CONSTRAINED SPLIT RATE ESTIMATION BY MOVING HORIZON [1]

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Abstract: The paper presents a method for the constrained Moving Horizon Estimation of origin-destination (OD) matrices for traffic networks. In traffic systems split ratios are important parameters for OD estimation and for control problems as well. The paper treats the general constrained Moving Horizon Estimation problem for traffic systems modelled in terms of linear time varying system and solves the split rate estimation process. The estimation is subjected to equality and inequality constraints. A numerical example is solved to demonstrate the Moving Horizon Estimation of split variables.

Keywords: state estimation, moving horizon, constrained, traffic

1. INTRODUCTION

Traffic engineering makes dynamic or static analysis and synthesis of automotive vehicle technologies possible. The main goal of engineering is planning and managing traffic systems.

The examination of the dynamic aspect of traffic control in a traffic system needs the previously measured or estimated volumes of vehicles. Since measuring of certain variables in the dynamic description is rather costly, one tries to predict them.

The permanently varying demand on road networks, regarding mainly intersections or motorways, needs to be estimated. Firstly, because traffic volumes can only be measured at the input or at the output point of the section, not in the intersection itself. Secondly, the most important task in course of traffic planning is the coordinate control of more than one intersection, which is based on estimated variables. The need could be defined by the dynamic OD (Origin Destination) matrix, which shows the links among the entries and exits in a given intersection.

One divides the intersection into three parts such as entry, exit and internal flows. The measurement of both the entry and the exit flows might be assumed. Traffic density cannot be measured without error, so the idealized flows play role in theoretical aspects only. A model setup of entry-exit travel demands regarding an intersection allows estimation methods to determine the internal link flows. The key of the model buildup is split parameter ratios. The split rate determines the percentage of turning vehicle entering the traffic system. If one assumes that these turning rates are slowly varying split probabilities, the methods to determine probabilities are called split ratio

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methods (Cremer and Keller, 1981), (Nihan and Davis, 1987). The split rates define a turning proportion. The stochastic view on creating the model was elaborated in (Nihan and Davis, 1989).

There exist many estimation techniques, for giving reliable estimation on dynamic OD matrix, their results, however, could be different. The short review on OD estimation begins with the Least Squares (constrained or not), based on statical methods such as Likelihood methods(Nihan and Davis, 1989) and Kalman filtering (Cremer and Keller, 1987), (Cremer and Keller, 1981), (Cremer, 1983,Baden-Baden, Germany), or Bayesian estimator (van der Zijpp, 1997, Greece). Sometimes, combined estimators (using constraints or apriori knowledge about the intersection) can be applied.

Not only static constraints can be given, but also dynamic ones. In certain lanes for example traffic jams may occur, at intersections some of the split rates may be temporarily zero. The goal of the approach presented in the paper was to find a solution that makes it possible to give constraints during the estimation process. Constraints must be taken into account in course of dynamic OD estimation. A class of optimal state estimation methods is called Moving Horizon Estimation (MHE) method (Findeisen, 1997), (Rao, 2000), (Tyler and Morari, 1996). The MHE can be considered as the dual of the Model Predictive Control, though some special assumptions must be given regarding filter stability. Another advantage of Moving Horizon Estimation can be the fact that constraints assumption can be combined in the estimation process. In the following section the Moving Horizon state estimation method is applied to a basic intersection model(Kulcsár and Varga, 2004; Varga *et al.*, 2004).

The paper is divided into 5 chapters. Following a short introduction, the general problem is described in the first section. The second section briefly summarizes the MHE principles and shows how they can be applied to traffic systems. The third part gives a numerical example. The conclusion contains further research issues.

2. THE PROBLEM

One of the basic elements in traffic network systems are intersections. A general intersection is given in Figure 1. It is supposed that the proportions of entry-flow split, according to the destinations, are variant. No traffic lights at this intersection and the right hand side of road is regularized, since from the point of view of estimation, one only takes the time varying input and output volumes into account. Traffic regulation, however can be applied in model description. In this case the mathematical model for the dynamic process of exit volume is rather elementary.

