} \mathbf{x}_0 & \mathbf{0} & \ldots & \mathbf{0} \\ \mathbf{x}_1 & \mathbf{x}_0 & \ldots & \mathbf{0} \\ \vdots & \vdots & \ddots & \vdots \\ \mathbf{x}_{n-1} & \mathbf{x}_{n-2} & \ldots & \mathbf{x}_0 \end{bmatrix}.$$

In the case of an LTI system S, we will denote by $\mathbf{T}_S^n$ the lower Toeplitz matrix $\mathbf{T}_S$ associated with the first n elements of its impulse response.

2.2 Background Results

In this section we summarize, for ease of reference, several results that will be used to recast the FDI problem into a convex optimization form. We begin by recalling an algorithm, developed in (Lagoa *et al.* 2001), that generates uniformly distributed finite impulse responses $\{h^i\}_{i=1}^{N_s}$ with $h^i = \{\mathbf{H}_0^i, \mathbf{H}_1^i, \ldots, \mathbf{H}_N^i\}$ so that the function $H^i(\lambda) \doteq \sum_{k=0}^N \mathbf{H}_k^i \lambda^k$; $\mathbf{H}_k^i \in \Re^{m\times s}$ *can be completed* to belong to $\mathcal{BH}_\infty$. It will used to to obtain a convex, computationally tractable stochastic relaxation of the robust FDI problem in the case of structured uncertainty and noisy measurements.

Algorithm 1.

Let $k = 0$. Generate N_1 samples uniformly distributed over the set

$$\{\mathbf{H}_0 : \overline{\sigma}(\mathbf{H}_0) \leq 1\}. \tag{1}$$

(1) Let $k := k+1$. For every generated sample $(\mathbf{H}_0^i, \mathbf{H}_1^i, \ldots, \mathbf{H}_{k-1}^i)$, consider the partition

$$\begin{bmatrix} \mathbf{H}_k^i & \cdots & \mathbf{H}_1^i & \mathbf{H}_0^i \\ \mathbf{H}_{k-1}^i & \cdots & \mathbf{H}_0^i & \mathbf{0} \\ \vdots & & \ddots & \vdots \\ \mathbf{H}_0^i & \mathbf{0} & \cdots & \mathbf{0} \end{bmatrix} = \begin{bmatrix} \mathbf{H}_k^i & \mathbf{B} \\ \mathbf{C} & \mathbf{A} \end{bmatrix} \tag{2}$$

and let the matrices $\mathbf{Y}$ and $\mathbf{Z}$ be a solution of the linear equations

$$\begin{aligned} \mathbf{B} &= \mathbf{Y}(\mathbf{I} - \mathbf{A}^T\mathbf{A})^{\frac{1}{2}}; \\ \mathbf{C} &= (\mathbf{I} - \mathbf{A}\mathbf{A}^T)^{\frac{1}{2}}\mathbf{Z}, \end{aligned} \tag{3}$$

(2) Let $\mathbf{J}(\mathbf{H}_0, \mathbf{H}_1, \ldots, \mathbf{H}_{k-1}) \doteq |(\mathbf{I}-\mathbf{Y}\mathbf{Y}^T)^{\frac{1}{2}}|^m |(\mathbf{I}-\mathbf{Z}^T\mathbf{Z})^{\frac{1}{2}}|^s$. Generate

$$\left\lfloor N_s \mathbf{J}(\mathbf{H}_0^i, \mathbf{H}_1^i, \ldots, \mathbf{H}_{k-1}^i) \right\rfloor, \tag{4}$$

samples uniformly over the set $\{\mathbf{W} : \overline{\sigma}(\mathbf{W}) \leq 1\}$ and for each of those samples $\mathbf{W}^i$, compute

$$\mathbf{H}_k^i = -\mathbf{Y}\mathbf{A}^t\mathbf{Z} + (\mathbf{I} - \mathbf{Y}\mathbf{Y}^T)^{\frac{1}{2}}\mathbf{W}^i(\mathbf{I} - \mathbf{Z}^T\mathbf{Z})^{\frac{1}{2}}. \tag{5}$$

(3) If $k \leq N$ go to step 1. Otherwise, stop.

