REDUCING THE EFFECT OF UNMODELED DYNAMICS BY MRAC CONTROL LAW MODIFICATION

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Abstract: Solving a tracking problem does not always give desired results even when the adaptive control methods are used. Some difficulties may occur when the apriori assumptions laid down for the problem solution are not satisfied. One of the serious issues is the existence of unmodeled dynamics in the tracking problem. The proposed solutions are mainly based on robustification of the adaptation law. In this paper we propose to reduce the effect of unmodeled dynamics using the MRAC control law modification so that the standard adaptation law ensures the sufficiently small tracking error.

Keywords: model reference adaptive control, Lyapunov methods, tracking systems

1. INTRODUCTION

The model reference control structures can be successfully used to solve the tracking problem in case some ideal conditions regarding the controlled plant model are satisfied (Narendra and Annaswamy, 1989). However, these ideal conditions are not often satisfied. A frequent violation of the ideal assumptions is an incorrect estimation of the plant model structure that leads to the existence of unmodeled dynamics in the tracking problem. The unmodeled dynamics influence in adaptive systems represents a serious problem. The unmodeled dynamics can significantly deteriorate the control parameters adaptation or it can even destabilize this process. Especially in case of MRAC structures the existence of the unmodeled dynamics reduces their applicability to real processes.

An intensive research activity has been devoted to solve the problem of reducing the effect of unmodeled dynamics (Gonos, *et al*., 2004; Rohrs, *et al.*, 1985; Sastry and Bodson, 1989) but it still has not led to satisfactory results. The majority of the unmodeled dynamics problem solutions are based on the adaptation law robustification (Sastry and Bodson, 1989; Ioannou and Sun, 1996).

The aim of our paper is to demonstrate that the unmodeled dynamics problem in standard MRAC scheme can also be solved by the control law modification. The proposed approach is based on the general theory of stability in vector form (Šiljak, 1978).

2. PROBLEM FORMULATION

The tracking problem is always a nonlinear task, because the adaptation error dynamics is nonlinear. The let the problem solution plain the tracking problem with the linear model of controlled system and the linear reference model has been chosen.

Consider that the linear system with unmodeled dynamics is given by the state space equations in the form

$$\dot{x} = Ax + bu \quad (1)$$
$$y = c^T x$$

where $x \in R^p$ represents the plant state, $y \in R$ is the plant output and $u \in R$ denotes the control signal. The state variables of system (1) can be divided into two vectors: state variables of modeled dynamics $x_1 \in R^n$ and state variables of unmodeled dynamics $x_2 \in R^m$, with m+n=p.

The system (1) can then be decomposed so that the modeled and unmodeled dynamics is separated

$$\begin{aligned} \dot{x}_1 &= A_{11}x_1 + A_{12}x_2 + b_1 u \\ \dot{x}_2 &= A_{21}x_1 + A_{22}x_2 \end{aligned} \quad (2)$$

where A_{11} is a nxn Frobenius matrix of the modeled part of dynamics, A_{22} is a mxm matrix of the unmodeled part of dynamics, A_{12} (nxm) and A_{21} (mxn) are matrices representing the interactions. For the reasons of simplicity we consider the class of systems where only modeled part of dynamics is directly influenced by the control signal.

Assume that the desired closed loop dynamical behavior is described by the reference model in the form

$$\dot{x}_m = A_m x_m + b_m r \qquad x_m \in R^n \quad (3)$$

with the reference model state vector $x_m \in R^n$ and reference signal $r \in R$.

In standard MRAC schemes the control law is of the form

$$u = k_r r - k_1^T x_1 \quad (4)$$

where k_1^T denotes a feedback gain vector and k_r is a feedforward gain. However, in case of these control structures the unmodeled dynamics provokes the considerable deterioration of adaptive system tracking performances.

