CASCADED NONLINEAR RECEDING-HORIZON CONTROL OF INDUCTION MOTORS

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Abstract: In this paper the optimal nonlinear receding-horizon control structure is presented with application to induction motors in cascade structure, which provides global asymptotic tracking of smooth speed and flux trajectories. The controller is based on a finite horizon continuous time minimization of nonlinear tracking errors. Both the rotor flux and load torque are estimated by Kalman filter. The robustness properties with respect to electrical parameter variations and load disturbance are presented. Finally computer simulations show the flux-speed tracking performances and the disturbance rejection capabilities of the proposed controller in the nominal and mismatched parameters case.

Keywords: Nonlinear continuous receding-horizon control, Cascaded structure, Robust control, Induction motor.

1. INTRODUCTION

Induction motors are the most widespread systems in industrial application, but they represent a highly coupled and nonlinear multivariable system. In recent years, to increase performance of classical control, e.g. field oriented torque control (Novotny and Lipo, 1996), many control strategies have been proposed to achieve better dynamic performance and induction motors have been gradually replacing the DC motors. Among these control strategies, typical approaches include input-output linearization (Bodson *et al.*, 1994; Chiasson, 1993). Marino *et al.* (1999) have proposed a speed/torque and flux tracking adaptive controller without measurements of the rotor fluxes or load torque while adapting to the changing rotor resistance. This paper examines the nonlinear receding-horizon control approach based on a finite horizon dynamic minimization of tracking errors, to achieve torque and rotor flux amplitude tracking objectives. In the application context of motion control (robotics, machine tool), an extension to speed control is realized with a cascaded structure. This is a slightly modified version of the Ping's method (Ping, 1998). Note that this approach cannot be applied to induction motor since the derivative of the control signal will appear in the cost function. The advantages of the proposed control law include good tracking performance and good robustness property with respect to both load torque and resistances variations. It is noted that when the field oriented control was used, the variation in load torque and rotor resistance deteriorate the transient response. Moreover, the flux weakening operation cannot be done in field-oriented control since this operation will excite the coupling between flux and speed, causing undesirable speed fluctuation and perhaps instability (Bodson *et al*, 1994). However, with the proposed control strategy, the weakening operation has no effect on the speed behavior.

The paper is organized as follows. After the mathematical model of the induction motor developed in section 2, a brief overview of the optimal nonlinear receding-horizon control theory is presented in section 3. In section 4 we extend the previous scheme to speed control by a cascaded nonlinear control structure. Significant simulation results are given in section 5 for the nominal and mismatched model of the induction motor with load disturbance. The paper ends up with the concluding remarks and suggestions in section 6.

2. MATHEMATICAL MODEL

An induction motor is built up around three stator windings and three rotor windings. Using the Park's transformation, a two phases equivalent machine representation with two rotor windings and two stator windings is obtained. In this paper, the stator fixed α-β frame is chosen to represent the model of the machine and under the assumption of equal mutual inductance and linear magnetic circuit, the dynamics of the induction motor are given by a fifth-order model, (Hedjar *et al.*, 2004; Novotny and Lipo, 1996)

$$\dot{\mathbf{x}} = \mathbf{f}(\mathbf{x}) + \mathbf{g}\,\mathbf{u} \tag{1}$$

With: $\mathbf{x} = \left[i_{s\alpha}\ i_{s\beta}\ \varphi_{r\alpha}\ \varphi_{r\beta}\ \Omega\right]^{\mathrm{T}}$

$\mathbf{u} = \left[u_{s\alpha}\ \ u_{s\beta}\right]^{\mathrm{T}}$

Where: $i_{s\alpha},\ i_{s\beta}$: stator currents,

$\phi_{r\alpha},\ \phi_{r\beta}$: rotor fluxes,

Ω : speed,

$u_{s\alpha},\ u_{s\beta}$: stator voltages.

Vector function $\mathbf{f}(\mathbf{x})$ and constant matrix $\mathbf{g}$ are defined as follows:

$$\mathbf{f}(\mathbf{x}) = \begin{bmatrix} -\gamma\, i_{s\alpha} + \dfrac{k}{T_r}\varphi_{r\alpha} + p\,\Omega\, k\,\varphi_{r\beta} \\ -\gamma\, i_{s\beta} + \dfrac{k}{T_r}\varphi_{r\beta} - p\,\Omega\, k\,\varphi_{r\alpha} \\ \dfrac{L_m}{T_r} i_{s\alpha} - \dfrac{1}{T_r}\varphi_{r\alpha} - p\,\Omega\,\varphi_{r\beta} \\ \dfrac{L_m}{T_r} i_{s\beta} - \dfrac{1}{T_r}\varphi_{r\beta} + p\,\Omega\,\varphi_{r\alpha} \\ p\dfrac{L_m}{JT_r}\left(\varphi_{r\alpha}\, i_{s\beta} - \varphi_{r\beta}\, i_{s\alpha}\right) - \dfrac{\left(T_L + f\,\Omega\right)}{J} \end{bmatrix}$$

