OBSERVER DESIGN FOR LUR'E SYSTEMS WITH MONOTONE MULTIVALUED MAPPINGS [1]

W.P.M.H. Heemels * **A.Lj. Juloski** ** **B. Brogliato** ***

* *Embedded Systems Institute*
PO Box 513, 5600 MB Eindhoven, The Netherlands
e-mail: maurice.heemels@embeddedsystems.nl
** *Eindhoven University of Technology*
EH 4.15, PO Box 513, 5600 MB Eindhoven,
The Netherlands
e-mail: a.juloski@tue.nl
*** *INRIA Rhone-Alpes*
ZIRST Montbonnot, 655 avenue de l'Europe,
38334 Saint Ismier, France
e-mail:Bernard Brogliato@inrialpes.fr

Abstract: In this paper we present a constructive observer design procedure for a class of nonsmooth dynamical systems, namely systems of Lur'e type with a monotone multivalued mapping in the feedback path. Examples of such systems include various classes of hybrid systems. Under the assumption that the observed system is well behaved, we prove that the proposed observers are well-posed (i.e. that there exists a unique weak solution to the observer dynamics), and that the observer asymptotically recovers the state of the observed system, under the assumption that the weak solutions of the observer are absolutely continuous. The results are illustrated on an example of a deep sea oil drilling assembly with a string. *Copyright 2005 IFAC*

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1. INTRODUCTION

In this paper an observer design procedure for systems of Lur'e type with a maximal monotone multivalued mapping in the feedback path (see figure 1) is developed. The requirements that the mapping is maximal and monotone generalize the usually considered concept of continuous, sector bounded nonlinearity (Vidyasagar, 1993).

Examples of systems obtained by interconnecting linear dynamics in a feedback configuration with maximal monotone mapping, as in figure 1, include various classes of hybrid systems:

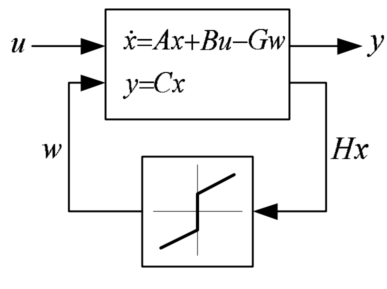

Fig. 1. Lur'e type system with maximal monotone multivalued mapping

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certain piece-wise linear systems (Sontag, 1981) (fig. 2a), linear relay systems (Johansson *et al.*, 1999) (figure 2b), linear complementarity systems (van der Schaft and Schumacher, 1998), (Heemels *et al.*, 2000), (figure 2c), and electric circuits with switching elements (e.g. ideal diodes, fig. 2c, MOS transistors, characteristic in fig. 2d).

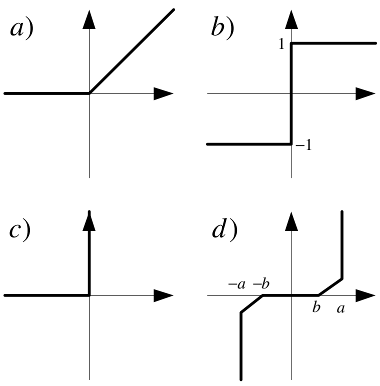

Fig. 2. Examples of maximal monotone mappings

We propose a Luenberger observer structure based on rendering the linear part of the error dynamics strictly positive real (SPR). As the considered systems can be non-Lipschitz, existence and uniqueness of solutions (i.e. well-posedness) of the system and observer is not guaranteed automatically. An observer design methodology for Lur'e type systems with *locally Lipschitz* slope restricted nonlinearities was studied before in (Arcak and Kokotović, 2001). However, since nonsmooth and non-Lipschitz nonlinearities are allowed in the system studied here, the results of (Arcak and Kokotović, 2001) are not applicable, and we have to resort to a framework of convex analysis, to establish an observer design procedure for the considered class of systems.

