INFORMATIVE EXPERIMENT DESIGN AND NON-PARAMETRIC IDENTIFICATION OF 6-DOF MOTION PLATFORM

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Abstract: In this paper the input design and non-parametric identification of a 6-input, 6-output Stewart Platform is considered. The approach takes into account the information matrix for the choice of realization combination of experiments. It considers as well a least squares recovery of frequency domain components. This approach makes maximum use of periodic excitations taking into account the constraints on the experimental conditions imposed over the system such as limited velocity and acceleration in order to recover the frequency response of the system from closed-loop data.

Keywords: System identification, Non-parametric identification, Multi-input/multi-output systems, Frequency domain, Closed-loop Identification.

1. INTRODUCTION

The SIMONA research project (SIMONA stands for the Institute for Research in SImulation, MOtion and NAvigation) has as one of its objectives the study in the field of simulation and pilot training by use of an advanced flight-simulator. This simulator (Figure 1) is composed of a light-weight cockpit (3500 kg) mounted over a six degree of freedom motion platform driven by six 1.20m-stroke hydraulic servo actuators. It is desired that this simulator has a wider bandwidth than conventional simulators to simulate special conditions. Therefore, the simulator is lighter and more flexible introducing important flexible dynamics to be considered for high performance control. At present, an approximation of the real system considering only the rigid body motion platform dynamics has been used for an initial control design (Koekebakker, 2001),(Valk, 2004). Better approximations of the dynamics of the whole system are necessary for increasing the performance in the entire working envelope to fulfil bandwidth requirements. For identification and control studies also a scaled (3:1) Stewart platform has been built.

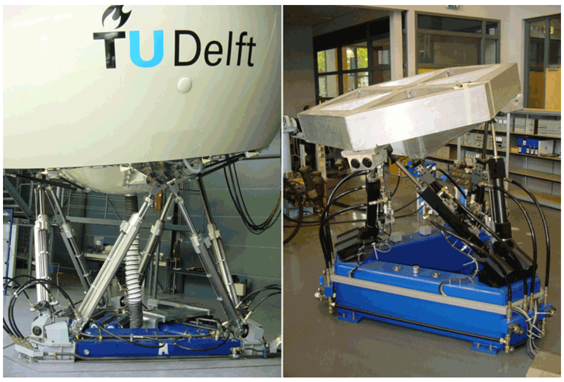

Fig. 1. Left: SIMONA Research Simulator, Right: Smaller Platform

The derivation of accurate dynamical models for mechanical systems based on black box system identification techniques has acquired more and more importance in recent years where high performance objectives are needed for high-precision mechanical servo systems. In this framework many system identification techniques have been addressed by several authors (Ljung, 1987), (McKelvey, 2002), (Pintelon and Schoukens, 2001). Additionally, in the last decade, a lot of research activity is to be found in the frequency domain system identification ((Pintelon and Schoukens, 2001), (McKelvey, 2002)). Having advantages such as delay modelling, shorter data sequences and straightforward superposition of experiment results which are preferred in many cases. However, much less research is found on multi-input, multi-output mechanical systems and most of the multivariable identification applications on real systems have a limited number of inputs and outputs and only two or three degrees-of-freedom (Callafon *et al.*, 1996), (Ninness and Gomez, 1995), (Liu and Wu, 1982). Frequency domain multi-variable identification provides a viable approach for high dimensional mechanical systems with efficient data processing with the advantages of good superposition possibilities. Moreover, combined with the use of periodical excitation, it is considered highly appropriate and optimally informative when properly designed for the identification of multivariable systems ((Schoukens *et al.*, 1994)).

Considering the above observations this paper addresses the estimation of a non-parametric multivariable frequency response of the 6-input, 6-output SIMONA Research Simulator motion system that has a Stewart Platform configuration. It is chosen to have a representation prior to model fitting enabling the reduction or elimination of error sources prior to fitting. The errors due to model structure selection and order thus are isolated from errors in excitation and measurement. For this purpose the following steps will be considered: First, the input signal design will be considered reaching towards informative experiments. Then, general experimental considerations will be addressed. Followed by a procedure for processing of the measured data and the estimation of frequency domain closed-loop responses. Additional analysis will be performed over some of the data sequences to give a measure of the approximated non-linear behaviour. Finally the open-loop response for the flight simulator system will be recovered from the previous found responses and conclusions will be drawn.

2. INFORMATION MATRIX

Considering the identification of systems it is of importance that the information that can be extracted from an experiment is as good as possible. The information that is related to the variance of the parameters of the final model, is defined by the Cramer Rao lower bound and it depends on parameters that can be adjusted when designing the input signals for the system identification experiment and when performing the experiment.