It can be shown (Lagoa *et al.* 2001) that the probability distribution of the samples generated by this algorithm converges with probability one to a uniform distribution as $N_s \to \infty$. Moreover, for a finite N_s, the difference between this probability density and a true uniform one is $\mathcal{O}(\frac{1}{N_s})$.

The next result gives a necessary and sufficient condition for the existence of an LTI bounded ℓ^2 operator mapping two given sequences.

Lemma 1. (Carathéodory-Fejér). (Foias and Frazho 1990). Given two sequences $\mathbf{u} = \{u(0), u(1), \cdots, u(l-1) \in R^n\}$ and $\mathbf{y} = \{y(0), y(1), \cdots, y(l-1) \in R^m\}$, there exists a stable, causal, linear time-invariant operator Δ with $||\Delta||_\infty \leq \gamma$ such that $\Delta\mathbf{u} = \mathbf{y}$ if and only if $T_y^{'}T_y \leq \gamma^2 T_u^{'}T_u$.

3. ROBUST FDI

3.1 Problem Formulation

In this paper we consider the problem of fault detection and isolation for systems represented by the following parameterized fault model which includes both dynamic uncertainty and disturbances:

$$\mathbf{y} = \left[G_0(\lambda, \Delta) + \sum_{i=1}^{r} f_i G_i(\lambda, \Delta_i) \right] \mathbf{u} + \mathbf{d} \qquad (6)$$

Here the transfer matrices $G_0(\lambda, \Delta_o)$ and $G_i(\lambda, \Delta_i), i = 1, \cdots, r$ represent the plant under normal (e.g. non failure) conditions and dynamic fault models, respectively, $\Delta_i \in \mathbf{\Delta}_i \subset \mathcal{BH}_\infty$ represent (structured) model uncertainty and $d, \ \|d\|_2 \leq \delta$ represents an unknown but ℓ^2 bounded disturbance. The scalars $f_i \in [0, 1]$ are fault indicators, with $f_i = 0$ corresponding to the case of no failure and $f_i = 1$ corresponding to the extreme case of total failure. Note that this formulation allows for the uncertainty to enter the dynamics in an arbitrary way.

In this context, the FDI problem can be stated as:

Problem 1. Given a model of the plant under normal conditions $G_o(\lambda, \Delta_o)$, failure dynamics $G_i(\lambda, \Delta_i)$, a bound δ on the measurement noise, uncertainty sets $\mathbf{\Delta_i}$, and n input/output experimental measurements determine: (i) whether a fault has occurred, and (ii) in that case isolate it and determine its strength.

Note that in general, due to the presence of uncertainty and noise, there may exist more than one set $\{\Delta_i, d, f\}$ that explains the experimental input/output data. In that case, to avoid ambiguities, we will select, among all possible solutions, the one corresponding to the minimum value of $\|f\|_2$. This choice minimizes the number of false alarms, since it tries to explain, whenever possible, the experimental data as being produced by the normal (non–failure) dynamics, possibly affected by dynamic uncertainty and measurement noise. With this choice, Problem 1 can be recast in the following (infinite–dimensional) optimization form:

Problem 2. Given the *a priori* information $G_i(\lambda, \Delta_i)$, δ and the experimental data $\mathbf{u}$ and $\mathbf{y}$ find:

$$\min_{\substack{\Delta_i \in \mathbf{\Delta}_i \\ \mathbf{d}, \|\mathbf{d}\|_2 \leq \delta}} \|f\|_2$$

subject to: (7)

$$\mathbf{y} = \left[G_0(\lambda, \Delta) + \sum_{i=1}^{r} f_i G_i(\lambda, \Delta_i) \right] \mathbf{u} + \mathbf{d}$$

3.2 Problem solution.

Unfortunately, as stated Problem 2 is not jointly convex in all the variables involved (Δ_i, f_i, d_i). Indeed, by appealing to Carathéodory-Fejér and Schur complement arguments (Shim and Sznaier 2003), it can be shown that even the simpler case of multiplicative unstructured uncertainty leads to a bilinear matrix inequality (BMI) in d, f. These problems are generically NP–hard (see for example (Tuan and Apkarian 1999)) and thus computationally expensive to solve.

