ROBUSTNESS OF THE LINEAR WORKSHOP TO THE CHANGE OF RATIO

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Abstract: This study concentrates on linear flow-shop whose organization is generally optimized for a well defined production in term of quantity and quality
In front of the economic needs which require a constant adaptation of the product, an analytical evaluation of the robustness to the changes of the ratios of production is presented. This evaluation is based on the assumption of a cyclic operation, which allows the modeling of the system by a strongly connected event graph. A modular decomposition allows the insulation of the ratio indeterminism in module. It thus makes possible to convert the model into an event graph with associated constraints. The robustness to ware ratio changes is evaluated. It's important to notice that during this study, we evolve in static mode (ratios do not change throughout the production) It is relevant information for piloting the workshop.
Systems with time constraint usually not allow this quality degradation ensuing from specification violation. In this case, a study of mode transient remains to be developed. We will use a toy manufacturing flow-shop throughout the paper as an illustration.

Keywords: Timed Petri nets, Robustness, Ratio of production.

1. INTRODUCTION

Market requires constant adaptations of the production. However the need does not change radically in its nature and quantity. Consequently, it is important for a production system to be able to manufacture various categories of products, while having the ability to make adjust in a slight way the nature of a product or the production ratios. In the presented paper, small and local variations will be studied. Obviously, the flexibility of considered process is very restricted compared to the general notion of flexibility.

The presented approach tries to study flow shops in order to prove and quantify a given degree of flexibility. On one hand, the constraints to be respected in order to maintain the initial performances of the workshop are established. On the other hand, the freedom degrees of the system are analytically characterized in order to define the quality and ratio changing possibilities.
The global methodology provides some sufficient conditions for dynamic ratio changing. Nevertheless, these conditions may be decomposed into two subsets. The first ones derive from the reachable rate conditions (Campos 1991). The second ones are robustness properties for ratio

changing from a cyclic functioning point to another. These last conditions will not be detailed in this paper.

In a first section timed Petri Nets will be presented. Then the subclass of strongly connected event graph will be detailed. Then, a decomposition approach is introduced. Its main quality is to preserve the analysis capacities of the model. This allows applying the rate conditions (Campos 1991). A particular assumption is used: the bottleneck machine is supposed to be outside of the module where a variation is introduced. A toy manufacturing workshop is used as an illustration throughout this paper.

Approach presentation on an example:
The workshop is composed of three working stations as well as two robots routing the parts in the various stations. The workshop carries out an assembling operation of two pieces of wood. Operations are: loading of a part in the pallets, part position recognition, deposit of adhesive, positioning of a ware feet, and unloading. Let us point out that the sequencing of the pallets is fixed at the scheduling level.

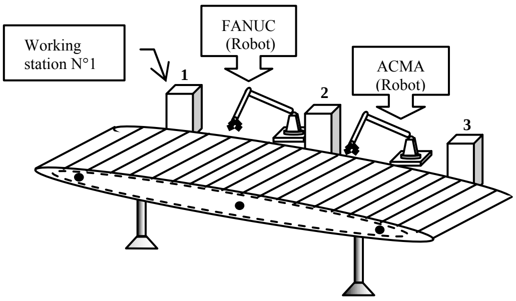

Fig. 1: Real representation of the workshop.

The general procedure includes three steps:
At first, the performance bounds are computed. They are used to make the assumption of a cyclic functioning. Then a subpart of the system is selected to contain a new variation. The bottleneck machine is assumed to be out of the considered sub-system. Finally, the ratio constraints which maintain the global production rate are established. The aim is to build a modular structure which is able to take decision concerning the ratio variations, without studying and without disturbing the global workshop. The transient mode problem is not studied in these lines.

Timed Petri Nets
Petri nets are graphic and mathematical tools able to model a broad variety of discrete systems. They are well fitted for descriptions and the studies of parallel systems.