3. REDUCING THE EFFECT OF UNMODELLED DYNAMICS

Using the matching conditions

$$\begin{aligned} A_m &= A - bk_1^{*T} \\ b_m &= bk_r^* \end{aligned} \quad (5)$$

the following dynamics of adaptation error (defined as $e = x_1 - x_m$) can be derived

$$\begin{bmatrix} \dot{e} \\ \dot{x}_2 \\ \dot{x}_m \end{bmatrix} = \begin{bmatrix} A_m & A_{12} & 0 \\ A_{21} & A_{22} & A_{21} \\ 0 & 0 & A_m \end{bmatrix} \begin{bmatrix} e \\ x_2 \\ x_m \end{bmatrix} - \begin{bmatrix} b_1 \\ 0 \\ 0 \end{bmatrix} \left(\Theta^T \omega\right) + \begin{bmatrix} 0 \\ 0 \\ b_m \end{bmatrix} r \quad (6)$$

where $\Theta = \begin{bmatrix} \kappa^T & -\rho \end{bmatrix}^T$, $\omega = \begin{bmatrix} x_1 & r \end{bmatrix}$,

$$\kappa = \left(k_1^T - k_1^{*T}\right),\ \rho = \left(k_r - k_r^*\right).$$

The stability of the system (6) is ensured by the tracking problem convergence. The stability proof will be based on the vector Ljapunov function methodology (Šiljak, 1978). The isolated subsystems of system (6) are

$$\begin{aligned} \dot{e} &= A_m e - b_1 \Theta^T \omega \\ \dot{x}_2 &= A_{22} x_2 \\ \dot{x}_m &= A_m x_m + b_m r \end{aligned} \quad (7)$$

When r = 0, the equilibrium point of system (6) is $e = 0,\ x_2 = 0,\ x_m = 0$. To analyze the equilibrium point stability the Lyapunov function candidates for each isolated subsystem have to be chosen as the functions of the corresponding subsystem variables

$$\begin{aligned} V_1 &= e^T P_1 e + \alpha \Theta^T \Theta \\ V_2 &= x_2^T P_2 x_2 \\ V_3 &= x_m^T P_3 x_m \end{aligned} \quad (8)$$

The conditions of continuity and positive definiteness are satisfied for the functions V_1, V_2, V_3. The vector Lyapunov function methodology is based on aggregation, where it is necessary to find the boundaries of the V_1, V_2, V_3 time derivatives along the relevant subsystems trajectories. The time derivatives of the Lyapunov functions candidates (8) along the subsystem (7) trajectories are

$$\begin{aligned} \frac{dV_1}{dt} &= \dot{e}^T P_1 e + e^T P_1 \dot{e} + 2\Theta^T \Theta = \\ &= \left[e^T A_m^T - \omega^T \Theta b_1^T\right] P_1 e + \\ &\quad + e^T P_1 \left[A_m e - b_1 \Theta^T \omega\right] + 2\Theta^T \Theta = \\ &= e^T \left(A_m^T P_1 + P_1 A_m\right) e - 2\omega^T \Theta b_1^T P_1 e \end{aligned} \quad (9)$$

$$\begin{aligned} \frac{dV_2}{dt} &= \dot{x}_2^T P_2 x_2 + x_2^T P_2 \dot{x}_2 = \\ &= x_2^T A_{22}^T P_2 x_2 + x_2^T P_2 A_{22} x_2 = \\ &= x_2^T \left(A_{22}^T P_2 + P_2 A_{22}\right) x_2 \end{aligned} \quad (10)$$

$$\begin{aligned} \frac{dV_3}{dt} &= \dot{x}_m^T P_3 x_m + x_m^T P_3 \dot{x}_m = \\ &= \left(x_m^T A_m^T + r^T b_m^T\right) P_3 x_m + x_m^T P_3 \left(A_m x_m + b_m r\right) = \\ &= x_m^T \left(A_m^T P_3 + P_3 A_m\right) x_m + 2r^T b_m^T P_3 x_m \end{aligned}$$

(11)

Choosing a suitable adaptation law

$$\frac{d\Theta}{dt} = f(e, x_1, r) \tag{12}$$

so that

$$\omega^T \Theta b_1^T P_1 e = \Theta^T \Theta \tag{13}$$

it is possible to introduce the following boundary for the V_1 time derivative

$$\frac{dV_1}{dt} = e^T (G_1) e \le -\lambda_m (G_1) \|e\|^2 \tag{14}$$

where $\left(A_{11} - b_1 k_1^{*T}\right)^T P_1 + P_1 \left(A_{11} - b_1 k_1^{*T}\right) = -G_1$.