To avoid this difficulty, in the sequel we propose to use a stochastic relaxation of the original problem that has polynomial, rather than exponential, computational complexity growth with the problem data (Khargonekar and Tikku 1996). The main idea of the method is to uniformly sample the set of admissible uncertainties $\mathbf{\Delta_i}$, in an attempt to find at least one element $\tilde{\Delta}_o \in \mathbf{\Delta}_o$ and r pairs $\left\{\tilde{\Delta}_i, \tilde{f}_i\right\} \in \mathbf{\Delta}_i \times [0, 1], i = 0, r$ so that model $G_o(\lambda, \tilde{\Delta}_o) + \sum f_i G_i(\lambda, \tilde{\Delta}_i)$ together with an admissible noise $\tilde{d}, \|\tilde{d}\|_2 \leq \delta$ can explain the experimental data y. As we show next, this removes the interpolation constraint that renders the problem non–convex in (f, d, Δ).

Lemma 2. For fixed $\Delta_i, \ i = 0, r$, Problem 2 is equivalent to the following LMI optimization problem:

$$\min \alpha$$

subject to:

$$\begin{bmatrix} \alpha & \mathbf{f}^T \\ \mathbf{f} & \mathbf{I} \end{bmatrix} \geq 0, \qquad \begin{bmatrix} \delta^2 & X' \\ X & I \end{bmatrix} \geq 0 \qquad (8)$$

$$X = \mathbf{y} - \left[T_{G_{0,\Delta_o}} + \sum f_i T_{G_{i,\Delta_i}} \right] \mathbf{u}$$

where $T_{G_{i,\Delta_i}}$ denotes the Toeplitz matrix associated with the impulse response of $G_i(\lambda, \Delta_i)$, $\mathbf{u} = \left[u_o^T, \ldots, u_{n-1}^T\right]^T$ and $\mathbf{y} = \left[y_o^T, \ldots, y_{n-1}^T\right]^T$.

Proof: Follows from (6) by applying a Schur complement argument to the inequalies:

$$\alpha \geq \mathbf{f}^T \mathbf{f}, \qquad \delta^2 \geq \mathbf{d}^T \mathbf{d} = X^T X \qquad (9)$$

The main difficulty with the approach outlined above is that the sets $\mathbf{\Delta}_i$ are *infinite dimensional*. However, since Δ_i are causal operators *only their first* n *Markov parameters* affect the output $\mathbf{y}$. Thus, rather than having to sample $\mathcal{BH}_\infty$, we only need to (i) sample the set $\mathcal{BH}_\infty^n$, which can be efficiently accomplished using Algorithm 1, and (ii) combine the samples [1]. This observation leads to the following robust FDI algorithm:

Algorithm 2. Given n (noisy) output measurements $\{y\}_{i=0}^{n-1}$ and nominal and failure dynamics $G(\lambda, \Delta)_i$, choose N_1 and generate $N_t(N_1)$ samples $\{\Delta^j(\lambda)\}_{j=1}^{N_t}$ from the set $\mathcal{BH}_i^r$ using Algorithm 1.

0.- Set $f_{min} = \infty$.

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[1] In the case of structured uncertainty, the same construction can be used block–wise.

1.- For each Δ^j, solve the following convex problem in f:

$$\min \|f\| \text{ subject to } \begin{bmatrix} \delta^2 & X' \\ X & I \end{bmatrix} \geq 0$$

$$X = \mathbf{y} - \left[T_{G_{0,\Delta_o^j}} + \sum f_i T_{G_{i,\Delta_i^j}} \right] \mathbf{u}$$

2.- If $\|f\| < \|f_{min}\|$ set $f_{min} = f$.
3.- Set $i = i + 1$. If $i \leq N_t$ go back to step 1.

Remark 1. Let (ϵ, ν) be two positive constants in $(0, 1)$, and, for a fixed Δ, denote by $f(\Delta)_{min}$ the minimum norm solution to the LMIs (8). Then direct application of Theorem 3.1 in (Tempo *et al.* 1996) shows that if N_1 in Algorithm 1 is chosen to satisfy

$$N_1 \geq \frac{\ln(1/\nu)}{\ln(1/(1-\epsilon))}, \tag{10}$$

then

$$\text{Prob}\left\{ \text{Prob}\left[\|f(\Delta)_{min}\|_2 < \|f_{min}^{N_1}\|_2 \right] \leq \epsilon \right\} \geq (1-\delta), \tag{11}$$

where $f_{min}^{N_1}$ denotes the solution found by Algorithm 2. Roughly speaking, with confidence $1 - \delta$, the algorithm will find, with probabilty $1 - \epsilon$, the solution to Problem 2. Moreover this bound is independent of the number of uncertainty blocks, their size and their probability distribution.