1.1 Definition

A Petri net is a graph whose nodes are places and transitions, and in which the arcs connect places to transitions or transitions to places. A PN is often represented by a 4 – uple N = (P, T, I, O), where:
- $P = \{P_1, P_2, ..., P_m\}$ is the set of the places.
- $T = \{T_1, T_2, ..., T_n\}$ is the set of the transitions.
- I is an input function type $P \times T \rightarrow N$ corresponding to the arcs who connect the place to the transition. $I(p, t) > 0$ means that there is an arc going from the place p to the transition T; $I(p, t) = 0$ means that this arc does not exist.
- O is an output function of type $T \times P \rightarrow N$ and corresponds to the arcs going of a transition to a place. $O(t, p) > 0$ means that there is an arc going from the transition t to the place p; $O(t, p) = 0$ indicates the non-existence of such an arc.

A timed PN is a pair (R, Θ) where:
R is a marked Petri net.
Θ is a function which associates one duration of firing of each transition. This duration can be deterministic or random.

1.2 Functioning of a timed Petri Net

The conditions of t transition firing are the same ones as subjacent PN but the transition firing are not instantaneous and proceeds in three stages (Laftit91).
The initialization, which begin when each input place p contains at least the same marks as the value of the arc (p, t). I(p, t) tokens is then removed from each place p of entry of the transition t.
The execution, which spends Θ(t) units of time. During this time, tokens are frozen in transitions.
End of firing, at the moment of end of execution. O(t,p) tokens is then added in each place of exit of the transition t.

1.3 Events Graphs and SCEG

Definition 1: event graph is a particular PN in which each place has exactly an input transition and an output transition. In other words a graph of event is PN N = {P, T, I, O} such as:

$$\forall (t,p) \in P\times T, |I(p,t)| = |O(p,t)| = 1$$

Definition 2: an event graph is strongly connected if there is a directed path which connects any node (a place or a transition) to any other node.

Theorem 1 (Chretienne 1983): The asymptotic behavior of SCEG who operate as soon as possible is K-periodical with a frequency of firing of the transitions equal to 1/Cmax. In other words, there is n and K such as for all, transition t, the t firing dates checks:

$$S_t\,(n+K) = S_t\,(n)+K\ C_{max}$$

$S_t(n)$ is the n^{th} firing of the transition t

Definition 3: We call elementary circuit any directed path who connecting a node to itself without going more than one time through the same node.

Theorem 2 (Ramamoorthy 1980): The minimal cycle time of a timed SCEG is given by C_{max}, where C_{max} is the maximum of the cycle times of the elementary circuits of the net.
The cycle of the circuit Γ, noted C(Γ), is computed by: $C(\Gamma_i) = \mu(\Gamma_i)/M(\Gamma_i)$, where $\mu(\Gamma_i)$ is total duration of firing of the transitions from the circuit Γ_i. and $M(\Gamma_i)$ is the number of tokens in the elementary circuit Γ_i.

The global cycle time of the SCEG is C_{max} such as:

$$C_{max} = \max \{C(\Gamma_i)\}.$$

Property 1 (Laftit 1991): For a given SCEG the marking of an optimal solution using one-periodic functioning constraints is also a marking of an optimal solution to the same system running in K-periodic mode.

The global topology of the workshop is a about flow-shop one, where some operations have many alternatives. As there are no critical resources sharing, the general structure of the net is then a free choice net. In this case, in spite of the indeterminism introduced on the level of the choices, we can calculate a reachable upper bound of the performances (Campos 1991).

Definition 4 : Free choice net (Diaz 2001) are such:
$\forall(p,q)\in(P\cap P\times T)$, $O(p,t)=\{q\} \vee I(p,t)=\{p\}$.
It is easy to see that SCEG are a subclass of free choice nets.

2. DECOMPOSTION AND PERFORMANCES EVALUATION

2.1 Introduction of a process flexibility

We include now a new operation (station of varnishing). The type of flexibility introduced here is process flexibility.

