μ-ANALYSIS OF INDIRECT SELF CONTROL OF AN INDUCTION MACHINE

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Abstract: Robust stability and performance of an induction machine controlled with Indirect Self Control (ISC) are examined through μ-analysis. The analysis indicates poor robust performance at higher rotor speeds. Using reasonable parameter deviations between plant and controller, the predicted bad performance is however hard to verify through simulations. The tested parameter variations seem not to generate the worst case model errors. In simulations, the ISC therefore outperforms linear controllers optimized with respect to the robust performance criterion (at dispense of nominal performance). The large predicted sensitivity to model errors however explains the large impact on performance due to seemingly small differences in the implementation of the ISC control.
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Keywords: Motor control, control applications, stability analysis, robustness, H-infinity control.

1. INTRODUCTION

Field-oriented controllers (FOC) are frequently used non-linear controllers for induction machines. The idea with field-orientation was originally to mimic control of DC-machines for induction motors. It has later been shown that the classical field-oriented controller actually performs asymptotic linearization and decoupling (Marino, *et al.*, 1993). Stability of FOC has been investigated for example in (De Wit *et al.*, 1995) and (Bazanella and Reginatto, 2002), where focus is on robustness regarding errors in the estimate of the rotor resistance.

A different class of controllers for induction machines is formed by the so called Direct Torque Control (DTC) techniques; see (Buja and Kazmierkowski, 2004) for an overview. One method within this family, which is run at constant switching frequency, is the Indirect Self Control (ISC). With ISC, torque and stator flux magnitude are controlled by PI controllers in closed-loop, see (Jänecke *et al.*, 1989) and (Maischak, 1995). The linearized closed-loop dynamics of an induction motor drive using ISC was examined in (Mosskull, 2002), where also tuning rules for the controller parameters were given to achieve desired bandwidth and stability margins. The robustness analysis was, however, performed under the simplifying assumption of perfect knowledge of parameters, for example used for stator flux estimation. One way to include flux estimation in the robustness analysis of the closed loop drive is to apply μ-analysis, as in done for FOC in (Thomas, 1993). As the analysis is still performed on linear models (as opposed to the work in (De Wit *et al.*, 1995) and (Bazanella and Reginatto, 2002)), the results may only be valid for small perturbations around an operating point. On the other hand, not only robustness in terms of stability but also robust performance can be considered. To further evaluate the ISC, μ-synthesis controllers are designed for some fixed operating points and the performance is compared to that of the ISC.

2. INDUCTION MACHINE

The induction machine can be described by the following set of complex-valued *space vector* equations, where space vectors are indicated by superscripts *s*, (Steimel, 2000):

$$\dot{\psi}_\mu^s(t) = -R_s i_s^s(t) + u_s^s(t) \qquad (1)$$

$$\dot{\psi}_r^s(t) = \frac{R_r}{L_\sigma}\psi_\mu^s(t) + \left(jn_p\omega_m(t) - \frac{R_r}{L_\sigma}\right)\psi_r^s(t) \qquad (2)$$

Here $\psi_\mu(t)$ and $\psi_r(t)$ represent the stator and rotor fluxes, respectively, and $\omega_m(t)$ is the mechanical rotor speed. The number of pole pairs of the induction machine is denoted by n_p, whereas R_s and R_r stand for the resistance in the stator and rotor windings. Finally, the machine inductances are denoted L_μ and L_σ. This representation, where all leakage inductance is put in the rotor mesh, is called the Γ-model and can be visualized by the equivalent circuit diagram (ECD) shown in Fig. 1.

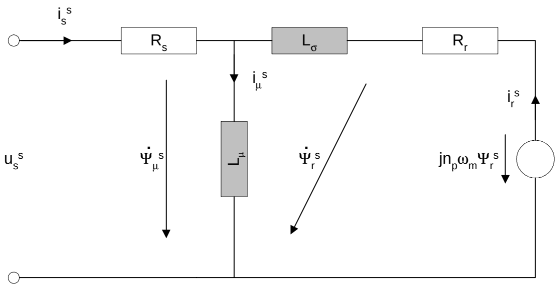

Fig. 1. Induction machine Γ-ECD.