$$\mathbf{g} = \left[g_1\ \ g_2\right] = \begin{bmatrix} \dfrac{1}{\sigma\, L_s} & 0 & 0\ 0\ 0 \\ 0 & \dfrac{1}{\sigma\, L_s} & 0\ 0\ 0 \end{bmatrix}^{\mathrm{T}}$$

All required parameters above have the following meanings:

$$\sigma = 1 - \frac{L_m^2}{L_s L_r}\ ;\ k = \frac{L_m}{\sigma L_s L_r}\ ;\ \gamma = \frac{1}{\sigma L_s}\left(R_s + R_r\,\frac{L_m^2}{L_r^2}\right)$$

Where: L_s, L_r are stator and rotor inductances,
L_m is the mutual inductance,
R_s, R_r are stator and rotor resistances,
$T_r = L_r/R_r$ is the rotor time constant,
p is the pole pair number,
J is the inertia of the machine,
f is the friction coefficient,
T_L is the load torque considered as an unknown disturbance.

Considering the torque and squared rotor flux modulus as outputs of the a.c. drive, the following equations can be derived, with y_1 as the torque and y_2 as the rotor flux norm:

$$\begin{cases} y_1 = h_1(\mathbf{x}) = p\dfrac{L_m}{L_r}\left(\varphi_{r\alpha} i_{s\beta} - \varphi_{r\beta} i_{s\alpha}\right) \\ y_2 = h_2(\mathbf{x}) = \varphi_{r\alpha}^2 + \varphi_{r\beta}^2 = \varphi_r{}^2 \end{cases} \tag{2}$$

It assumed that only stator currents ($i_{s\alpha}$, $i_{s\beta}$) and rotor speed (Ω) are measurable. Rotor fluxes ($\phi_{r\alpha}$, $\phi_{r\beta}$) and load torque (T_L) are estimated by Kalman Filter.

3. NONLINEAR RECEDING-HORIZON CONTROL LAW

In the receding-horizon control strategy, the following control problem is solved at each $t > 0$:

$$\begin{aligned} \underset{u(t)}{Min}\, J(\mathbf{x}(t), t, \mathbf{u}(t)) &= \frac{1}{2}\int_t^{t+T} L(\tau)\, d\tau \\ &= \underset{u(t)}{Min}\frac{1}{2}\int_t^{t+T}\left[\mathbf{x}(\tau)^T\,\mathbf{Q}\,\mathbf{x}(\tau) + \mathbf{u}(\tau)^T\,\mathbf{R}\ \ \mathbf{u}(\tau)\right] d\tau \end{aligned} \tag{3}$$

subject to the equation (1) and $x(t+T) = 0$ for $T>0$, where $\mathbf{Q}$ is positive definite and $\mathbf{R}$ positive semi-definite. To solve a nonlinear dynamic optimization problem with equality constraints is highly computationally intensive, and in many cases it is impossible to be performed within a reasonable time limit, especially for systems with very fast dynamics like induction motor. Furthermore, the global optimization solution cannot be guaranteed in each optimization procedure since, in general, it is a non-convex, constrained nonlinear optimization problem.

To avoid the computational burden, we shall approximate the above receding- horizon control problem by Simpson's rule:

$$\begin{aligned} J &= \frac{1}{2}\int_t^{t+T} L(\tau)\, d\tau = \frac{T}{6}\left[L(t) + 4L\left(t + \frac{T}{2}\right) + L(t+T)\right] \\ &= \frac{h}{3}\left[L(t) + 4L(t+h) + L(t+2h)\right] \end{aligned}$$

with $T = 2\,h$ is the prediction horizon.