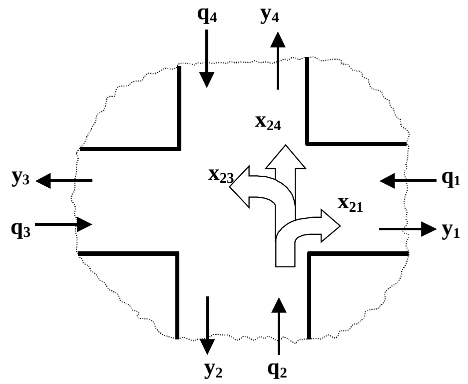

Fig. 1. A simple intersection

To show the problem the following variables are defined:

- $q_i(k)$ traffic volume (the number of vehicles) entering the intersection from entrance i, during time interval $k = 1, 2, ..N$
- $y_j(k)$ traffic volume (the number of vehicles) leaving the intersection from exit j, during time interval $k = 1, 2, ..N$
- $x_{ij}(k)$ the percentage of $q_i(k)$ (split rate) that is destinated to exit j, $k = 1, 2, ..N$.

Let us consider the following intersection model

$$y_j(k) = \sum_{i=1}^{m} q_i(k) x_{ij}(k) + v_j(k), \quad (1)$$

where $i = 1, ..n$ and $j = 1, ..m$. $v_j(k)$ is a zero mean noise term. The input measurement is a noisy term, since $q_i(k) = \tilde{q}_i(k) + \zeta_i(k)$, with the same assumption for the noise $\zeta_i(k)$ as above.

Split variables are independent trials. The model and its constraints are given by

$$x_{ij}(k+1) = x_{ij}(k) + w_{ij} \quad (2)$$

$$1 \geq x_{ij} \geq 0 \quad (3)$$

$$\sum_{j=1}^{m} x_{ij}(k) = 1. \quad (4)$$

The random variation in split parameter is small, and $w_{ij}(k)$ is a zero mean random component. All random components ζ, v, w are mutually independent terms. The scheme of the MH observer is given in Fig. 2. For the sake of simplicity, let us arrange all the elements of the OD matrix in a single vector and use the following notations:

$$x_k = [x_{ij}(k)]^T$$
$$w_k = [w_{ij}(k)]^T$$
$$v_k = [v_j(k)]^T$$

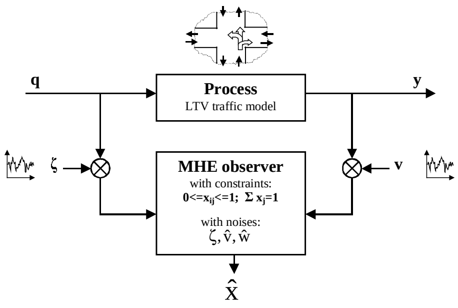

Fig. 2. The MHE observer

The problem is to observe states x_k under certain conditions. The latest estimation of the split parameters can be treated as a filtering problem. The fact that constraints have to be taken into consideration makes the task difficult. In the presented case, two types of constraints are applied (inequality and equality), but further constraints may be implemented. When using state estimation, it is extremely difficulty to constraints put on the observer. In the following section one tries to emphasize the effectiveness of the constraint Moving Horizon Estimation (MHE) method as a reliable state observer of split ratios.

3. CONSTRAINED SPLIT RATE ESTIMATION

Let us consider the following discrete linear time-variant system

$$x_{k+1} = Ax_k + Gw_k \tag{5}$$

$$y_k = C_k x_k + +v_k \tag{6}$$

with x_0 given. W_k is the random state noise vector, v_k the measurement noise vector, $x(t)$ the state vector. A, G are constant parameter matrices, C_k is time-variant output map of the dynamic system.

In our case one may neglect the control input, since the split rate, and $G = A = I_n$ where n is the number of states (i.e. the number of turning rates). These parameters are unknown, because in most cases only input and output detectors are installed in intersections.

In reality, the states of a dynamic system cannot be measured directly. Usual state estimation methods either for linear, or for nonlinear cases are available from the literature.