Using a polar notation with magnitudes and angles for the fluxes, i.e.

$$\psi_\mu^s(t) = m_\mu(t)e^{j\chi_\mu(t)}, \quad \psi_r^s(t) = m_r(t)e^{j\chi_r(t)} \qquad (3)$$

the electrical torque can be expressed as, (Steimel, 2000),

$$T = \frac{3}{2}\frac{n_p}{L_\sigma} m_\mu(t) m_r(t) \sin\left(\chi_\mu(t) - \chi_r(t)\right) \qquad (4)$$

3. INPUT CONSTRAINTS

Modern drives often use voltage source inverters to feed the induction machines. The inverter converts a DC-voltage, denoted U_d, to three AC stator voltages with variable frequency and amplitude. The fundamentals of the stator voltages are upper limited by, (Kovacs, 1984),

$$m_u(t) \triangleq \left|u_s^s(t)\right| \le \frac{2}{\pi}U_d \qquad (5)$$

This is a constraint on the stator voltage amplitudes, but one should also put restrictions on the stator voltage frequency to prevent the steady state slip frequency, ω_{slip}^0, from exceeding the pull-out slip frequency (Steimel, 2000), i.e.

$$\left|\omega_{slip}^0\right| = \left|\omega_u^0 - n_p\omega_m^0\right| \le \frac{1}{T_\sigma} \qquad (6)$$

Here the stator frequency, $\omega_u(t)$, is the time derivative of the angle of the stator voltage space vector $u_s^s(t)$. This angle will be denoted $\chi_u(t)$, cf. (3). Further, $T_\sigma = L_\sigma/R_r$ is the *rotor stray time constant*.

4. LINEAR PROCESS MODEL

The induction machine is described by equations (1), (2) and (4). Inputs are the three stator voltages, represented by the magnitude and frequency of the stator voltage space vector $u_s^s(t)$, and outputs are the control variables torque and stator flux magnitude. To obtain a linear model of the process, these equations are preferably rewritten in the form of non-linear state space equations in the following way

$$\dot{m}_\mu = -R_s\left(\frac{1}{L_\mu} + \frac{1}{L_\sigma}\right)m_\mu + \frac{R_s}{L_\sigma}m_r\cos\delta + m_u\cos\delta_{u\mu} \qquad (7)$$

$$\dot{m}_r = \frac{1}{T_\sigma}m_\mu\cos\delta - \frac{1}{T_\sigma}m_r \qquad (8)$$

$$\dot{\delta}_{u\mu} = \frac{R_s}{L_\sigma}\frac{m_r}{m_\mu}\sin\delta - \frac{m_u}{m_\mu}\sin\delta_{u\mu} + \omega_u \qquad (9)$$

$$\dot{\delta} = -\left(\frac{R_s}{L_\sigma}\frac{m_r}{m_\mu} + \frac{1}{T_\sigma}\frac{m_\mu}{m_r}\right)\sin\delta + \frac{m_u}{m_\mu}\sin\delta_{u\mu} - n_p\omega_m \qquad (10)$$

where

$$\delta_{u\mu}(t) = \chi_u(t) - \chi_\mu(t), \quad \delta(t) = \chi_\mu(t) - \chi_r(t) \qquad (11)$$

From (4) and (7)-(10), a linear model **G** from stator voltage magnitude and frequency to torque and stator flux magnitude can now be derived, i.e.

$$\begin{pmatrix} T \\ m_\mu \end{pmatrix} = \mathbf{G}\begin{pmatrix} m_u \\ \omega_u \end{pmatrix} \triangleq \mathbf{G}\mathbf{u_s} \qquad (12)$$

5. SCALING

To facilitate the controller analysis, the system is scaled as proposed in (Skogestad and Postlethwaite, 1996). Introducing diagonal scaling matrices $\mathbf{D_u}$ and $\mathbf{D_e}$, with the maximum expected values of the inputs and control errors along the diagonals respectively, the induction machine transfer function changes to

$$\mathbf{G} = \mathbf{D_e^{-1}}\hat{\mathbf{G}}\mathbf{D_u} \qquad (13)$$

Here the original transfer function is denoted with a hat. The limit on the stator voltage magnitude follows from (5), and (6) will be used to scale the stator voltage frequency, although it only represents the maximum steady state frequency. Hence, by introducing the *stator frequency* ω_μ as the time derivative of χ_μ, we get (only considering upper magnitude limitation)

$$u_{1\max} \approx \max\left\{\frac{2}{\pi}U_d^0 - m_\mu^0\left|\omega_\mu^0\right|, \quad 0\right\}, \quad u_{2\max} = \frac{1}{T_\sigma} \qquad (14)$$

Here the approximation $m_u^0 \approx m_\mu^0/\omega_\mu^0/$ valid for all but very low stator frequencies was used. The matrix $\mathbf{D_e}$ is determined by the maximum control errors set to *10%* of maximum torque and rated flux (see the appendix).