In order to find the current control that improves tracking error along a fixed interval, the output tracking error $\mathbf{e}(\tau)$ is used instead of the state vector $\mathbf{x}(\tau)$ in the above receding control problem. The above performance index can be written as:

$$\begin{aligned} J = \frac{h}{3}\Big[&\mathbf{e}^{\mathrm{T}}(t)\,\mathbf{Q}\,\mathbf{e}(t) + \mathbf{u}^{\mathrm{T}}(t)\mathbf{R}\,\mathbf{u}(t) \\ &+ 4\mathbf{e}^{\mathrm{T}}(t+h)\mathbf{Q}\,\mathbf{e}(t+h) + 4\mathbf{u}^{\mathrm{T}}(t+h)\mathbf{R}\,\mathbf{u}(t+h) \\ &+ \mathbf{e}^{\mathrm{T}}(t+2h)\,\mathbf{Q}\,\mathbf{e}(t+2h) + \mathbf{u}^{\mathrm{T}}(t+2h)\,\mathbf{R}\,\mathbf{u}(t+2h)\Big] \end{aligned} \tag{4}$$

Where $L(\tau) = \mathbf{e}^{\mathrm{T}}(\tau)\,\mathbf{Q}\,\mathbf{e}(\tau) + \mathbf{u}^{\mathrm{T}}(\tau)\,\mathbf{R}\,\mathbf{u}(\tau)$.

Thus, the problem consists in elaborating a control law $\mathbf{u}(\mathbf{x},t)$ that improves tracking accuracy along the interval $\left[t; t+T\right]$, such that $\mathbf{y}(t+T)$ tracks $\mathbf{y}_{ref}(t+T)$. Note that the desired output trajectory is specified by a smooth function $\mathbf{y}_{ref}(t+T)$ for $t \in \left[t_0\,; t_f\right]$. That is, the tracking error is defined by:

$$\mathbf{e}(t+T) = \mathbf{y}(t+T) - \mathbf{y}_{ref}(t+T)$$

A simple and effective way of predicting the influence of $\mathbf{u}(t)$ on $\mathbf{y}(t+T)$ is to expand it into a r_i^{th} order Taylor series expansion, in such a way to obtain, for each component of the vectors:

$$y_i(t+T) = h_i(t) + TL_f h_i + \frac{T^2}{2!} L_f^2 h_i + + \dots\dots\dots + \frac{T^{r_i}}{r_i!} L_f^{r_i} h_i + \frac{T^{r_i}}{r_i!} L_g L_f^{r_i} h_i \, u \quad \text{for } i = 1,..,m \quad (5)$$

Where $L_f^k \, h_i$ denote the k^{th} order Lie derivative of h_i with respect to $\mathbf{f}(\mathbf{x})$. r_i is the relative degree of the output y_i, defined to be the nonnegative integer j such that the j^{th} derivative of y_i along the trajectory of equation (1) explicitly depends on $\mathbf{u}(t)$ for the first time.

The expansion of the motor outputs $\mathbf{y}(t+T)$ in a r^{th} (with $r_1 = 1$ and $r_2 = 2$) order Taylor series in compact form is:

$$\mathbf{y}(t+T) = \mathbf{y}(t) + \mathbf{V}_y(\mathbf{x},T) + \mathbf{\Lambda}(T)\, \mathbf{W}(\mathbf{x})\, \mathbf{u}(t) \quad (6)$$

Where:

$$\mathbf{y}(t) = \begin{vmatrix} y_1 \\ y_2 \end{vmatrix}; \quad \mathbf{W}(\mathbf{x}) = \begin{vmatrix} L_{g_1} h_1 & L_{g_2} h_1 \\ L_{g_1} L_f h_2 & L_{g_2} L_f h_2 \end{vmatrix}$$

$$\mathbf{\Lambda}(T) = \begin{vmatrix} T & 0 \\ 0 & \dfrac{T^2}{2} \end{vmatrix}; \quad \mathbf{V}_{\mathbf{y}}(\mathbf{x},T) = \begin{vmatrix} TL_f h_1 \\ TL_f h_2 + \dfrac{T^2}{2} L_f{}^2 h_2 \end{vmatrix}.$$

Similarly, $\mathbf{y}_{ref}(t+T)$ may be expanded in a same r^{th} order Taylor series:

$$\mathbf{y}_{ref}(t+T) = \mathbf{y}_{ref}(t) + \mathbf{d}(t,T) \quad (7)$$

Where:

$$\mathbf{y}_{ref}(t) = \begin{vmatrix} y_{ref_1} \\ y_{ref_2} \end{vmatrix}, \quad \mathbf{d}(t,T) = \begin{vmatrix} T\, \dot{y}_{ref_1} \\ T\, \dot{y}_{ref_2} + \dfrac{T^2}{2} \ddot{y}_{ref_2} \end{vmatrix}$$

The tracking error at the next instant $(t+T)$ is then predicted as function of the input $\mathbf{u}(t)$ by:

$$\begin{aligned} \mathbf{e}(t+T) &= \mathbf{y}(t+T) - \mathbf{y}_{ref}(t+T) = \\ &= \mathbf{e}(t) + \mathbf{V}_y(\mathbf{x},T) - \mathbf{d}(t,T) + \mathbf{\Lambda}(T)\mathbf{W}(\mathbf{x})\, \mathbf{u}(t) \end{aligned} \quad (8)$$