Under the natural assumption that there exists a solution of the observed system, it is proven that there exists a weak solution of the proposed observer, and that this solution is unique. Proving the existence of strong solutions for the proposed observer structure is an open research problem. Some sufficient conditions will be given in the paper. Well-posedness of the system is an important theoretical question, and, from a practical standpoint, if an observer is to be numerically implemented, well-posedness is necessary to ensure the proper behavior of the implementation.

It is further shown that the observer recovers asymptotically the state of the observed system. These results are applied to a simplified model of a deep sea oil drilling assembly with a string, with the set-valued dry friction with Stribeck effect (Mihajlović *et al.*, to appear).

Stability of Lur'e type systems with SPR linear part and a discontinuous nonlinearity has been studied in (Yakubovich, 1964,1965), but the problem of existence and uniqueness of solutions for this systems was not considered. The main results in this paper generalize results from (Brogliato, 2004) to the case of systems with external inputs.

2. PRELIMINARIES

The material in this section is taken from (Aubin and Cellina, 1984), (Brezis, 1973), (Tyrell Rockafellar, 1970).

With $\mathcal{L}^1_{loc}[0,\infty)$ and $\mathcal{L}^2_{loc}[0,\infty)$ we denote the Lebesgue spaces of locally integrable and square integrable functions functions defined on $[0,\infty)$.

A mapping $\rho : \mathbb{X} \to \mathbb{Y}$, where $\mathbb{X}, \mathbb{Y} \subseteq \mathbb{R}^l$, is said to be *multivalued* if it assigns to each element $x \in \mathbb{X}$ a subset $\rho(x) \subset \mathbb{Y}$ (which may be empty). The domain of the mapping $\rho(\cdot)$, $\operatorname{dom}\rho$ is defined as $\operatorname{dom}\rho = \{x | x \in \mathbb{X}, \rho(x) \neq \emptyset\}$. We define the graph of the mapping ρ as:

$$\operatorname{Graph}\rho = \{(x, x^*) \mid x^* \in \rho(x)\}. \quad (1)$$

Multivalued mapping ρ is said to be *monotone* if

$$\forall x_1, x_2 \in \operatorname{dom}\rho, \quad \forall x_1^* \in \rho(x) \forall x_2^* \in \rho(x_2) \quad \langle x_1^* - x_2^*, x_1 - x_2 \rangle \geq 0, \quad (2)$$

where $\langle \cdot, \cdot \rangle$ denotes the inner product.

A multivalued mapping ρ is said to be *maximally monotone* if its graph is not strictly contained in the graph of any other monotone mapping. In other words, maximality means that new elements can not be added to the $\operatorname{Graph}\rho$ without violating the monotonicity of the mapping. All the examples in figure 2 are maximal monotone mappings.

A *differential inclusion* (DI) is given by an expression of the form

$$\dot{x} \in F(t, x) \quad (3)$$

where F is a set-valued mapping, that associates to the state x of the system and time t the set of admissible velocities. An *absolutely continuous* (AC) function x is considered to be a *strong solution* of the DI (3) if (3) is satisfied almost everywhere. A point x_0 is a *fixed point (equilibrium)* of the DI (3) if $0 \in F(t, x_0)$, $\forall t$.

An important result concerning differential inclusions of the form

$$\dot{x}(t) \in -A(x(t)), \quad x(0) \in \operatorname{dom} A \quad (4)$$

where A is a maximal monotone mapping is that there exists a unique strong solution x, defined on $[0,\infty)$ (Brezis, 1973, section 3.1), (Aubin and Cellina, 1984, chapter 3).

To generalize the previous result to nonautonomous DIs we consider the system of the form:

$$\dot{x}(t) \in -A(x(t)) + u(t), \quad x(0) \in \operatorname{dom} A \quad (5)$$

where A is a multivalued mapping and the external input signal $u \in \mathcal{L}^1_{loc}[0,\infty)$. Following

(Brezis, 1973, section 3.2) we define a continuous function x to be a *weak solution* to (5) if there exist sequences $u_n \in \mathcal{L}^1_{loc}[0,\infty)$ and $x_n \in \mathcal{C}[0,\infty)$ such that x_n is a strong solution to

$$\dot{x}_n \in -A(x_n(t)) + u_n,$$

$u_n \to u$ in $\mathcal{L}^1_{loc}[0,\infty)$ sense and $x_n \to x$ uniformly on every interval $T \subseteq [0,\infty)$.