Thus, by introducing an (arbitrarily small) risk of a false alarm, we can substantially alleviate the computational complexity entailed in robustly detecting and isolating faults in plants subject to structured uncertainty and measurement noise. In addition, it can be argued that a purely deterministic approach to FDI could be potentially overly optimistic, since the system will be deemed to be operating under no-fault conditions even if there exist a *single* combination of uncertainty and noise such that the corresponding $\|f\|_2 = 0$. On the other hand, in such cases the approach proposed here will indicate, (with probability close to 1) the existence of a fault [2].

3.3 A Deterministic Convex Relaxation for Multiplicative Uncertainty

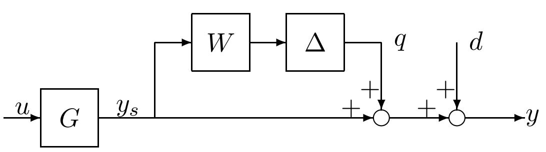

Fig. 1. Setup for Robust Analytic FDI with multiplicative uncertainty.

Consider the special case of Problem 2 shown in Figure 1, where the nominal and failure dynamics

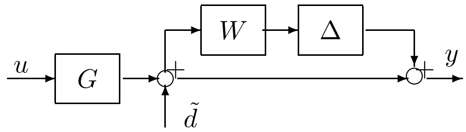

Fig. 2. Jointly convex FDI setup

are subject to *multiplicative, unstructured* uncertainty. While the problem is still not jointly convex in all the variables involved, a convex relaxation can be obtained by considering the alternative setup shown in Figure 2, where measurement noise is also affected by the unknown error dynamics Δ:

$$y = (I + \Delta W)\left[(G_0 + \sum_{i=1}^{r} f_i G_i)u + d \right] \tag{12}$$

When compared to the original setup shown in Figure 1, it can be easily seen that the only difference is in the measurement noise level. Specifically, assume that there exists a triple $(\mathbf{f}, \tilde{\mathbf{d}}, \Delta)$ satisfying (12) with $\|\tilde{\mathbf{d}}\|_2 \leq \tilde{\eta} \doteq \frac{\eta}{1+\|W\|_{\ell^2\to\ell^2}\|\Delta\|_{\ell^2\to\ell^2}}$, and let $(\mathbf{d} \doteq (1 + \Delta)\tilde{\mathbf{d}}$. Then the triple $(\mathbf{f}, \mathbf{d}, \Delta)$ satisfies

$$y = (I + \Delta W)(G_0 + \sum_{i=1}^{r} f_i G_i)u + \mathbf{d} \tag{13}$$

and $\|\mathbf{d}\|_2 \leq \eta$. Thus, one can attempt to find a solution to the original problem by searching for a solution to the model (in)validation problem shown in Figure 2, with noise level $\tilde{\eta}$. As we show in the sequel this leads to a convex optimization problem. In addition, one will expect that if $\|\Delta\| \ll 1$ then this approximation is not too conservative. This conjecture will be substantiated in section 4.

Theorem 1. There exist a feasible tripe $(\mathbf{f}, \tilde{\mathbf{d}}, \Delta)$ that satisfies equation (12) if and only if there exists at least an admissible vector $\mathbf{f}$, $0 \leq f_i \leq 1$ and a finite sequence $\mathbf{q} = \{\mathbf{q}_0, \mathbf{q}_1, \cdots, \mathbf{q}_n\}$ such that the following set of LMIs hold:

$$\begin{aligned} \mathbf{A}_1(q) &\doteq \begin{bmatrix} \mathbf{X}(\mathbf{q}) & (\mathbf{T}_\mathbf{q}^n)^T \\ \mathbf{T}_\mathbf{q}^n & \left[\frac{\mathbf{I}}{\gamma^2} - (\mathbf{T}_W^n)^T \mathbf{T}_W^n \right]^{-1} \end{bmatrix} \geq 0 \\ \mathbf{A}_2(q) &\doteq \begin{bmatrix} \eta^2 & \mathbf{Y}^T(q) \\ \mathbf{Y}(q) & \mathbf{I} \end{bmatrix} \geq 0 \end{aligned} \tag{14}$$