6. INDIRECT SELF CONTROL (ISC)

The continuous time ISC control law is given by

$$u_{sref}^{s}(t) = \left(w_{\psi}(t) + jw_{T}(t)\right)\psi_{\mu}^{s}(t) + R_{s} i_{s}^{s}(t) \quad (15)$$

where $w_{\psi}(t) = F_{\psi}\, e_{\psi}$ is the output of a PI controller for the stator flux magnitude and $w_T(t)$ is the output of a torque controller, given by

$$\begin{aligned} w_T(t) &= n_p \omega_m(t) + \frac{2}{3}\frac{R_r}{n_p m_r^2(t)} T_{ref}(t) + \\ &+ F_T(p)\frac{2}{3}\frac{R_r}{n_p m_r^2(t)} e_T(t) \end{aligned} \quad (16)$$

Here F_T is a PI-controller and e_T is the error in torque. The factor in front of the torque reference is the steady state conversion to slip frequency. From the analysis in (Mosskull, 2002) it follows that, for zero torque, the linearized ISC control law can be represented as

$$\mathbf{u_s}(t) = \mathbf{G^{-1}}\left(\mathbf{Ce}(t) + \mathbf{C_{fw}} T_{ref}(t)\right) \quad (17)$$

where $\mathbf{e}(t) = (e_T(t)\ e_{\psi}(t))^T$ and

$$C(s) = \begin{pmatrix} \dfrac{F_T(s)}{sT_{\sigma}+1} & 0 \\ 0 & \dfrac{m_{\mu}^{0}}{s} F_{\psi}(s) \end{pmatrix}, \quad C_{fw}(s) = \begin{pmatrix} \dfrac{1}{sT_{\sigma}+1} \\ 0 \end{pmatrix} \quad (18)$$

For the μ-analysis we will focus on the feedback part of the controller, which will be denoted **K**, i.e.

$$\mathbf{K} = \mathbf{G^{-1}C} \quad (19)$$

Due to pulse width modulation (PWM), the stator voltage in (17) will be delayed by T_d s.

7. CONTROL REQUIREMENTS

The requirements on the closed loop system will be set in terms of the transfer functions **KS** and **S**, the mixed-sensitivity approach (Skogestad and Postlethwaite, 1996). The sensitivity function is given by $\mathbf{S} = (\mathbf{I}+\mathbf{GK})^{-1}$.

7.1 Input Requirements - KS.

The magnitude of the stator voltage is restricted by (5) whereas the stator frequency is only restricted in steady state. Neglecting the steady state constraint on the frequency therefore motivates the following requirement (note that the absolute values of the scaled inputs are limited by one):

$$\bar{\sigma}\left(\mathbf{W_{P1}}(j\omega)\mathbf{K}(j\omega)\mathbf{S}(j\omega)\right) < 1 \quad ,\forall \omega \quad (20)$$

where

$$\mathbf{W_{P1}}(j\omega) = \begin{pmatrix} 1 & 0 \\ 0 & 0.01 \end{pmatrix} \quad (21)$$

7.2 Performance Requirements - S

Performance requirements on the closed loop system are often given in terms of torque step response times, or equivalently, on the torque control bandwidth, ω_B. One way of assuring such a requirement is to put a constraint on the sensitivity function. Thus, we demand that

$$\bar{\sigma}\left(\mathbf{S}(j\omega)\right) < \frac{1}{\left|w_{P2}(j\omega)\right|} \quad ,\forall \omega \quad (22)$$

where

$$w_{P2}(j\omega) = \frac{\dfrac{j\omega}{M} + \omega_B}{j\omega + \omega_B Q} \quad (23)$$