Proposition 2.1. (Brezis, 1973, theorem 3.4) For the case when the mapping A in (5) is maximal monotone mapping there exists a unique weak solution x to (5) for every $u \in \mathcal{L}^1_{loc}[0,\infty)$.

A difference between weak and strong solutions is that a weak solution, while continuous, is not necessarily absolutely continuous. However, the following holds:

Proposition 2.2. (Brezis, 1973, proposition 3.2): For the case when the mapping A in (5) is maximal monotone mapping we have the following properties:

- *If a strong solution to (5) exists, it is unique*
- *Any AC function x which is a weak solution to (5) is also a strong solution to (5).*

Following (Wen, 1988, theorem 1), we call a linear system given by (A, B, C), where B has full column rank (i.e. $\mathrm{Ker}\{B\} = \emptyset$), strictly positive real (SPR) if there exist a $P = P^\top > 0$ and a $Q = Q^\top > 0$ such that:

$$PA + A^\top P = -Q \tag{6a}$$
$$B^\top P = C \tag{6b}$$

3. PROBLEM STATEMENT

Consider the system that is given by the following differential inclusion (see figure 1):

$$\dot{x} = Ax - Gw + Bu \tag{7a}$$
$$w \in \varrho(Hx) \tag{7b}$$
$$y = Cx \tag{7c}$$

where $Hx(0) \in \mathrm{dom}\,\varrho$ and $A \in \mathbb{R}^{n\times n}$, $B \in \mathbb{R}^{n\times m}$, $G \in \mathbb{R}^{n\times l}$ is full column rank, $H \in \mathbb{R}^{l\times n}$ and $C \in \mathbb{R}^{p\times n}$. The mapping $\varrho : \mathbb{R}^l \to \mathbb{R}^l$ is assumed to be maximally monotone.

Assumption 3.1. For all initial states $x(0)$ such that $Hx(0) \in \mathrm{dom}\,\varrho$ and inputs $u \in \mathcal{L}^1_{loc}[0,\infty)$ of interest, we assume that the system (7) has a strong solution.

The proposed observer has the following form:

$$\dot{\hat{x}} = (A - LC)\hat{x} - G\hat{w} + Ly + Bu \tag{8a}$$
$$\hat{w} \in \varrho((H - KC)\hat{x} + Ky) \tag{8b}$$
$$\hat{y} = C\hat{x} \tag{8c}$$

where $K \in \mathbb{R}^{l\times p}$ and $\hat{x}(0)$ are such that $(H - KC)\hat{x}(0) + Ky(0) \in \mathrm{dom}\,\varrho(\cdot)$.

4. MAIN RESULTS

The problem of observer design consists in finding the gains L, K which will guarantee that there exists a unique solution $\hat{x}$ to the observer dynamics on $[0,\infty)$, and that $\hat{x}(t) \to x(t)$ as $t \to \infty$. In this section we will prove that if L and K are chosen such that the triple $(A - LC, G, H - KC)$ is SPR the obtained observer (8) will satisfy the mentioned requirements.

Before we prove this we will first show how the gains L and K can be computed such that $(A - LC, G, H - KC)$ is SPR. This can be achieved by solving the matrix inequality:

$$(A - LC)^\top P + P(A - LC) < 0 \tag{9a}$$
$$G^\top P = H - KC. \tag{9b}$$

Inequality (9) is a linear matrix inequality in P,K,$L^\top P$. For necessary and sufficient conditions for the existence of solutions for (9), see for instance, (Arcak and Kokotović, 2001).

To prove that SPR property of $(A - LC, G, H - KC)$ guarantees the proper behavior of the observer, we start of with a theorem on well-posedness.