The boundaries for the V_2 and V_3 time derivatives are

$$\frac{dV_2}{dt} = x_2^T (G_2) x_2 \le -\lambda_m (G_2) \|x_2\|^2 \tag{15}$$

where $A_{22}^T P_2 + P_2 A_{22} = -G_2$ and

$$\frac{dV_3}{dt} = x_m^T (G_3) x_m \le -\lambda_m (G_3) \|x_3\|^2 \tag{16}$$

where $A_m^T P_3 + P_3 A_m = -G_3$.

It is also necessary to set bounds on the subsystem interactions

$$e^T P_1 A_{12} x_2 \le \|e\| \|P_1 A_{12}\| \|x_2\| \tag{17}$$

where $\|P_1 A_{12}\| = \lambda_M^{1/2} \left(A_{12}^T P_1^T P_1 A_{12}\right) > 0$

$$x_2 P_2 A_{21} e \le \|P_2 A_{21}\| \|e\| \|x_2\| \tag{18}$$

where $\|P_2 A_{21}\| = \lambda_M^{1/2} \left(A_{21}^T P_2^T P_2 A_{21}\right)$ and

$$x_2 P_2 A_{21} x_m \le \|P_2 A_{21}\| \|e\| \|x_m\| \tag{19}$$

The aggregated system is of the form

$$\dot{z} = Wz \tag{20}$$

where z is the aggregated system state vector and W denotes the aggregation matrix

$$W = \begin{bmatrix} w_{11} & w_{12} & 0 \\ w_{21} & w_{22} & w_{23} \\ 0 & 0 & w_{33} \end{bmatrix} \tag{21}$$

with

$$\begin{aligned} w_{11} &= -\lambda_m (G_1) \\ w_{22} &= -\lambda_m (G_2) \\ w_{33} &= -\lambda_m (G_3) \end{aligned} \tag{22}$$

$$\begin{aligned} w_{12} &= \lambda_M^{1/2} \left(A_{12}^T P_1^T P_1 A_{12}\right) \\ w_{21} &= \lambda_M^{1/2} \left(A_{21}^T P_2^T P_2 A_{21}\right) \\ w_{23} &= w_{21} \end{aligned} \tag{23}$$

where $\lambda_m(.)$ denotes the minimal matrix eigenvalue, $\lambda_M(.)$ is the maximal matrix eigenvalue and G_1, G_2, G_3 are the solutions of the following equations

$$\begin{aligned} &\left(A_{11} - b_1 k_1^{*T}\right)^T P_1 + P_1 \left(A_{11} - b_1 k_1^{*T}\right) = -G_1 \\ &A_{22}^T P_2 + P_2 A_{22} = -G_2 \\ &A_m^T P_3 + P_3 A_m = -G_3 \end{aligned} \tag{24}$$

The stability conditions are as follows

$$\begin{aligned} &\text{i)} && -w_{11} > 0 \\ &\text{ii)} && w_{11} w_{22} - w_{21} w_{12} > 0 \\ &\text{iii)} && -w_{33}(w_{11} w_{22} - w_{21} w_{12}) > 0 \end{aligned} \tag{25}$$

Let us now analyze the possibilities of satisfaction of the conditions (25):

The condition i) is satisfied if the reference model is stable.
The condition ii) is essential and its satisfaction will be analyzed in the following.
The condition iii) requires the satisfaction of the condition ii) as well as the stability of the reference model.

Let us define the stability measure $L(w_{11,} w_{12,}, w_{21}, w_{22})$ in the form

$$L(w_{11,} w_{12,}, w_{21}, w_{22}) = w_{11} w_{22} - w_{21} w_{12} \tag{26}$$

then the condition ii) of (25) is

$$L(w_{11}, w_{12}, w_{21}, w_{22}) > 0 \tag{27}$$

which after introducing (22) and (23) into(26) can be rewritten into the following form

$$\begin{aligned} &\lambda_m (G_1) \lambda_m (G_2) - \\ &- \lambda_M^{1/2} \left(A_{12}^T P_1^T P_1 A_{12}\right) \lambda_M^{1/2} \left(A_{21}^T P_2^T P_2 A_{21}\right) > 0 \end{aligned} \tag{28}$$