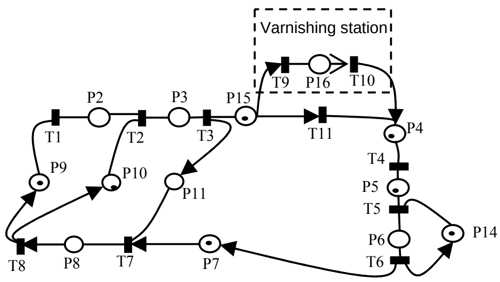

Fig.2: PN model of the process

Indeed there is now the choice between two operations requiring different resources. In other words, a standard ware or a modified one can be produced (Fig 2).
It's easy to check that the PN model of the workshop before the addition of the new station is a SCEG.

2.2 Decomposition procedure (Proth 1997)

The introduction of a new procedure into the operating sequence makes that the obtained graph is no more an event graph or a state graph. Then it is no more possible to apply the theorem 2 on the modified model (fig2).
It is necessary to apply a new approach to evaluate the performances of the system. Therefore, a modular decomposition of the process is proposed.
The first step of decomposition consist in showing that our process has only one entry and one exit.
All the manufacturing processes have the same input points and output points, this indicate that the PN model is controllable output PN (Proth 1997), then we determine the ranges of products contained in the process.

Definition 5:

Let W be a t-invariant of a PN named N. A PN named NW is W-derived from N, which contains output and input place if:
The set of transition of NW is $\|W\|$,
$\forall$ t $\in\|W\|$,$\forall$p $\in$P, I(p,t) and O(t,p) are the same for N and NW.

Definition 6:
Let us introduce W-CFIO of N, where W is a t-invariant of N. NW is the PN W-derived from N when it verifies the following properties:
Each place of NW has exactly one output transition.
$\exists(t1, t2) \in \|W\|\times \|W\| \mid \forall p\in P, I(p,t1)=O(t2,p) = \varnothing$.
NW contains no elementary circuits.

Definition 7: Output controllable Petri nets (Proth 1997)
A Petri Net N = $(P\cup R, T, A, M_0)$ where :
P (resp. R) is a set of places known as process place (resources places); the sets P and R are disjoined.
T is the whole of the transitions,
$A \subset ((P\cup R) \times T) \cup (T \times (P\cup R))$ is a set of arcs,
M_0 is initial marking,
All the arcs values are equal to 1,
There is a controllable output PN, if the following conditions are checked:
$(t, p) \in A \Leftrightarrow (p, t) \in A$, $\forall t \in T$ et $p \in R$,
$M0(p) \geq 1$, $\forall p \in R$
The subnet N1 = (P, T, A1, M1) where :
A1 is a restriction of A to $(P \times T) \cup (T \times P)$, M1 is a restriction of M_0 to P, is a decomposable PN,
N1 has transitions sources and transitions well, but does not have any place source or well,
There is a unit W = (W1..., Wk) of T-invariants of N1 such as the networks NWi Wi derived from N1 are a Wi-CFIO which cover N1 and have only one transition well

We can say that PN (fig. N°2) is an Output controllable Petri nets, and thus determine the transition sources (T_1) and the well transition (T_8), as well as the sets: P (places of process) and R (places of the resources).
The sets are: P = $\{P_2, P_3, P_4, P_5, P_6, P_7, P_8, P_{15}, P_{16}\}$
R = $\{P_9$, P10, P11, P12, P13, P17$\}$

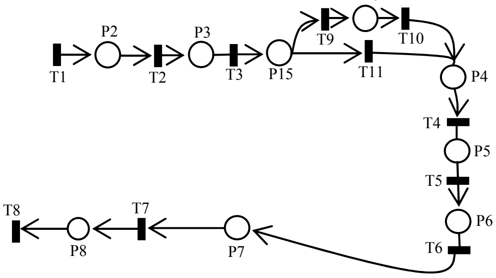

Fig.3 : underlying-network N_1

Integration process

Let us consider a system {N1, N2,…, Nn}of output controllable Petri nets.
$N_i = (P_i \cup R_i, T_i, A_i, M_{0,i})$, let us denote T_e^i (resp. T_s^i) the transitions sources (resp. well) from N_i.