Theorem 4.1. Consider the system (7), under assumption 3.1, and the observer (8). If the triple $(A - LC, G, H - KC)$ is SPR, the observer dynamics (8) has a unique weak solution on $[0,\infty)$.

Proof. Since the triple $(A - LC, G, H - KC)$ is SPR and G has full column rank there exist P, Q that satisfy (6). Introduce the change of variables:

$$z = R(\hat{x} + g), \tag{10}$$

in (8), where $RR = P$ and define

$$g = (H - KC)^\top((H - KC)(H - KC)^\top)^{-1}Ky. \tag{11}$$

From the SPR condition it follows that $H - KC$ has full row rank (as $H - KC = G^\top P$), and hence the inverse in (11) exists. Define the mapping $f : \mathbb{R}^n \to \mathbb{R}^n$ as $f(z) = R^{-1}(H - KC)^\top \varrho((H - KC)R^{-1}z)$. Using SPR condition (6b), (8) transforms into:

$$\begin{aligned}\dot{z} \in\, & R(A - LC)R^{-1}z - f(z)\\ & + R(Bu + Ly + (A - LC)g + \dot{g})\end{aligned} \tag{12}$$

where $z(0) \in \mathrm{dom}\,f(\cdot)$. From the SPR condition (6b) it follows that $H - KC$ and $(H - KC)R^{-1}$ have full row rank, and together with the fact that ϱ is maximal monotone we have that f is maximal monotone as well (Tyrell Rockafellar and Wets, 1998, theorem 12.43).

From the SPR condition (6a) it follows that $R(A - LC)R^{-1} + R(A - LC^\top)R^{-1}$ is negative definite. Hence the mapping $z \to -R(A - LC)R^{-1}z +$

$f(z)$ is maximal monotone (Tyrell Rockafellar and Wets, 1998, corollary 12.44). By assumption we have $u \in \mathcal{L}^1_{loc}[0,\infty)$, $y \in \mathcal{L}^1_{loc}[0,\infty)$. Moreover, y is AC and thus it follows that $\dot{g} \in \mathcal{L}^1_{loc}[0,\infty)$. By virtue of proposition 2.1, (12) and hence (8) posses a unique weak solution. $\diamondsuit$

To ensure the existence of strong solutions more stringent assumptions have to be imposed on the original system and proposed observers. If we consider again the inclusion (12), conservative sufficient conditions for existence of strong solutions are given for instance in (Brezis, 1973, theorem 3.6), which state that:

- $u \in \mathcal{L}^{loc}_2[0,\infty)$, $\dot{y} \in \mathcal{L}^{loc}_2[0,\infty)$
- there exists a proper convex lower semicontinuous function ξ with $\xi(z) \geq \delta$, for all z and some δ, such that:

$$-R(A-LC)R^{-1}z + f(z) = \partial\xi(z) \tag{13}$$

  where $\partial\xi$ denotes the subdifferential of ξ (see (Brezis, 1973),(Tyrell Rockafellar, 1970), (Tyrell Rockafellar and Wets, 1998) for the details). For (13) to hold, it is required that the mapping $z \to -R(A-LC)R^{-1}z + f(z)$ satisfies a property called maximal cyclic monotonicity. In our case this would mean that $R(A-LC)R^{-1}$ is symmetric positive semi-definite as shown in (Tyrell Rockafellar, 1970, chapter 24), and that ϱ can be written as $\varrho = \partial\varphi$ for some proper lower semicontinuous function φ (Tyrell Rockafellar, 1970, theorems 24.8, 24.9).

For particular choices of maximal monotone mappings (e.g. relay or complementarity characteristics) some results are available in the literature ((Heemels *et al.*, 2000; Çamlibel *et al.*, 2003; van der Schaft and Schumacher, 1998; Lootsma *et al.*, 1999)). The question of existence of strong solutions for the general observer structure is an open research problem, and will be considered in the future work. Here, we make the following assumption.