The parameter Q defines tolerable values for the sensitivity function at steady state and M sets the limit at high frequencies. In this work the parameters are set to

$$M = 2,\ Q = 1e\text{-}6,\ \omega_B = 100\ rad/s. \quad (24)$$

7.3 Uncertainty

In practice there is always a time delay connected to digital controllers. The controller time delay will be modeled as an uncertainty at the plant input. That is

$$\mathbf{G} \cdot e^{-j\omega T_d} = \mathbf{G}\left(\mathbf{I} + \mathbf{W_I \Delta_I}\right) \quad (25)$$

where $\boldsymbol{W}_I = diag(w_I, w_I)$ and

$$w_I = \frac{j\omega T_{d\max}}{j\dfrac{\omega T_{d\max}}{2} + 1}, \quad \left\|\Delta_I\right\|_{\infty} < 1 \quad (26)$$

The maximum time delay, T_{dmax}, is set to *2 ms*, which is reasonable in for example traction applications.

8. MODEL FOR CONTROLLER ANALYSIS

The system can now be put on the standard form used for μ-analysis/synthesis, see Fig. 2.

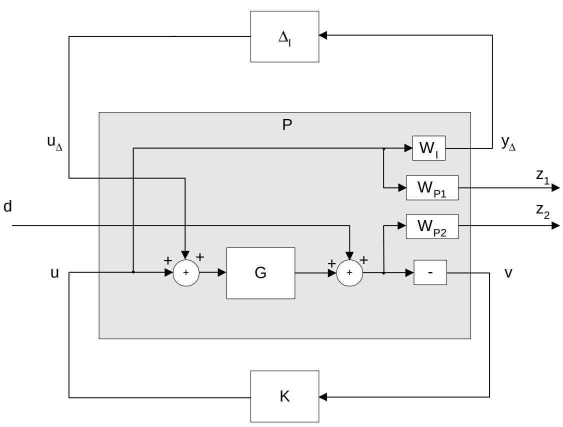

Fig. 2. Plant model for μ-analysis.

The system **P** is given by

$$\mathbf{P} = \begin{pmatrix} \mathbf{0} & \mathbf{0} & \mathbf{W_I} \\ \begin{bmatrix} \mathbf{0} \\ \mathbf{W_{p2}G} \end{bmatrix} & \begin{bmatrix} \mathbf{0} \\ \mathbf{W_{p2}} \end{bmatrix} & \begin{bmatrix} \mathbf{W_{p1}} \\ \mathbf{W_{p2}G} \end{bmatrix} \\ \mathbf{-G} & \mathbf{-I} & \mathbf{-G} \end{pmatrix} \quad (27)$$

where the following notation was introduced $\mathbf{W}_{P2} = diag(w_{P2}, w_{P2})$.

8.1 Evaluation Criteria

Controller evaluation is done by closing the loop and introducing a fictitious performance uncertainty $\mathbf{\Delta_P}$, see Fig. 3.

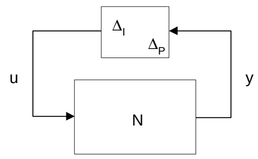

Fig. 3. Plant model for controller evaluation.

Now four criteria can be defined in terms of the structured singular value μ, (Skogestad and Postlethwaite, 1996):

Nominal Stability (NS): **N** internally stable

Nominal Performance (NP):

$$\mu_{\Delta_P}\left(\mathbf{N}_{22}\left(j\omega\right)\right) < 1 \quad ,\forall\omega \text{(and NS)} \tag{28}$$

Robust Stability (RS):

$$\mu_{\Delta_I}\left(\mathbf{N}_{11}\left(j\omega\right)\right) < 1 \quad ,\forall\omega \text{ (and NS)} \tag{29}$$

Robust Performance (RP):

$$\mu_{\Delta}\left(\mathbf{N}\left(j\omega\right)\right) < 1 \quad ,\forall\omega \text{ (and NS)} \tag{30}$$

where $\Delta = diag(\mathbf{\Delta_I}, \mathbf{\Delta_P})$.