N_i; i = 1..., n are integrated by the intermediary of a Q unit of interfaces places and Γ unit of inter arcs modules. Resulting PN noted N is such as:

$$N = (\bigcup_{i=1}^{n}(P_i \cup R_i) \cup Q, \bigcup_{i=1}^{n} T_i, \bigcup_{i=1}^{n}(A_i) \cup \Gamma, \bigodot_{i=0}^{n} M_{0,i} 0 M_{0,Q})$$

Where: $\Gamma \subset (\bigcup_{i=1}^{n} T_s^i \times Q) \cup (Q \times \bigcup_{i=1}^{n} T_e^i))$

O represents the concatenation. Each $t \in T_s^i$, such as the set of output places of t has at most one element (Proth 1997).
The following figure illustrates the integration process:

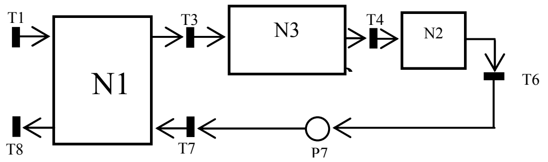

Fig.4: Integrated model

With Q = {P7} et Γ = {(T_6, P_7), (P_7, T_7)}.
Let us recall that the aim of the modular decomposition is solving conflicts issue from the addition of a modified ware in the production. On figure 3, the P_{15} place has two output transitions T_{11} and $T_{10.}$ Each of the two path leads to a different manufacturing range.

This conflict prevents an evaluation of performance since the PN graph is no more a graph of event.
The modular decomposition enables us to isolate the conflict (in this case, it is the N3 module). By carrying out this decomposition we gathered all the actions which proceed before the conflict in the same module and the actions which proceed after the conflict in the same module too. We have at all three modules. By gathering them we obtain a macro model which does not present any more a conflict and whose temporal study will be simpler.

Transformation from a macro model towards a SCEG

The N1 module has several inputs and outputs; this property complicates the assembling the various modules. To solve this problem we carries out a decomposition of the module N1, we extracts two submodules which represent a controllable output PN according to definition 6 (Proth 1997).

Here two submodule$_s$ N1' et N1'':

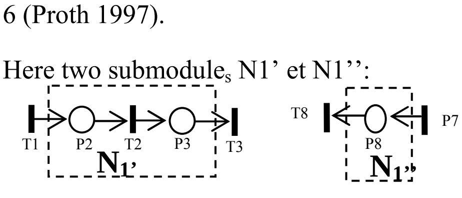

Fig.5 : the submodules de N_1 ($N_{1'}$ et $N_{1''}$)

Reduction techniques
Substitution of one or several places: A p_i place can be substituted if it fulfilled some well known conditions presented in (David 1992).

Simplification of the modules:

Each module has a single input transition (source) and a single output transition (well). It was established that the PN model have a periodic cycle. Each transition from the PN model has a periodic firing instant. Let us consider t_i, t_j as two transitions and St_i, St_j their respective firing moments. According to the equation of Laftit which defines the transition firing moment (Laftit 1991):
$St(n) = St(1)+(n-1)C_{max} \Rightarrow St_j(n) - St_i(n) = \Theta$

The last equation represents the sojourn time between the firing moment of the well transition t_i and the output transition t_j. If the PN model has a periodic cycle, Θ remains constant.

It results that each module is represented by a timed place of duration Θ. This time is determined by the difference between the firing moments of the output and input transitions from each module, where Θ is called interfering time of transition (Campos 1991)

General decomposition procedure:

The objective of this procedure is to solve the conflicts which appear in the PN model after the insertion of the new product. This addition makes possible the modifying the manufacturing range.