By using the predicted tracking error equation (6), at the time $T = h$ and $T = 2h$, the performance index (4) can be written in the conventional quadratic form:

$$\bar{J} = \frac{3}{2h} J = \frac{1}{2} \mathbf{U}^T \mathbf{P}(\mathbf{x},h)\mathbf{U} + \mathbf{U}^T \mathbf{G}(\mathbf{x}) + m(\mathbf{e},\mathbf{Q},h)$$

Where: $\mathbf{U}(t) = \left| \mathbf{u}(t) \;\; \mathbf{u}(t+h) \;\; \mathbf{u}(t+2h) \right|^{\mathrm{T}}$;

$$\mathbf{P}(\mathbf{x},h) = diag\left(\mathbf{R} + \mathbf{W}^{\mathrm{T}}(\mathbf{x})\, \mathbf{K}(\mathbf{x},h)\, \mathbf{W}(\mathbf{x}), 4\mathbf{R}, \mathbf{R} \right)$$

$$\mathbf{G}^{\mathrm{T}}(\mathbf{x}) = \left| \mathbf{W}^{\mathrm{T}}(\mathbf{x}) \left(\mathbf{\Gamma}(h)\, \mathbf{Q}\, \mathbf{e} + \mathbf{Z}(\mathbf{x},h) \right) \; 0 \;\; 0 \right|;$$

$$\mathbf{\Gamma}(h) = 4\, \mathbf{\Lambda}(h) + \mathbf{\Lambda}(2h);$$

$$\mathbf{K}(\mathbf{Q},h) = 4\mathbf{\Lambda}(h)\, \mathbf{Q}\, \mathbf{\Lambda}(h) + \mathbf{\Lambda}(2h)\, \mathbf{Q}\; \mathbf{\Lambda}(2h);$$

$$\begin{aligned} \mathbf{Z}(\mathbf{x},h) = {} & 4\mathbf{\Lambda}(h)\, \mathbf{Q} \left(\mathbf{V}_y(\mathbf{x},h) - \mathbf{d}(t,h) \right) + \\ & \mathbf{\Lambda}(2h)\, \mathbf{Q} \left(\mathbf{V}_y(\mathbf{x},2h) - \mathbf{d}(t,2h) \right) \end{aligned};$$

$m(\mathbf{e},\mathbf{Q},h)$ are terms that are independent of $\mathbf{U}(t)$.

The minimization of $\bar{J}$ with respect to $\mathbf{U}(t)$, by setting $\partial \bar{J} / \partial \mathbf{U} = 0$, yields to the optimal control :

$$\mathbf{U}(t) = -\mathbf{P}^{-1}(\mathbf{x},h)\, \mathbf{G}(\mathbf{x},h) \quad (9)$$

The applied control signal to nonlinear system at time t is given by:

$$\begin{aligned} \mathbf{u}(t) = & -\left[\mathbf{R} + \mathbf{W}^{\mathrm{T}}(\mathbf{x})\, \mathbf{K}(\mathbf{x},h)\, \mathbf{W}(\mathbf{x}) \right]^{-1} \mathbf{W}^{\mathrm{T}}(\mathbf{x}) \times \\ & \left(\mathbf{\Gamma}(h)\, \mathbf{Q}\, \mathbf{e} + \mathbf{Z}(\mathbf{x},h) \right) \end{aligned} \quad (10)$$

We notice that the previous output-tracking control law only affects the torque (y_1) and the rotor flux (y_2). In the induction machine, the aim is to control speed and flux, thus an extension to speed control is achieved, in the next section, looking at a cascaded nonlinear receding-horizon control structure.

4. CASCADED STRUCTURE OF THE NONLINEAR RHC

Cascaded control (Boucher *et al.*, 1996) is typically prescribed for linear systems involving time-scale separation assumption. That is, the inner loop is designed to have a faster dynamic than the outer loop. In this paper, the nonlinear continuous receding-horizon control scheme is extended to control speed by using the cascaded structure (fig.1). The mechanical equation of the motor is given by:

$$\dot{\Omega}(t) = \frac{1}{J} y_1(t) - \frac{f}{J} \Omega(t) - \frac{1}{J} T_L \quad (11)$$

Note that the load torque T_L will induce a steady state error in the external loop, therefore a linear observer is used to estimate it and it is noted $\hat{T}_L$.

The equation (11) allows controlling the speed by acting on the torque y_1. Thus, the initial system can be decomposed into two sub-systems in a cascaded form (Figure 1). The inner loop incorporates torque-flux model and the external loop is the velocity transfer function deduced from the mechanical equation given above.