The most popular estimation methods are based on least squares method (LS) with a recursive form. Recursive algorithm formulation allows the step-by-step calculation of the estimated state.

Regarding dynamic systems, the most obvious optimal state estimation can be batch estimation (full information). This approximation can be applied to nonlinear systems without linearization. The general Batch Estimator offers the possibility to threat constraints during estimation.

The batch estimator is an infinite horizon state estimator. When applying batch estimation, the entire past behavior of the system is known.

$$\min_{(\bar{x}_0, \hat{w}_{-1|k}, \ldots, \hat{w}_{k-1|k})} \Psi_k$$

$$\begin{aligned} \Psi_k = \; & \hat{w}_{-1|k}^T Q_0^{-1} \hat{w}_{-1|k} + \\ & + \sum_{j=0}^{k-1} \hat{w}_{j|k}^T Q^{-1} \hat{w}_{j|k} + \\ & + \sum_{j=0}^{k} \hat{v}_{j|k}^T R^{-1} \hat{v}_{j|k}, \end{aligned}$$

subject to:

$$\begin{aligned} \hat{x}_{0|k} &= \bar{x}_0 + \hat{w}_{-1|k} \\ \hat{x}_{j+1|k} &= A\hat{x}_{j|k} + G\hat{w}_{j|k} \\ y_j &= C\hat{x}_{j|k} + \hat{v}_{j|k} \\ 0 \leq x_k &\leq 1 \\ \sum_{j=1}^{m} x_{jk} &= 1. \end{aligned}$$

However, Batch Estimator, even for a small state space is intractable from the point of view of numerical computation, as the Batch Estimator window is infinite.

The Moving Horizon Estimation scheme can be seen in Figure 3.

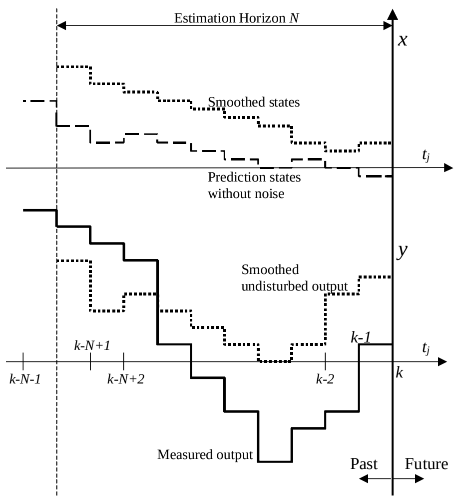

Fig. 3. General Moving Horizon Estimation process

Let the generalized MHE optimization criteria be defined by the following functional

$$\min_{(\bar{x}_{k-N-1},\hat{w}_{k-N-1|k},\ldots,\hat{w}_{k-1|k})} \Psi_k$$

$$\begin{aligned}\Psi_k &= \hat{w}^T_{k-N-1|k} Q_0^{-1} \hat{w}_{k-N-1|k} + \\ &+ \sum_{j=k-N}^{k-1} \hat{w}^T_{j|k} Q^{-1} \hat{w}_{j|k} + \\ &+ \sum_{j=k-N}^{k} \hat{v}^T_{j|k} R^{-1} \hat{v}_{j|k} + \Psi^*_{k-N},\end{aligned}$$

subject to:

$$\begin{aligned}\hat{x}_{k-N|k} &= \bar{x}_{k-N} + \hat{w}_{k-N-1|k} \\ \hat{x}_{j+1|k} &= A\hat{x}_{j|k} + G\hat{w}_{j|k} \quad j = k-N-1,\ldots,k-1 \\ y_j &= C\hat{x}_{j|k} + \hat{v}_{j|k} \quad j = k-N-1, \\ & 0 \le x_k \le 1 \\ & \sum_{j=1}^{m} x_{jk} = 1,\ldots,k\end{aligned}$$

with R^{-1}, Q^{-1} which are symmetric positive semidefinite noise weighting matrices. While $Q_{-N|k}$ penalizes initial state $\bar{x}_{k-N}$, R^{-1} weights the output prediction error and Q^{-1} penalizes all estimated state noise.