Assumption 4.2. Weak solutions for the observer (8) are AC (and thus, weak solutions are strong solutions by proposition 2.2).

For the observer (8) the observation error $e := x - \hat{x}$ dynamics can be formed as:

$$\dot{e} = (A-LC)e - G(w-\hat{w}) \tag{14a}$$
$$w \in \varrho(Hx) \tag{14b}$$
$$\hat{w} \in \varrho(H\hat{x} + K(y-\hat{y})) \tag{14c}$$

Note that the point e_0 is a fixed point (equilibrium) of system (14) for a given x-trajectory if it satisfies the following inclusion for all $t > 0$:

$$0 \in (A-LC)e_0 - G[\varrho(Hx(t)) - \varrho(H\hat{x}(t) + KCe_0)] \tag{15}$$

where $\hat{x}(t) = x(t) - e_0$.

Theorem 4.3. Consider the observed system (7) *under assumption 3.1, the extended observer* (8), *where the triple* $(A-LC, G, H-KC)$ *is SPR, under assumption 4.2, and the observation error dynamics* (14). *The point* $e = 0$ *is the unique fixed point of the observation error dynamics* (14) *and is globally exponentially stable.*

Proof. Note that $e_0 = 0$ is a fixed point of (14), since it satisfies the inclusion (15).

Next, we show that $e_0 = 0$ is the only fixed point. From $(A-LC)e_0 \in G(\varrho(Hx) - \varrho(H\hat{x} + KCe_0))$ for all $t \geq 0$ it follows that $P(A-LC)e_0 \in PG(\varrho(Hx(t)) - \varrho(H\hat{x}(t) + KCe_0))$ for some t. Using the SPR condition (6b) we get the following condition for the fixed point e_0:

$$e_0^\top P(A-LC)e_0 = ((H-KC)e_0)^\top(w-\hat{w})$$

where $w \in \varrho(Hx(t))$ and $\hat{w} \in \varrho(H\hat{x}(t) + KCe_0)$. From the SPR condition (6a) it follows that $e_0^\top P(A-LC)e_0 \leq 0$. From the monotonicity condition (2) for $\varrho(\cdot)$ it follows that $e_0^\top P(A-LC)e_0 = ((H-KC)e_0)^\top(w-\hat{w}) \geq 0$. Hence, $e_0 = 0$ is the only solution of the inclusion (15).

To show that the unique fixed point $e_0 = 0$ is globally exponentially stable consider the Lyapunov function $V = \frac{1}{2}e^\top Pe$. Since by assumption 3.1 x is AC, and by assumption 4.2 $\hat{x}$ is AC it follows that e is also AC, and $\dot{e}$ exists almost everywhere. Hence, V is also AC, and the derivative $\dot{V}$ exists almost everywhere. $\dot{V}$ satisfies:

$$\begin{aligned}\dot{V} &= e^\top P\dot{e} = e^\top P((A-LC)e - G(w-\hat{w})) \\ &= -\frac{1}{2}e^\top Qe - e^\top(H-KC)^\top(w-\hat{w}) \qquad (16a) \\ &\leq -\frac{1}{2}e^\top Qe\end{aligned}$$

for some $w, \hat{w}$ satisfying (14b),(14c). From $V(t) \leq V(0) - \frac{1}{2}\int_0^t e^\top(\tau)Qe(\tau)d\tau$ it follows that the AC function of time V is nonincreasing, and $\frac{1}{2}\lambda_{min}(P)e^\top(t)e(t) \leq V(0) - \frac{1}{2}\int_0^t \lambda_{min}(Q)e^\top(\tau)e(\tau)d\tau$ where $\lambda_{min}(\cdot)$ denotes the minimal eigenvalue. From Gronwall's lemma (Vidyasagar, 1993):

$$\frac{1}{2}\lambda_{min}(P)e^\top(t)e(t) \leq V(0)\exp\left(-\frac{\lambda_{min}(Q)}{\lambda_{min}(P)}t\right). \tag{17}$$

$\diamondsuit$

5. EXAMPLE

A simplified scheme of a deep see oil drilling equipment is depicted in figure 3. The assembly consists of the drilling tool (depicted by a small disc), rotary table (big disc) which acts as a reservoir of kinetic energy, DC motor, and a drill string, which is used to transmit the energy from the surface to the drilling tool.