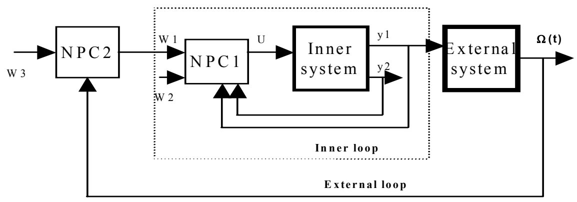

Fig.1. Cascaded control configuration.

The desired reference models, chosen in continuous time, are given by:

- For the torque trajectory (y_1):

$$\frac{y_{ref_1}(s)}{w_1(s)} = \frac{\omega_0}{s + \omega_0}$$

- For the flux trajectory (y_2):

$$\frac{y_{ref_2}(s)}{w_2(s)} = \frac{\omega_f{}^2}{s^2 + 2\xi_f \omega_f s + \omega_f{}^2}$$

- For the velocity trajectory (Ω):

$$\frac{\Omega_{ref}(s)}{w_3(s)} = \frac{\omega_v^2}{s^2 + 2\xi_v\omega_v s + \omega_v^2}.$$

Assuming that the torque y_1 tracks the reference signal $\mathbf{y}_{ref1}$, the global prediction model of the external loop is calculated, including the torque closed loop, in the following manner:

$$\Omega(s) = \frac{1}{Js+f} y_1(s) \approx \frac{\omega_0}{(Js+f)(s+\omega_0)} w_1(s)$$

Then from the above equations, the predicted tracking error of the rotor speed can be expressed by:

$$e_v(t+T) = e_v(t) + V_v(\Omega,T) - d_v(t,T) + W_v(T)W_1(t) \quad (12)$$

Where: $e_v(t) = \Omega(t) - \Omega_{ref}(t)$,

$$V_v(\Omega,T) = \frac{f}{J}\left(\frac{5f}{2J}T^2 - 3T\right)\Omega + \frac{1}{J}\left(3T - \frac{5}{2}T^2\left(\frac{f}{J} + w_0\right)\right)y_{ref1} + \frac{1}{J}\left(\frac{f}{2J}T^2 - T\right)\hat{T}_L$$

$d_v(t,T) = T\dot{\Omega}_{ref} + \frac{T^2}{2}\ddot{\Omega}_{ref}$ and $W_v(T) = \frac{w_0}{2J}T^2$.

The control objective is the tracking of Ω to a desired reference Ω_{ref} and the tracking of y_1 and y_2 to desired reference signals $\mathbf{y}_{ref1}$ and $\mathbf{y}_{ref2}$. The performance indexes for the nonlinear system are:

- Inner loop:

$$J_1 = \frac{1}{2}\int_t^{t+T} L(\tau)\,d\tau = \frac{1}{2}\int_t^{t+T} \mathbf{e}^T(\tau)\mathbf{Q}\,\mathbf{e}(\tau) + \mathbf{u}^T(\tau)\mathbf{R}\,\mathbf{u}(\tau)\,d\tau \quad (13)$$

- External loop:

$$J_2 = \frac{1}{2}\int_t^{t+T_v} L_v(\tau)\,d\tau = \frac{1}{2}\int_t^{t+T_v} q_v\, e_v^2(\tau) + r_v\, W_1^2(\tau)\,d\tau = \frac{h_v}{3}\left|L_v(t) + 4\,L_v(t+h_v) + L_v(t+2h_v)\right| \quad (14)$$

Also in this case, the cost function (14) can be expressed in quadratic form:

$$J_2 = \frac{h_v}{3}\left| m_v(e_v, q_v, h_v) + \overline{\mathbf{W}}_1^T \mathbf{G}_v(\Omega,h_v) + \frac{1}{2}\overline{\mathbf{W}}_1^T \mathbf{P}_v(\Omega,h_v)\overline{\mathbf{W}}_1 \right|$$

Where:

$\overline{\mathbf{W}}_1(t) = \left|W_1(t)\ \ W_1(t+h_v)\ \ W_1(t+2h_v)\right|^{\mathrm{T}}$,

$\mathbf{G}_v(\Omega,h_v) = \left|4q_v W_v(h_v)\left(2e_v + K_v - \overline{d}\right)\ 0\ 0\right|^{\mathrm{T}}$,

$\mathbf{P}_v(\Omega,h_v) = diag\left(r_v + 20\ q_v\ W_v^2(h_v), 4r_v, r_v\right)$;

$K_v(\Omega,h_v) = V_v(\Omega,h_v) + V_v(\Omega,2h_v)$,

$\overline{d}(\Omega,h_v) = d_v(t,h_v) + d_v(t,2h_v)$.