To solve the problem, the conflict part is isolated. All the parts which present a choice are integrated in only one module. This module has an input transition (source) and an output transition (well). To determine these transitions, the follows procedure is applied:
For timed PN, we define:
$I(P_i)$: the set of the arcs entering the place pi,
$O(P_i)$: the set of the outgoing arcs of place pi.
The symbols (°t) and (t°) respectively represent the set of upstream places of transition t, and the set of the output places of t.
- If there exists $t_i° = P_i$ such as $I(P_i) = 1$ and $O(P_i) > 1$ then t_i is an input transition ,
- If there exists $°t_j = P_j$ such as $I(P_j) > 1$ and $O(P_j) = 1$ then t_j is an output transition,
- If $I(P_i) = O(P_j)$

Then we integrate all the directed paths between t_i and t_j in only one timed place, the interfering time represents the temporization associated to the place. The functioning is assumed to be periodic into the new choice structure, which enables us to assimilate the module to a timed place and to calculate a general cycle time. When the cyclic conditions are fulfilled, according to the property of Laftit (property 1 and theorem 1), the integration procedure preserves the system optimality.

Moreover, we can say that the graph resulting from the assembly of all the modules is an event graph with associated constraints ensuing from the cyclic functioning assumptions.

2.3 Performance evaluation of the model

When an event graph is obtained, the performance evaluation is made using the theorem 2. It is necessary however to keep in mind that the model studied for the calculation of the cycle time is a meta model. This model is only correct for a given cyclic functioning. It is thus not planned to calculate transient states on this level of modelling.

In the followed section, choice possibilities on the ratios of production defined inside the module is studied. A modelling based on the works of Hillion was removed because this last fixes the production ratios before building the PN (Hillion 1989).

Study of the flexible part:

Let us focus on the module containing the flexible part.

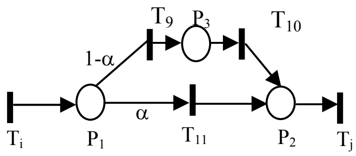

Fig.6: PN including a choice of range of products

The cycle time of the system is equal to:

$$C_1 = \alpha\, C_{max1} + (1-\alpha)\, C_{max2}$$

C_1 is the cycle time of a composite sequential process corresponding to a linear combination of the two possible sequential processes (Campos 1991).
C_{max1} corresponds to the cycle time of the sequential process associated to the normal product (which passes by T11). C_{max2} corresponds to the cycle time of the modified sequential process (passing by T9 and T10). α is the production ratio. It can be proved that there is no need to integrate other elementary circuits than the one of the new sequential process. The demonstration is based upon decomposition into a minimal base using the graph theory (Morioka 1991).

The addition of a new operation in the industrial process may modify system functioning rate. The initial system is periodic and has a cycle time C_0. It is a constraint to keep the same critical cycle, facing some modifications of the manufacturing process. Then, C_0 must be greater than C_1
There are two different scenarios:
- $C_0 \geq C_{max2}$: System performances remain the same.
- $C_0 < C_{max2}$: the system becomes slower than previously. Then the corresponding value of α is not allowed

It is obvious that the first case does not introduce a problem. Consequently, the study focuses on the second case. We will try to establish criteria to be respected in order to guarantee the optimality of the new system.

The assumption is made that the critical circuit which fix the maximum rate of the workshop is not included in the new considered model:

$$C_{max2} > C_{max1} \text{ and } C_1=\alpha\, C_{max1}+(1-\alpha)C_{max2}$$

Obviously, the α value conditions system performances. The values of C_{max1} and C_{max2} are known. In order to set C_1 lower than the olden critical time, a correct value of α has to be applied:

$$C_{max2} > C_{max1} \Rightarrow \begin{cases} C_{\max 1} = C_0 - \Delta_1 \\ C_{\max 2} = C_0 + \Delta_2 \end{cases}$$

Thus:

$$C_0 = \alpha.(C_0 - \Delta_1) + (1-\alpha).(C_0 + \Delta_2)$$
$$C_0 = C_0 - \alpha.(\Delta_1 + \Delta_2) + \Delta_2$$
$$\Rightarrow \alpha = \frac{\Delta_2}{\Delta_1 + \Delta_2}$$

We define Δ, as a variation in the length of the corresponding circuit.
α depends mainly on the value on Δ_2.
α is a ratio such as $0 \leq \alpha \leq 1$
$\alpha = 0 \Rightarrow$ Only the standard wares are manufactured.
$\alpha = 1 \Rightarrow$ Only the varied wares are manufactured.