An experimental setup mimicking the drilling equipment was realized by Mihajlović et al.

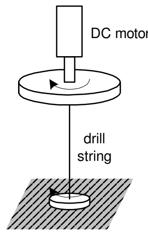

Fig. 3. Drilling assembly with a string

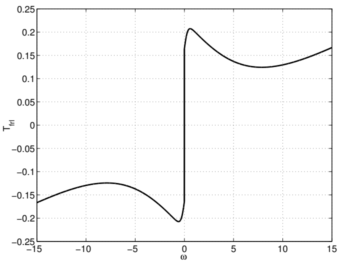

Fig. 4. Dry friction characteristic

(Mihajlović *et al.*, to appear). It was shown that the dynamics of the experimental setup can be accurately described by the following model:

$$\dot{x}_1 = x_2 - x_3 \tag{18a}$$

$$\dot{x}_2 = \frac{k_m}{J_u}u - \frac{k_\theta}{J_u}x_1 - \frac{1}{J_u}T_{fru}(x_2) \tag{18b}$$

$$\dot{x}_3 \in \frac{k_\theta}{J_l}x_1 - \frac{1}{J_l}T_{frl}(x_3), \tag{18c}$$

where x_1 is the difference in angular positions of the discs, x_2 is the angular velocity of the upper disc and x_3 is the angular velocity of the lower disc. The measured variable is taken to be $y = x_1$.

$T_{fru}(\cdot)$ and $T_{frl}(\cdot)$ denote the friction moments at the upper and the lower disc, respectively. $T_{fru}(\cdot)$ is dominated by the viscous friction, and for simplicity, is here taken to be equal to $b_{up}x_2$. The friction moment at the lower disc $T_{frl}(\cdot)$ is a dry friction with the Stribeck effect, i.e. negative damping appears at a certain range of angular velocities. To describe this friction torque a set-valued characteristic based on neural networks is used in (Mihajlović *et al.*, to appear):

$$T_{frl}(x_3) = \begin{cases} (T_{stickl} + T_1(1 - \dfrac{2}{1+e^{w_1|x_3|}}) + \\ T_2(1 - \dfrac{2}{1+e^{w_2|x_3|}}))\,\mathrm{sign}(x_3) + b_l x_3 \\ \qquad \text{for } x_3 \neq 0 \\ [-T_{stickl}, T_{stickl}] \\ \qquad \text{for } x_3 = 0 \end{cases} \tag{19}$$

Numerical values of the parameters in (18) and (19) are given in table 1. The set valued friction law (19), with parameter values from table 1 is depicted in figure 4.

Table 1. Parameter values of the model

J_u	$0.4765\frac{\text{kgm}^2}{\text{rad}}$	T_{stickl}	0.1642Nm
J_l	$0.0326\frac{\text{kgm}^2}{\text{rad}}$	T_1	0.0603Nm
k_m	$3.9950\frac{\text{Nm}}{\text{V}}$	T_2	−0.2267Nm
k_θ	$0.0727\frac{\text{Nm}}{\text{rad}}$	w_1	5.7468
b_{up}	$2.2247\frac{\text{kgm}}{\text{rads}}$	w_2	0.2941
b_l	$0.0109\frac{\text{kgm}}{\text{rads}}$		

For the purpose of simulation the input signal u in (18) is chosen to be a constant signal, $u = 2\text{V}$. It is easy to check that for the chosen input signal the system (18) satisfies the conditions of (Filippov, 1988, theorem 2.7.1), and hence has a solution on a arbitrarily long time interval for every initial condition x_0, i.e. the system (18) satisfies assumption 3.1.