From the minimization of the performance indexes (J_1 and J_2), and by taking only the control signal applied at time t (receding-horizon principle), we obtain:

- For the external loop:

$$W_1(t) = -\frac{4\,q_v W_v\left(2\,e_v + K_v(\Omega,h_v) - \overline{d}_v(\Omega,h_v)\right)}{r_v + 20\,q_v\,W_v^2(h_v)}$$

$$W_2 = \phi_{rnom}, \quad (15)$$

- For the inner loop:

$$\mathbf{u}(t) = -\left[\mathbf{R} + \mathbf{W}^{\mathrm{T}}(\mathbf{x})\,\mathbf{K}(\mathbf{x},h)\,\mathbf{W}(\mathbf{x})\right]^{-1}\mathbf{W}^{\mathrm{T}}(\mathbf{x})\times \left(\mathbf{\Gamma}(h)\,\mathbf{Q}\,\mathbf{e} + \mathbf{Z}(\mathbf{x},h)\right) \quad (16)$$

Tracking performance:

- For the external loop: the equation (15) with $r_v = 0$, gives using the second order derivative of Ω the following speed tracking error dynamics:

$$\ddot{e}_v + \frac{6}{5h_v}\dot{e}_v + \frac{2}{5h_v^2}e_v = \frac{1}{J}\left(\frac{f}{J} - \frac{6}{5h_v}\right)\left(T_L - \hat{T}_L\right) \quad (17)$$

- For the internal loop: we assume that $\mathbf{W}(\mathbf{x})$ has a full rank. Let $\mathbf{Q} = q\,\mathbf{I}_2$, $\mathbf{R} = 0$ in the controller (16), we obtain:

$$\mathbf{u}(t) = -\mathbf{W}(\mathbf{x})^{-1}\mathbf{K}(\mathbf{Q},h)^{-1}\left(\mathbf{\Gamma}(h)\,\mathbf{Q}\,\mathbf{e}(t) + \mathbf{Z}(\mathbf{x},h)\right)$$

Differentiating the output y_1 one time and the output y_2 twice and by using the above control equation, we can show that the tracking errors dynamics are:

- For the torque:

$$\dot{e}_1(t) + \frac{3}{2h}e_1(t) = 0 \quad (18)$$

- For the flux:

$$\ddot{e}_2(t) + \frac{6}{5h}\dot{e}_2(t) + \frac{2}{h^2}e_2(t) = 0 \quad (19)$$

The above dynamics equations are linear and time invariant. Thus, the proposed tracking controller design technique leads to feedback linearization and we can easily verify the asymptotic stability of the tracking errors dynamics of the overall system. Moreover, if $\lim_{t\to\infty}\hat{T}_L \to T_L$ then the speed tracking error $e_v(t)$ converges towards the origin.

Zero dynamics: The error dynamics (17), (18) and (19) are linear and time invariant. Thus, the relative degrees are respectively 1 and 2. The sum is three, leading to a two order unobservable dynamics. By using the method given in (Bodson et *al.*, 1994; Hedjar et *al.*, 2004), we can show that these two zeros dynamics are stable provided the decoupling matrix $\mathbf{W}(\mathbf{x})$ is nonsingular. Notice that the decoupling matrix is only singular at the start up ($\det \mathbf{W}(\mathbf{x}) = -2pkR_r(\varphi_{r\alpha}^2 + \varphi_{r\beta}^2)$), this singularity can be avoided by using a flux observer with initial condition $\hat{\phi}(0) \neq 0$.

Flux and load torque observer: Rotor fluxes are difficult to measure and several papers are devoted to this problem. Since Kalman filter is less sensitive to noise and model inaccuracy. Thus, it has a good behavior in the presence of resistance variations. The

recursive form of the Kalman filter used in this paper is derived from the electrical equations of the induction motor model (1) (Hedjar et *al.*, 2004).
For the load torque estimator, by using the mechanical equation (11) one can estimate the load torque by a simple Kalman-Bucy filter with the rotor speed as the output and the estimated mechanical torque $\hat{y}_1$ as the input.

5. SIMULATION RESULTS

Computer simulations have been performed to check the behaviour of the proposed controller. The plant under control is a 1.1 kW induction machine used in (Ortega *et al.*, 2000) with the following parameters:

$R_r = 3.6\ \Omega$, $R_s = 8\ \Omega$, $L_r = 0.47$ H,

$L_s = 0.47$ H, $L_m = 0.452$ H, $J = 0.015$ kgm^2,

$p = 2$, $f = 0.005$, $T_{nom} = 5$N,

$\Omega_{nom} = 73.3$ rad/s $\phi_{r\alpha\beta} = 1.14$ Wb.