The symmetrical matrices $A_{12}^T P_1^T P_1 A_{12}$ and $A_{21}^T P_2^T P_2 A_{21}$ have positive eigenvalues and the product $\lambda_m (G_1) \lambda_m (G_2)$ is as well positive if the unmodeled dynamics is stable. The satisfaction of the condition (28) depends on the possibility of arbitrary increasing the value of $\lambda_m(G_1)$ by means of the adaptation. The problem is that the increase of $\lambda_m(G_1)$ provokes also an increase of $\|P_1\|$ and so an increase of $\lambda_m^{1/2}\left(A_{12}^T P_1^T P_1 A_{12}\right)$.

The mechanism of the conditions (25) satisfaction is very complicated and can not be analytically proved. In the control systems with adaptation the satisfaction of these conditions depends on the structure and the norm of interactions.

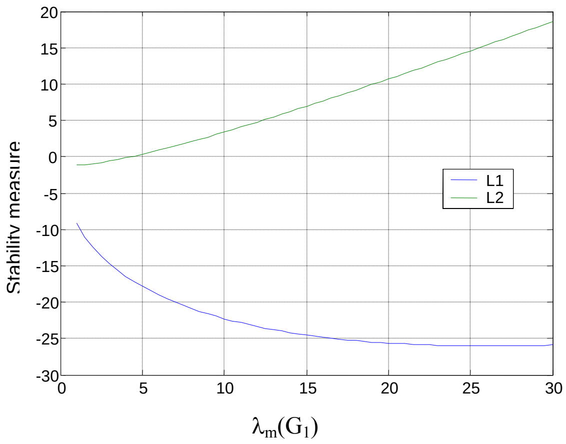

Fig. 1 The stability measure dependence on the value of $\lambda_m(G_1)$

Using a simple example of the 3rd order system consisting of the second order modeled part and of the first order unmodeled part we can illustrate the dependence of the stability measure on the value of $\lambda_m(G_1)$, that represents the influence of state controller gains in the presence of the relatively "strong" interactions.

In Fig. 1 the green line L2 corresponds to the reference model stability measure equal to -1 and the blue line L1 represents the reference model with the stability measure of -0.1. It can be seen that the adaptation error convergence can be influenced by the stability measure of the reference model. However, the reference model dynamics is given by the control performance requirements, so it is necessary to ensure the increase of the reference model stability measure indirectly during the adaptation error transient processes.

This indirect increasing of the stability measure can be obtained by a modification of the control law (4) to the form

$$u = -k_1^T x_1 + k_r r + k_2 e \tag{29}$$

where the feedback term $k_2 e$ ensures the increase of the A_m matrix stability measure.

After introducing (22) and (23) the stability condition ii) of (25) can be rewritten into the form

$$-\lambda_m\left(\left(A_m - b_1 k_2^T\right)^T P_1 + P_1\left(A_m - b_1 k_2^T\right)\right)\lambda_m(G_2) -$$
$$-\lambda_M^{1/2}\left(A_{12}^T P_1^T P_1 A_{12}\right)\lambda_M^{1/2}\left(A_{21}^T P_2^T P_2 A_{21}\right) > 0 \tag{30}$$

The symmetrical matrices $A_{12}^T P_1^T P_1 A_{12}$ and $A_{21}^T P_2^T P_2 A_{21}$ have positive eigenvalues and by means of the appropriate adaptation of k_2 it is possible to ensure the satisfaction of the ii) stability condition in (25).

4. EXAMPLE

Consider the controlled plant model in the form:

$$\dot{x}_1 = \begin{bmatrix} 0 & 1 \\ a_1 & a_2 \end{bmatrix} x_1 + \begin{bmatrix} 0 \\ 1 \end{bmatrix} x_2 + \begin{bmatrix} 0 \\ 1 \end{bmatrix} u$$
$$\dot{x}_2 = \begin{bmatrix} a_4 & 0 \end{bmatrix} x_1 + a_5 x_2 \tag{31}$$
$$y = \begin{bmatrix} 1 & 0 \end{bmatrix} x_1$$

where a_i $(i = 1,\ldots,5)$ are unknown slowly varying parameters.