with:

$$\begin{aligned} \mathbf{X}(q) &\doteq (\mathbf{T}_W^n \mathbf{T}_y^n)^T \mathbf{T}_W^n \mathbf{T}_y^n - (\mathbf{T}_W^n \mathbf{T}_y^n)^T \mathbf{T}_W^n \mathbf{T}_q^n \\ &\quad - (\mathbf{T}_W^n \mathbf{T}_q^n)^T \mathbf{T}_W^n \mathbf{T}_y^n \\ \mathbf{Y}(q) &\doteq \left[\mathbf{T}_y^n - \mathbf{T}_q^n - (\mathbf{T}_{G_o}^n + \sum_i f_i \mathbf{T}_{G_i}^n)\mathbf{T}_u^n \right] \end{aligned}$$

and the matrices $\mathbf{T}_{(.)}^n$ are defined in Section 2.1.

Proof: From equation (12) we have that

$$\mathbf{T}_z^n = \mathbf{T}_W^n \left(\mathbf{T}_y^n - \mathbf{T}_q^n \right), \quad \mathbf{T}_d^n = \mathbf{T}_y^n - \mathbf{T}_q^n - \mathbf{T}_G^n \mathbf{T}_u^n \tag{15}$$

[2] see also (Zhou 2000) for a similar argument used in the context of probabilistic model (in)validation.

From Lemma 1, we have that there exists $\Delta \in \mathbf{\Delta}$ mapping the input-output sequences (z, q) if and only if

$$(\mathbf{T}_z^n)^T\mathbf{T}_z^n \geq \frac{1}{\gamma^2}(\mathbf{T}_q^n)^T\mathbf{T}_q^n. \qquad (16)$$

Combining equations (15) and (16) and using Schur complements, gives the first LMI in (14). The second LMI is a simple restatement of $\|d\|_2 \leq \eta^2$.

Remark 2. From the results above it follows that finding minimum $\|\mathbf{f}\|$ such that (12) holds reduces to a convex LMI minimization problem.

4. ILLUSTRATIVE EXAMPLE

In this section we illustrate the potential of the proposed approach using a simplified model of the yaw damper system of a jet transport(Shim and Sznaier 2003). The system under consideration is given by

$$y = (I + \Delta W)(G_0 + \sum_{i=1}^{3} f_i G_i)u + d \qquad (17)$$

where

$$G_0 = \frac{1}{D_0(s)}\begin{bmatrix} -4.75s^3 - 2.48s^2 & 1.23s^3 + 0.30s^2 \\ -1.19s - 0.56 & +0.83s + 0.42 \\ 1.15s^2 - 2.00s & 10.73s^2 \\ -13.73 & +16.43s + 10.83 \end{bmatrix}$$

$$G_1 = \frac{1}{D(s)}\begin{bmatrix} 6.08s^6 + 4.69s^4 & -1.57s^6 - 1.65s^4 \\ +1.13s^2 & -0.59s^2 \\ 1.80s^4 + 9.63s^2 & -0.57s^4 - 1.43s^2 \end{bmatrix}$$

$$G_2 = \frac{1}{D(s)}\begin{bmatrix} 6.35s^5 + 0.36s & -1.21s^5 - 0.26s \\ -1.47s^5 + 4.69s & 0.38s^5 - 3.27s \end{bmatrix}$$

$$G_3 = \frac{1}{D(s)}\begin{bmatrix} 3.03s^3 + 0.086 & -1.37s^3 - 0.06 \\ 18.54s^3 + 2.08 & -4.46s^3 - 1.55 \end{bmatrix}$$

and

$$\begin{aligned} D_0(s) &= s^4 + 1.92s^3 + 1.61s^2 + 0.83s + 0.16 \\ D(s) &= s^8 + 2.55s^7 + 3.76s^6 + 4.16s^5 + 3.18s^4 \\ &\quad + 1.71s^3 + 0.58s^2 + 0.0826s + 0.0006 \end{aligned}$$

Here the inputs to the system are rudder and aileron deflections, measured in degrees, and the outputs are yaw rate and bank angle. In order to apply our theory, a discrete–time model of the system above was obtained by using samplers and zero order holds with a sampling time of 0.1 seconds.