The parameters values of the three reference models are chosen as follows:

$\xi_f = 1$, $\omega_f = 25$ rad/s for the flux trajectory

$\xi_v = 1$, $\omega_v = 10$ rad/s for the speed trajectory

$\omega_0 = 45$ rad/s for the torque trajectory.

To examine the flux and the speed tracking performances, it was considered that speed must reach the value $\Omega = 70$ rad/s in the interval of time 0.02-2 s; and $\Omega = 140$ rad/s in the interval 2-4 s; and $\Omega = 60$ rad/s for $t > 4$. The flux must reach the nominal value $\phi_{nom} = 1.14$ Wb in the interval of time 0-2s. As it is stated in (Bodson et *al.*, 1994; Marino et *al.*, 1999), the flux reference will need to be reduced from the nominal value as the speed reference is increased above the rated speed in order to keep the required field voltages within the limits. Thus, the flux is reduced to 0.5 Wb in the interval 2-4s. To test the disturbance rejection, a 5 Nm unknown load torque is applied between $t = 0.8$ s and $t = 1.4$ s, afterwards it is decreased to 2Nm. All initial conditions of the motor are set to zero except for the flux observer: $\phi_r(0) = 0.02$ Wb.

After trials, the control parameters are chosen as:

$\mathbf{Q} = 10^4\, \mathbf{I}_2$, $\mathbf{R} = 10^{-3}\, \mathbf{I}_2$, $h = 0.001$,

$q_v = 100$, $h_v = 0.001$, $r_e = 0.0001$.

Figure 2 shows that the behavior of the actual rotor flux is very close to the flux reference. It also appears that the rotor speed fits to the speed reference trajectory. The applied load torque has no effect on the flux and its effect on the speed is rapidly compensated since it is well estimated (Figure 4). Figure 3 depicts the variations of the admissible stator voltage ($u_{s\alpha}$, $u_{s\beta}$) and the stator current i_s that is also admissible, within the saturations limits (Ortega et *al.*, 2000).

In the mismatched case, Figure 5 shows the resistances variations (R_r and R_s), the load torque and the induced rotor time constant variation. The simulation results on Figure 6 shows that a good tracking performance is achieved and the above results demonstrate that the proposed controller has strong robustness properties in the presence of load disturbance and parameter variations.

Consequently, the use of the proposed feedback nonlinear receding-horizon scheme under cascade structure can solve the control problem of induction machines in the presence of uncertainties in load torque and resistance parameters variations without rotor resistance estimation. We remind that when a decoupling control algorithm or field oriented control is used, the variation in load torque and rotor resistance R_r causes the loss of input-output decoupling property and this can deteriorate the transient response. This calls for adaptive version of the algorithm where the convergence of the estimator is under persistency excitation of the induction machine (Marino et *al.*, 1999). Moreover, in order to keep the required field voltages within the limits, the flux reference was reduced from its nominal value as the speed reference was increased above the rated speed. Operation in this flux weakening regime will excite the coupling between flux and speed in classical field oriented control, causing undesirable speed fluctuation and perhaps instability (Bodson et *al.*, 1994). However, with the proposed control strategy, the flux weakening operation has no effect on the speed behavior.

6. CONCLUSIONS

In this paper, we have shown that the approximated nonlinear receding-horizon controller, used in cascaded structure, can be successfully applied to the control of induction machines. Based on simulation results, we have demonstrated that the proposed control law achieves speed and flux amplitude tracking objectives even with disturbance, thus presents sufficient robustness in case of electrical parameter variations. These results obtained with the particular trajectories used in motion control are very attractive in this field of applications. Additional research should be oriented first towards a nonlinear sensorless control scheme to reduce anymore the cost, secondly to discrete time-implementation of the proposed nonlinear receding-horizon controller. Analysis of the influence of sampling rate, truncation errors, measurement noise and saturations are all worthy of further investigation.

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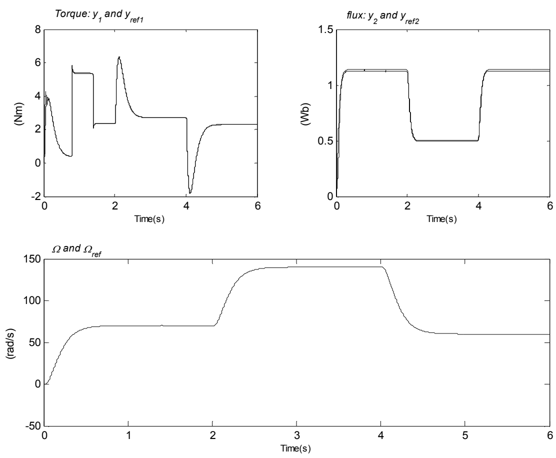

Fig. 2. Rotor torque, rotor flux and speed tracking performance.