If the expected output is small, R^{-1} has to be chosen large, compared to Q^{-1}, and the resulting sensor noise vector becomes small, compared to $\hat{w}_{j|k}$. On the other hand, if the measurements are not reliable, Q^{-1} should be chosen large, compared to R^{-1}.

Ψ^*_{k-N} is the arrival cost to the analogue of the *cost to go* in MPC technique. The arrival cost summarizes all knowledge about the best estimation before the N-th step. Regarding the unconstrained linear case, the arrival cost can be expressed explicitly. If state or noise inequality constraints, or nonlinearities are present, we do not have an analytic expression to generate the arrival cost. Though an analytic approach is unavailable, an *approximate* cost may be given. When inequality constraints are inactive, the approximation is exact. Therefore, the poor choosing of the arrival cost leads to the filter's instability. To find the initial condition of General MHE, we used a batch estimation for the first $N-1$ step estimates. The stability of the MHE filter is effected by the choosing of the initial condition and the weighting matrices, as well.

To slide between windows the filtered estimate update is preferred. The use of the Kalman Filter ensures another possibility of state estimation. The Kalman Filter has been widely applied in traffic systems. This has been published in numerous papers e.g. (Cremer and Keller, 1987),(Keller and Ploss, 1987, Amsterdam). This method, based on Gaussian distributions of random variables, is defined on a probability framework of the unknown split parameters. Kalman filter equations (Kalman, 1960) can be formulated as recursive ones started with an initial condition. The optimal estimation depends on the choosing of state noise covariance (Q) and on the output noise covariance (R) weights. The Kalman estimator can be applied, subject to inequality constraints, by using stochastic programming (van der Zijpp, 1997). The connection between Kalman filtering and full information estimation is known.
Applying the MHE method to a simple intersection with a moving horizon window with 1 step length, one can write as follow

$$\min_{(\bar{x}_0,\hat{w}_{k-2|k},\hat{w}_{k-1|k})} \Psi_k$$

$$\begin{aligned}\Psi_k &= \hat{w}^T_{k-2|k} Q_0^{-1} \hat{w}_{k-2|k} + \\ &+ \hat{w}^T_{k-1|k} Q^{-1} \hat{w}_{k-1|k} + \\ &+ \hat{v}^T_{k-1|k} R^{-1} \hat{v}_{k-1|k} \\ &+ \hat{v}^T_{k|k} R^{-1} \hat{v}_{k|k} + \Psi^*_{k-N},\end{aligned}$$

subject to:

$$\begin{aligned}\hat{x}_{k-1|k} &= \bar{x}_{k-1} + \hat{w}_{k-2|k} \\ \hat{x}_{k|k} &= A\hat{x}_{k-1|k} + G\hat{w}_{k-1|k} \\ y_{k-1} &= C\hat{x}_{k-1|k} + \hat{v}_{k-1|k} \\ y_k &= C\hat{x}_{k|k} + \hat{v}_{k|k} \\ & 0 \le x_k \le 1 \\ & \sum_{j=1}^{m} x_{jk} = 1\ldots,k\end{aligned}$$

where m depends on the layout of the intersection.

4. EXAMPLE

As an illustrative example, let us consider the following simple traffic system. Let us suppose we have two different input directions and two common output directions, with 4 split variables. The system (simplified intersection from Fig. 1)can be described in the following state-space form

$$\begin{aligned}x_{k+1} &= x_k + u_k + w_k \\ y_k &= C_k x_k + v_k,\end{aligned}$$

where $x_k = \begin{bmatrix} x_{13} & x_{14} & x_{23} & x_{24}\end{bmatrix}^T$ is the state vector containing the turning rates, w_k is the state noise term, a zero mean random signal,u_k denotes the systematic variation component in it, v_k is the measurement zero mean noise, $y_k = \begin{bmatrix} y_3 & y_4\end{bmatrix}^T$ is the output traffic volume, C_k is the time variant output map of the system, depending upon the geometry of the intersection. In our case C_k is given by

$$C_k = \begin{bmatrix} q_1 & 0 & q_2 & 0 \\ 0 & q_1 & 0 & q_2 \end{bmatrix}$$
$$q_{k+1} = \tilde{q}_k + \zeta_k$$

where $q_k = \begin{bmatrix} q_1 & q_1 \end{bmatrix}^T$ means the detected noisy input volume of the vehicles into the intersection. The permanently varying input sequence turns the intersection model in a discrete time varying one. ζ_k is the zero mean random signal.