- If $\alpha = \frac{\Delta_2}{\Delta_1 + \Delta_2}$ then

$$C_0 = \alpha.C_{\max 1} + (1-\alpha).C_{\max 2}$$

$\Rightarrow$ In this case, performances are conserved.

- If $\alpha < \frac{\Delta_2}{\Delta_1 + \Delta_2}$ then

$$C_0 < \alpha.C_{\max 1} + (1-\alpha).C_{\max 2}$$

$\Rightarrow$ In this case, the system performances are decreased.

- If $\alpha > \frac{\Delta_2}{\Delta_1 + \Delta_2}$ then

$$C_0 > \alpha.C_{\max 1} + (1-\alpha).C_{\max 2}$$

$\Rightarrow$ In this case, performances remain the same.
In conclusion; to preserve the system performance it is necessary that:

$$\frac{\Delta_2}{\Delta_1 + \Delta_2} \leq \alpha \leq 1$$

Generalization

Let us consider an integrated module, such as t_i and t_j are respectively its input and output transitions.

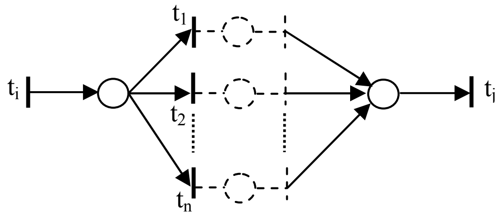

Fig.7: PN including N choice of ranges of products

If $A_S(t_i°) = A_E(°t_j) = n$ such as $n > 1$
Then the cycle time of the system is equal to:

$$C = \alpha_1 C_{\max 1} + \alpha_2 C_{\max 2} + \ldots + \alpha_n C_{\max n}$$

With $\sum_{i=1}^{n} \alpha_i = 1$

In this model there are "n" directed path of the place $t_i°$ towards the place $°t_j$, which give the following equation system:

$$\begin{cases} C_{\max 1} = C + \Delta_1 \\ C_{\max 2} = C + \Delta_2 \\ \dots \\ C_{\max n} = C + \Delta_n \end{cases}$$

Where $\Delta_2, \dots, \Delta_n$ are algebraic values

$$\Rightarrow \alpha_j = -\frac{1}{\Delta_j} \sum_{\substack{i=1 \\ i \neq j}}^{n} \alpha_i \Delta_i$$

In order to keep the system performances it is necessary to have:

$$-\frac{1}{\Delta_j} \sum_{\substack{i=1 \\ i \neq j}}^{n} \alpha_i \Delta_i \leq \alpha_j \leq 1$$

α_j the ratio which corresponds to the manufacturing process which has the longest way between the place $t_i°$ towards the place $°t_j$, $C_{maxj} > C_{maxi}$, for $i \in [0, n]$ $i \neq j$.

The presented inequation does not give the values of the process to be used to keep the same performances, because of couplings between the ratios α_i. we propose in our paper a means to establish if the ratios employed degrade or improve the performances of the flow shop.

The planning worker will have a mathematical tool which insures that the production of the necessary quantities of a specific product will be made within the deadlines, while respecting the initial global performances.

3. CONCLUSION

This study allows evaluating the impact of process flexibility on the flow-shop functioning on the system performances.

The cycle time of the initial flow-shop has to be maintained. This property constitutes a good performance criterion. In fact, the production constancy is a kind quality of service. It simplifies the production management of the downstream productive entities. Nevertheless this quality of service is possible under some necessary conditions. The constraints to fulfill on the production ratios, in order to a constant cycle time have been analytically established according to the structure of the workshop.

In the presented work, the nature of the flexibility is limited. It is process flexibility. Also on the number of choices in a given module is limited to one. Our study field may become broader, by allowing shared resources between the various alternative paths.

The possibility of on-line ratio changing has not been studied. It is however a natural prospective work. This involves characterizing the existence of transient states according to the physical constraints of the process and its periodicity, using the operating margins.

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