9. CONTROLLER EVALUATION

This section shows the evaluation criteria (28)–(30) when ISC is applied to an induction machine with the data given in the appendix. The controller is tuned according to recommendations in (Mosskull, 2002) to give a bandwidth of *100 rad/s*. The analysis is done for three operating points with nominal flux and zero torque, where the stator frequency is set to *10%* (OP1), *50%* (OP2) and *90%* (OP3) of base speed ω_0. The sampling time is *0.5 ms* for all operating points and the results are presented in Fig. 4, Fig. 6 and Fig. 8. Under the assumption of perfect parameters, the analysis in (Mosskull, 2002) indicates no difference in behavior between the different operating points. This also follows from Fig. 4. The reason that the nominal performance at operating point three deviates from the first two operating points is that the maximum singular value of **KS** gets close to the limit. It however stays less than the limit. Note that this is possible as $\|(1\ 1)^T\| = \sqrt{2}$, which is larger than one. The robust stability and robust performance criteria for the ISC all show peaks at frequencies corresponding to the operating point stator frequency, see Fig. 6 and Fig. 8. This was to be expected as a linearized model of the induction motor has large relative gain array (RGA) elements at higher stator frequencies, see (Mosskull, 2004). Inverse-based controllers, such as (19), are usually not recommended for plants with large RGA elements, (Skogestad and Postlethwaite, 1996).

To further evaluate the ISC, linear μ-synthesis controllers were designed for the three operating points. The resulting structured singular values with these controllers are shown in Fig. 5, Fig. 7 and Fig. 9. The μ-synthesis design shows that it is possible to reduce the peaks of the RP singular values at dispense of nominal performance. Note that μ -synthesis is with respect to RP.

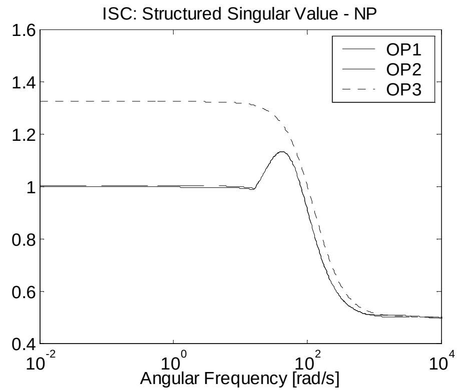

Fig. 4. Nominal performance criterion: ISC.

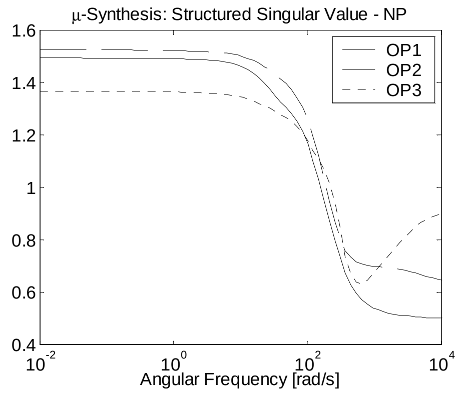

Fig. 5. Nominal performance criterion: μ-synthesis.

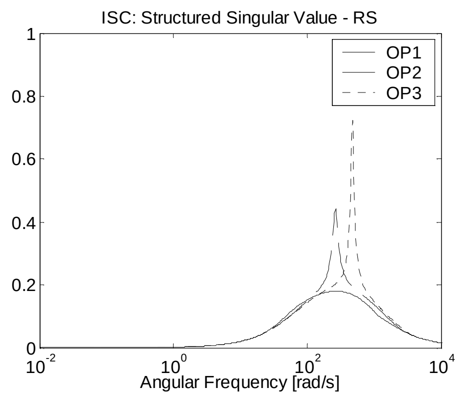

Fig. 6. Robust stability criterion: ISC.

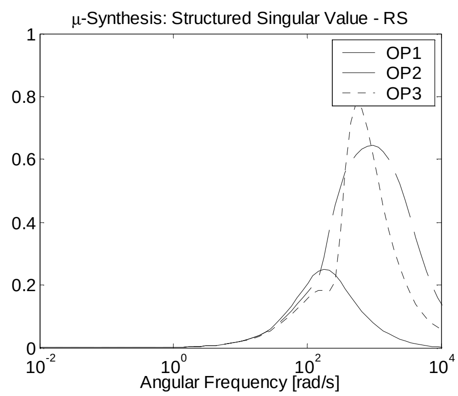

Fig. 7. Robust stability criterion: μ-synthesis.