One should point out that in split parameters usually two types of variation are allowed: the random variation term, and the systematic components. The time average of these terms is assumed to be zero. The simulation result can be seen in Fig. 4. Let us suppose we have 1 sample in every second,

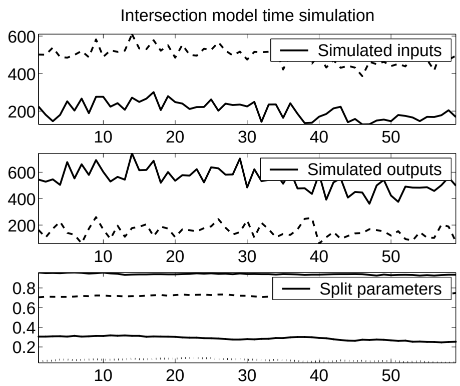

Fig. 4. Time simulation of the input and output measurements and the not smoothed turning rates

during 1 hour. For solving constrained MHE numerically, one may apply quadratic programming. It solves the problem above, while, additionally, satisfies equality and inequality constraints.

Let the horizon comprise 1 sample, and by applying diagonal R, Q and R_0, Q_0 weighting the following results are gained, see in Fig. 5. The results have been compared to those done by Kalman Filter estimation process.

By solving a one-step backward receding horizon problem, one should solve a quadratic functional at each step under the dynamic equality, the split inequality and equality constraint. To check the fulfil of the constraints, in Fig. 5 one can see , that unlike to Kalman filtering, in the MHE case the estimation process does not exceed either 0, or 1. Additionally, the estimated sum of the split parameters is equal to the one in Receding Horizon case (see in Fig. 6).

5. CONCLUSIONS

The paper summarizes the Moving Horizon Estimation (MHE) approach for a simple intersection. The turning rate estimation in like traffic systems gives space for further control problem solving.

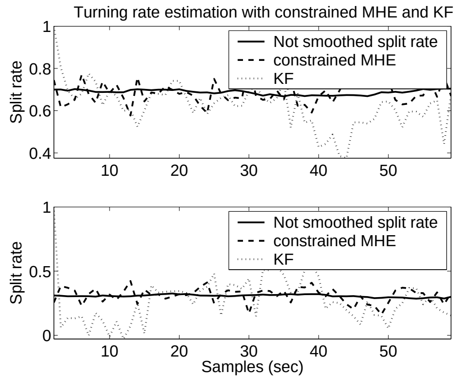

Fig. 5. Time simulation of the estimation processes regarding x_{23} and x_{24}

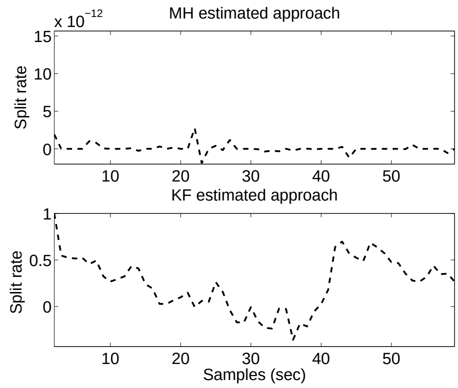

Fig. 6. Equality constraint regarding $x_{23}+x_{24}-1$.

The nonlinear turning rate estimation problem could be regarded a constraint estimation, and equality, respectively, inequality constraints are needed to be taken into consideration. The MHE process can be solved by quadratic problem formulation.

The paper shows a numerical example to demonstrate MHE based split parameter estimation. Furthermore, the general MHE technique for nonlinear traffic system processes can be applied in online traffic control.

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