The output y(t) is required to follow as close as possible the output $y_m(t)$ of the reference model

$$\dot{x}_m = \begin{bmatrix} 0 & 1 \\ -1 & -3 \end{bmatrix} x_m + \begin{bmatrix} 0 \\ 1 \end{bmatrix} r \tag{32}$$
$$y_m = \begin{bmatrix} 1 & 0 \end{bmatrix} x_m$$

Using the proposed modification of the control law (29) and the standard adaptation law (Murgaš, *et al.*, 1992)

$$\frac{d\Theta}{dt} = \alpha\omega\varepsilon \tag{33}$$

where $\alpha > 0$, $\varepsilon = b^T P e$ and P is the solution of the Lyapunov matrix equation

$$A_m^T P + P A_m = -I \tag{34}$$

the acceptable tracking error can be obtained, as illustrated in Fig.2 and Fig. 3.

It can be seen from Fig. 2 and Fig. 3 that the unmodeled dynamics influence reduction by means of the control law modification has been very efficient. The generalization of the proposed solution will be necessary in the future.

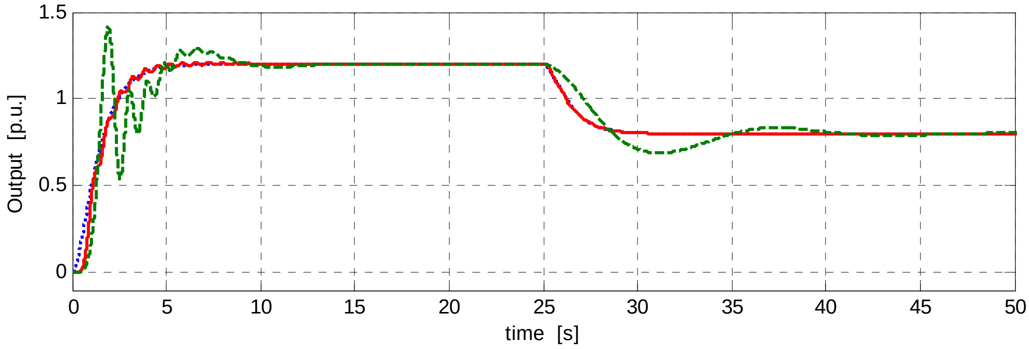

Fig. 2 Performances of the MRAC with the proposed control law modification
modif. MRAC, stand. MRAC, ref. model

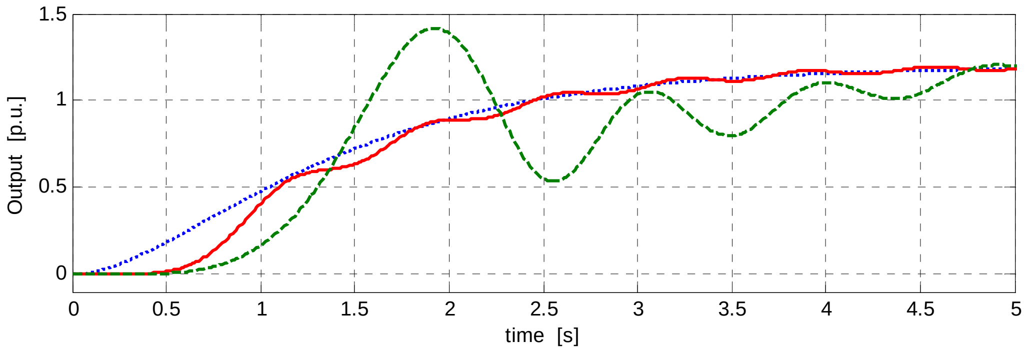

Fig. 3 Performances of the MRAC with the proposed control law modification (detail)
modif. MRAC, stand. MRAC, ref. model

5. CONCLUSION

The aim of the proposed paper has been to reduce the effect of unmodeled dynamics in MRAC tracking problems. The proposed modification of the standard control structure increases the tracking system robustness even in case when the standard adaptation law is used.

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