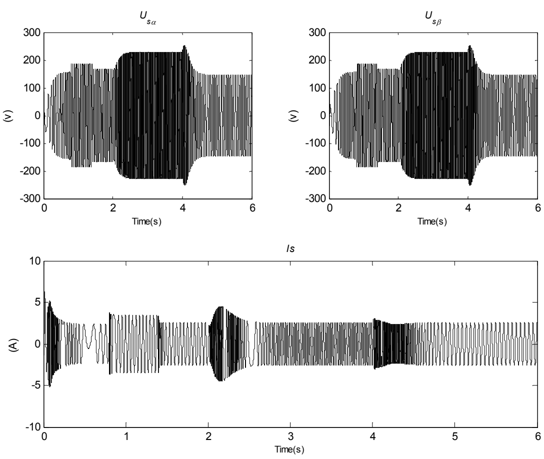

Fig. 3. Stator voltage ($u_{s\alpha}$, $u_{s\beta}$) and stator current i_s.

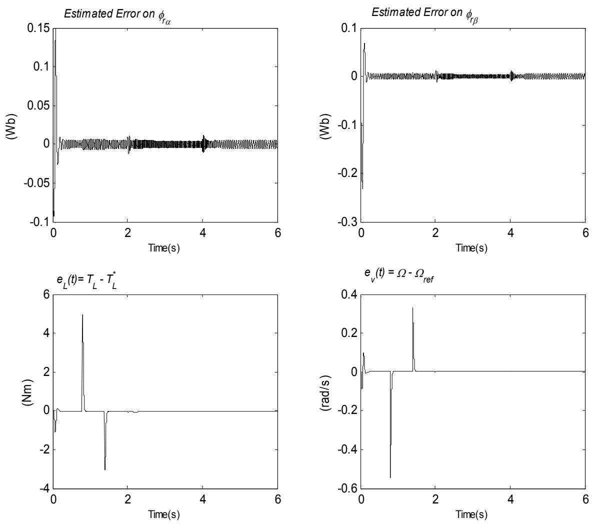

Fig. 4. Flux and Load torque estimated errors and rotor speed tracking error.

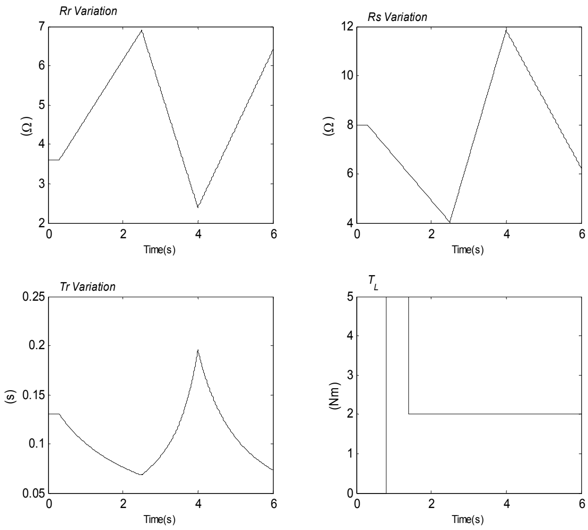

Fig. 5. Electrical parameter variations.

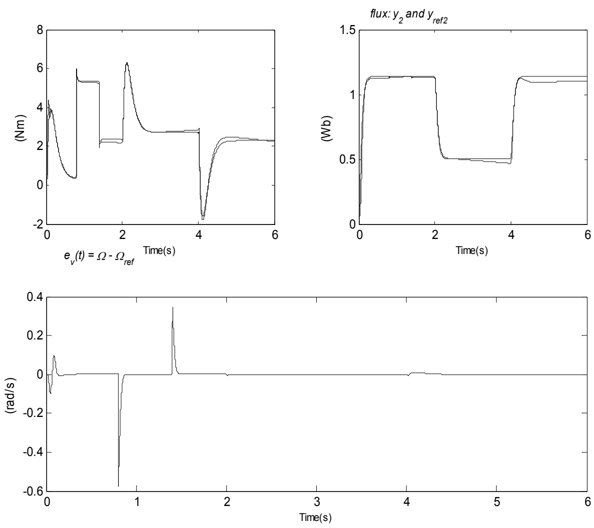

Fig. 6. Rotor torque, rotor flux and speed tracking performance in the mismatched case.