Assume that the only a priori information available is that the nominal and failure dynamics are subject to multiplicative model uncertainty with $\|\Delta\|_\infty \leq 0.1$, leading to the block diagram shown in Figure 1. Further assume that the energy of the measurement noise is bounded by 10% of the energy of the impulse response of the nominal plant, and that the dynamics of the actual plant (as opposed to the model used in algorithm are given by $G_{actual} = (I + \tilde{\Delta})G$ with [3] :

$$\begin{aligned} \tilde{\Delta} &= \frac{0.018}{D_\Delta}\begin{bmatrix} \Delta_{11} & \Delta_{12} \\ \Delta_{21} & \Delta_{22} \end{bmatrix} \\ \Delta_{11} &= 1.9z^4 + 2.5z^3 - 0.24z^2 - 1.04z - 0.25 \\ \Delta_{12} &= 0.5z^4 + 0.8z^3 + 0.25z^2 - 0.09z - 0.03 \\ \Delta_{21} &= 2.9z^4 + 3.0z^3 - 1.66z^2 - 2.3z - 0.51 \\ \Delta_{22} &= 3z^4 + 3.5z^3 - 1.12z^2 - 2.1z - 0.47 \\ D_\Delta &= z^4 + 1.87z^3 + 1.27z^2 + 0.37z + 0.04 \end{aligned} \qquad (18)$$

where $z = \lambda^{-1}$.

Table 1 and 2 show the results of several experiments with simulated faults for the risk–adjusted and convex relaxations respectively. For the risk–adjusted relaxation $N_t = 1500$ samples of the uncertainty where used, which guarantees, with confidence 0.99, a probability of 0.99 of finding the minimum $\|f\|_2$ that explains the experimental data. In all cases, the experimental data corresponded to 20 samples of the impulse response of $(I + \tilde{\Delta})G_f$ [4] , corrupted by noise with $\|d\|_2 = 0.50$. As shown there, both relaxations were able to establish the existence of a fault and to provide a good estimate of its indicators. Similar results, omitted for space reasons, where obtained with higher uncertainty and noise levels (both 20%). In this case, as expected, the risk–adjusted relaxation provided tighter estimates of the actual fault than the deterministic relaxation, but both procedures were able to correctly identify the presence of a fault and estimate its location.

5. CONCLUSION

In this paper we considered the problem of robust fault detection and isolation for systems described by a parameterized fault model and subject to dynamic uncertainty, entering the plant in an arbitrary way. In general this setup leads to non–convex, NP hard problems. To remove this limitation, we propose to pursue a risk–adjusted approach, where in return for an (arbitrarily small) probability of a false alarm one can obtain a substantial reduction of the computational complexity of the problem. As illustrated in the paper, the number of samples needed for reliable fault estimation is relatively small. Moreover, this number is independent of the size or number of blocks of the uncertainty and its actual probability distribution. Thus, it is feasible to generate and store these samples off–line, leading to further reduction of the computational complexity of the problem that needs to be solved on–line.

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[3] This corresponds to a randomly generated uncertainty with $\|\Delta\|_\infty \leq 0.5$.

[4] Here G_f denotes the transfer function corresponding to the failure mode under consideration.

Real Fault(RF) Mode			RF Norm	Estimated Fault(EF) Mode			EF Norm
0.0	0.0	0.0	0.0	10^{-6}*0.8263	10^{-6}*0.9219	10^{-6}*0.8964	10^{-6}*1.5285
1.0	0.0	0.0	1.0	0.8362	0.1092	0.0643	0.8458
0.0	1.0	0.0	1.0	0.0451	0.8060	0.0103	0.8073
0.0	0.0	1.0	1.0	0.0257	0.0000	0.8610	0.8614
0.8452	0.7728	0.0015	1.1452	0.7645	0.6210	0.0479	0.9861
0.4513	0.2136	0.5628	0.7524	0.4033	0.1775	0.4733	0.6466
0.1021	0.7533	0.0256	0.7606	0.1322	0.5667	0.0371	0.5831

Table 1. Estimates obtained sampling the uncertainty, with $\|\Delta\|_\infty \leq 0.1$ and 10% noise level