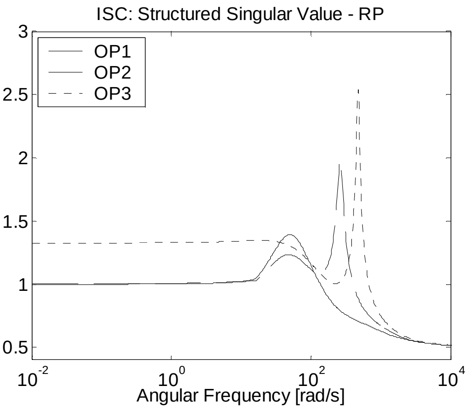

Fig. 8. Robust performance criterion: ISC.

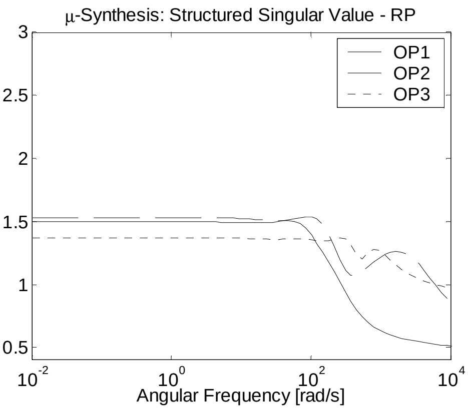

Fig. 9. Robust performance criterion: μ-synthesis.

10. SIMULATIONS

In this section some simulation results are shown to check the theoretical results obtained above. A discrete-time ISC is compared with the designed μ-synthesis controllers. The μ-synthesis controllers were discretized through zero-order holds on the inputs. The simulation consists of

- torque steps between 0 and 500 Nm
- a flux step from 100% to 90% of nominal flux at time 2.25 s

Step responses with the ISC are shown in Fig. 10 when the controller uses correct motor parameters. The step responses are scaled according to (13), i.e. one in the plots corresponds to *10%* of maximum torque and rated flux, respectively. The responses are more or less independent of the operating point and the cross-coupling is very small. There is no sign of the problems predicted by peaks in the structured singular values in Fig. 8, although the simulation loop contains time delays. The corresponding results with the μ-synthesis controller are shown in Fig. 11. We see a worse nominal performance as is indicated by Fig. 5, where especially the cross-coupling from torque reference to flux is large.

To see the effect of the peaks of the structured singular values, model errors are introduced by changing the parameters of the motor while keeping the parameters used by the controller fixed. The worst combination seems to be to increase the stator resistance and to decrease the leakage inductions of the motor. In Fig. 12 step responses at OP2 are shown where the stator resistance of the real machine is increased by a factor four and the leakage inductance is decreased by a factor four. Although these are unrealistically large model errors, the results are still not too bad with the ISC. As a matter of fact, the corresponding simulation with the μ-synthesis controller is unstable.

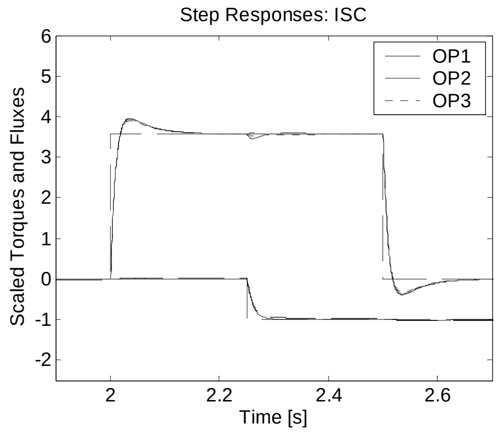

Fig. 10. Step responses with ISC.

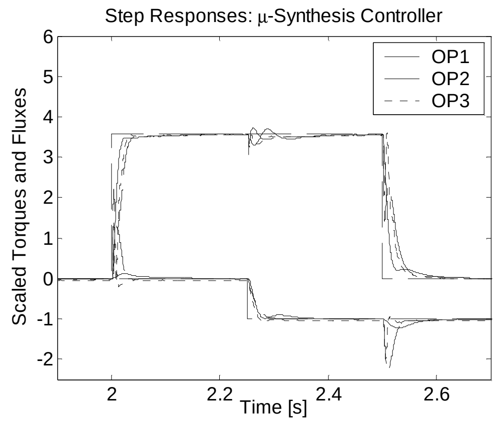

Fig. 11. Step responses with the μ-synthesis controller.