The friction mapping depicted in figure 4 is not monotone, but can be transformed into a monotone mapping using the technique of loop transformation (Vidyasagar, 1993). The new friction mapping is defined as $\tilde{T}_{frl}(\omega) = T_{frl}(\omega) - m\omega$, where $m = -0.02$ is the maximal negative slope of the graph in figure 4. The system matrix A is replaced by $\tilde{A} = A - mGH$.

We will design the observer (8). Observer design of the form (8) for system (18) entails finding gains L and K such that the triple $(\tilde{A} - LC, G, H - KC)$ is SPR. The following values for P, Q, L and K are found:

$$P = \begin{bmatrix} 0.804 & 0.029 & 0.066 \\ 0.029 & 0.110 & -0.000 \\ 0.066 & -0.000 & 0.032 \end{bmatrix}, \; Q = \begin{bmatrix} 0.548 & -0.000 & -0.000 \\ -0.000 & 0.970 & -0.036 \\ -0.000 & -0.036 & 0.092 \end{bmatrix},$$

$$L = [\, 2.476 \;\; 5.199 \;\; -26.220 \,]^\top, \; K = -2.025.$$

We will show simulations for the initial state for the system taken as $x(0) = [0 \;\; 0 \;\; 0]^\top$ and for the observer as $\hat{x}(0) = [3 \;\; 3 \;\; 3]^\top$. The solution of the system (18) is constructed using the dedicated technique for simulating friction based on the switched friction models presented in (Leine *et al.*, 1998). The solution of the observer (8) is computed using the implicit midpoint rule (Juloski, 2004).

The simulation results are depicted in figure 5 and the estimation error is depicted in figure 6. When a constant input voltage is applied (i.e. a constant torque is applied to the upper disc) slip-stick oscillations in the angular velocity of the second disc x_3 occur due to the negative damping in the friction law (19). During this oscillations the velocity of the third disc alternates between 0 (stick phase), and a positive value (slip phase). As guaranteed by the theory, the designed observer is able to provide the correct estimate of the state. Moreover, based on (17) we can provide a bound on the decrease of the squared estimation error. This is indicated by the dashed line in figure 6.

6. CONCLUSIONS

In this paper we consider an observer design for Lur'e type systems with maximal monotone multivalued mappings in the feedback path. In contrast with the previous work on nonlinear observer design, the considered class of systems in nonsmooth and the standard theory does not apply. Even the existence and uniqueness of solutions is not a priori guaranteed.

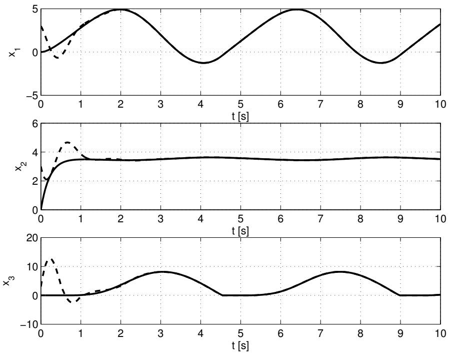

Fig. 5. Responses of the system (solid) and the observer (dashed): x_1 (upper), x_2 (in the middle), x_3 (lower) under the constant input voltage

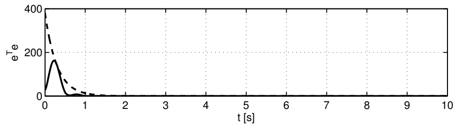

Fig. 6. The norm of the estimation error(solid) and the envelope of the error norm (17) (dashed)

We proposed an observer structure, together with a constructive design method. The approach taken in the paper is based on rendering the linear part of the observation error dynamics SPR, by choosing appropriate observer gains. Under the natural assumption that the observed system has a solution, and that the control input belongs to a certain admissible class, it is shown that there exists a unique solution for the estimated state, and that the observer recovers the state of the original system asymptotically. The relevance and applicability of the presented results is demonstrated on the example of the drilling system.

Future work will mainly investigate the issue of existence of strong solutions for the proposed observer.

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