Real Fault(RF) Mode			RF Norm	Estimated Fault(EF) Mode			EF Norm
0.0	0.0	0.0	0.0	10^{-3}*0.0942	10^{-3}*0.1022	10^{-3}*0.0957	10^{-4}*1.6879
1.0	0.0	0.0	1.0	0.8080	0.0839	0.0492	0.8138
0.0	1.0	0.0	1.0	0.0640	0.7820	0.0119	0.7848
0.0	0.0	1.0	1.0	0.0408	0.0000	0.8399	0.8409
0.8452	0.7728	0.0015	1.1452	0.7458	0.6099	0.0627	0.9655
0.4513	0.2136	0.5628	0.7524	0.3902	0.1449	0.4687	0.6268
0.1021	0.7533	0.0256	0.7606	0.1421	0.5460	0.0425	0.5658

Table 2. Estimates obtained using the deterministic relaxation, with $\|\Delta\|_\infty \leq 0.1$ and 10% noise level.

Research is currently under way seeking to extend these results to time–varying and classes of non–linear systems.

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REFERENCES

Collins, E. and T. Song (2000). Robust $\mathcal{H}_\infty$ estimation and fault detection of uncertain dynamic systems. *J. Guidance, Control and Dynamics* **23**(5), 857–864.

Emaimi-Naeimi, A., M. M. Akhter and S. M. Rock (1998). Effect of model uncertainty on fault detection: The threshold selector. *IEEE Trans. Autom. Contr.* **33**(12), 1106–1115.

Foias, C. and A. E. Frazho (1990). *The commutant lifting approach to interpolation problems, Operator theory: Advances and Applications*. Vol. 44. Birkhäuser.

Frank, P. and X. Ding (1997). Survey of robust residual generation and evaluation in observer based fault detection systems. *J. of Process Control* **7**(6), 403–429.

Gertler, J. (1998). *Fault Detection and Diagnosis in Engineering Systems*. Marcel Dekker.

Henry, D., A. Zolghadri, F. Castang and M. Monsion (2001). A new multi-objective filter design for guaranteed robust fdi performance. In: *Proc. IEEE Conf. Dec. Contr.*. pp. 173–178.

Jiang, B., J. Wang and Y. Soh (2002). An adaptive technique for robust diagnosis of faults with independent effects on system outputs. *Int. J. Control* **75**(11), 792–802.

Khargonekar, P and A. Tikku (1996). Randomized algorithms for robust control analysis and synthesis have polynomial complexity. In: 35^{th} *IEEE Conference on Decision and Control*. Kobe, Japan. pp. 3470 – 3475.

Lagoa, C., M. Sznaier and B. R. Barmish (2001). An algorithm for generating transfer functions uniformly distributed over h_∞ balls. In: *American Control Conference*. Vol. 5. pp. 5038–5043.

Patton, R. (1994). Robust model-based fault diagnosis:the state of the art. In: *Fault detection, Supervision and Safety for Technical Processes*. pp. 1–24.

Saberi, A., A. A. Stoorvogel, P. Sannuti and H. Niemann (2000). Fundamental problems in fault detection and identification. *Int. J. Robust and Nonlinear Control* **10**(14), 1209–1236.

Shim, D. K. and M. Sznaier (2003). A caratheodory-fejer approach to simultaneous fault detection and isolation. In: *Proc. 2003 ACC*. pp. 2979–2984.

Stoustrup, J. and H. Niemann (2003). Optimal threshold functions for fault detection and isolation. In: *Proc. 2003 ACC*. pp. 1782–1787.

Tempo, R., E. W. Bai and F. Dabbene (1996). Probabilistic robustness analysis: Explicit bounds for the minimum number of sampling points. In: *Proc. IEEE Conf. Dec. Contr.*. pp. 3418–3423.

Tuan, H. D. and P. Apkarian (1999). Low nonconvex rank bilinear matrix inequalities: Algorithms and applications. In: 38^{th} *Conference on Decision and Control*. Phoenix, Arizona, USA. pp. 1001–1006.

Zhong, M., S. X. Ding, J. Lam and H. Wang (2003). An lmi approach to design robust fault detection filters for uncertain lti systems. *Automatica* **39**, 543–550.

Zhou, T. (2000). Unfalsified probability estimation for a model set based on frequency domain data. *Int. J. of Contr.* **73**(5), 391–406.