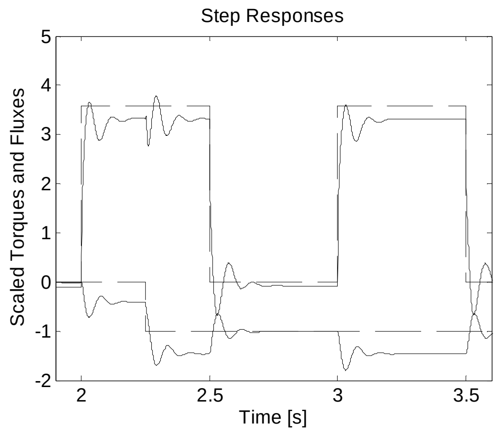

Fig. 12. Step responses with the ISC where the stator resistance is four times larger and the leakage inductance four times smaller compared to the values used by the controller.

Actually, the discrete time ISC proposed by (Maischak, 1995) uses the reference magnitude for the stator flux explicitly shown in (15) instead of the actual magnitude. The simulations as well as the analysis performed above have considered the case with the actual flux magnitude. By strictly following (Maischak, 1995), the simulation results in the ideal case with perfect motor parameters are shown in Fig. 13. Compared to Fig. 10 we see a very large disturbance in torque at a step in the flux reference. Although the deviation in flux is small (note only magnitude deviation), the error in torque is large and the error increases with the operating point rotor speed. This fits with the results of Fig. 8. In this case however the error is not caused by erroneous parameters but caused by an approximation in the control law, which gives an error in the magnitude of the stator flux.

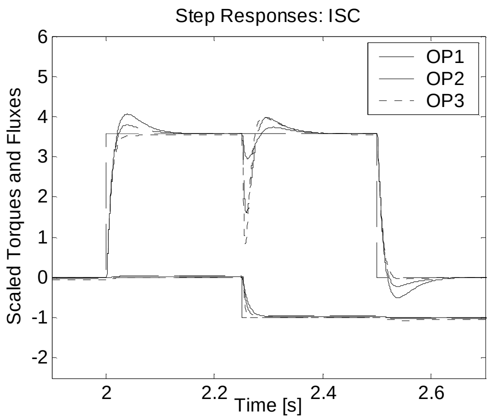

Fig. 13. Step responses with ISC as proposed in (Maischak, 1995).

11. CONCLUSION

In an attempt to properly analyze robustness of Indirect Self Control (ISC) of an induction machine, μ-analysis was applied to linearized models of the system. The analysis indicates decreasing robust stability as well as robust performance with increasing rotor speed. Problems are to be expected for frequencies around the operating point stator frequency. At simulations with different parameters of the induction machine and the controller, the ISC still performed surprisingly well. Even with large model errors it outperformed μ-synthesis controllers designed for optimal robust performance. It was concluded that the worst case model errors, giving large structured singular values during analysis, were not obtained by the tried variations of motor parameters. However, by slightly modifying the ISC control law, to use the reference stator flux magnitude instead of the actual value, small steps in the flux reference were shown to have large impact on the torque.

Compared to the analysis for classical field-oriented controllers, given in (Thomas, 1993), the structured singular values for ISC show sharper peaks. Otherwise, control methods relying on stator flux estimation, such as DTC techniques, are considered more robust to parameter variations compared to field-oriented controllers orienting to rotor flux. Under ideal conditions, stator flux estimation only depends on one motor parameter, the stator resistance; whereas rotor flux estimation involves more motor parameters, see e.g. (Xu *et al.*, 1993).

APPENDIX: Γ–MODEL MOTOR DATA

Stator resistance	R_s	=	18.5mΩ
Rotor resistance	R_r	=	17.3mΩ
Stator inductance	L_μ	=	6.2mH
Leakage inductance	L_σ	=	0.79mH
Number of pole pairs	p	=	2
Rated DC-link voltage	U_0	=	750V
Base speed	ω_0	=	528rad/s
Rated flux	Ψ_0	=	0.9Vs
Maximum torque	T_{max}	=	1400Nm

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