It has also to be considered that the input design has to take into account the constraints imposed in the experimental conditions. In the identification of the Stewart Platform the constraints are given by maximum acceleration and speed as a function of a position in the working envelope of the Stewart platform configuration.

In the multivariable case the matrix $\mathcal{R}_0(\omega)$ defining the Fourier Transform of different realization sequences of a exciting reference is defined as (1).

$$\mathcal{R}_0(\omega) = \left[\, \mathbf{R}_{0,1}(\omega)\ \mathbf{R}_{0,2}(\omega)\ \cdots\ \mathbf{R}_{0,n}(\omega) \,\right] \quad (1)$$

where $\mathbf{R}_{o,i}(\omega)$ is the Fourier transform of the exciting reference of the i_{th} experiment realization at frequency ω and n is the number of realizations. The determinant of matrix $\mathcal{R}_0(\omega)$ is related to the information matrix so that an optimal input design will have a maximum $|det(\mathcal{R}_0(\omega))|$ (Guillaume *et al.*, 1996). The maximization of the determinant of matrix $\mathcal{R}_0$ can be considered in two different parts as given by (2), where $a(\omega)$ is the maximum amplitude for a given frequency and $\mathbf{Q}$ is a matrix composed of -1 and 1 elements.

$$\mathcal{R}_0(\omega) = a(\omega)\mathbf{Q} \quad (2)$$

The maximization of $a(\omega)$ which will be constrained by the maximum energy and amplitude to be used in the experiment. The maximization of $det|\mathbf{Q}|$ will be given by the chosen combination sequence of experiment realizations that will provide better conditioning of the experiments as well as improved information for the same independent signal amplitude.

The maximization of $a(\omega)$ is done by using energy compressed signals that use efficiently the energy applied to the excitation for a given limit in the amplitude of the excitation signals. This is equivalent to having signals with a low crest factor. There are several methods for optimization of multisine signals in order to minimize the crest factor ((Morelli, 2003), (van den Bos, 1987), (Guillaume *et al.*, 1991)). However most of these non-linear optimizations result in non-acceptable local minima or involve high computational effort as it is shown by Table 1. Table 1 illustrates the average results of different self-implementation of the referred methods in Matlab applied to 20-component multisines with different frequency distributions.

In this application a minimization by selection of ϕ_1 as given in Schroeder's equation (3) provided

Table 1. Crest Factor minimization

Schroeder (Schroeder, 1970)	1.8622
Levenberg-Marquardt (Guillaume *et al.*, 1991)	1.7998
Simplex (Morelli, 2003)	1.8397
Iterative Least Squares (van den Bos, 1987)	1.7807

a acceptable crest factor in an almost direct way (Schroeder, 1970).

$$\phi_n = \phi_1 - \frac{\pi n^2}{N} \quad (3)$$

In (3), ϕ_n is the phase of the n_{th} frequency component in the Schroeder multisine, N is the total number of frequency components and ϕ_1 is the first component phase.

The second part considers a sequence combination matrix Q (Guillaume *et al.*, 1996) composed only of 1 and -1 elements. There are several choices that can be made, one of this choices is given by the combination matrix, Q_6 in (4), which will provide a maximum in $det|Q_6|$.

$$Q_6 = \begin{bmatrix} 1 & -1 & -1 & 1 & -1 & -1 \\ 1 & 1 & -1 & -1 & -1 & -1 \\ 1 & -1 & 1 & -1 & -1 & -1 \\ 1 & 1 & 1 & 1 & 1 & -1 \\ 1 & -1 & -1 & -1 & 1 & 1 \\ 1 & 1 & 1 & 1 & -1 & 1 \end{bmatrix} \quad (4)$$

3. EXPERIMENT CONSIDERATIONS

The signal design for the system identification of the flight simulator dynamics is based on multi sine use, the number of frequency components in the multi sine is enough to assure the persistence of excitation for the overall experiment. Several considerations have to be taken into account in order to make the approximations more reliable.

1. A base frequency is to be chosen, all other frequencies should be integer multiples of the base frequency. The selection of the base frequency is the maximum resolution that can be reached between successive frequency points. A small frequency will give a high resolution but will increase the experiment time.

2. Multiple frequency points are designed to cover the frequency range of interest. Frequency domain measurements can be adapted to particular frequency spectra resulting in good signal to noise ratios on the frequencies of interest (Schoukens *et al.*, 1994). In this paper a non uniformly spaced frequency distribution from 0.1 to 650 Hz with uniform amplitude spectra was applied.

3. Frequency domain experiment results can be superposed. This allows the choice of different resolutions for different experiments or frequency ranges. A good selection will reduce the experiment time considerably. In this paper a division of frequency ranges is considered. The low frequency range with high resolution (0.1 to 24) and the high frequency range with low resolution (25 to 650 Hz).

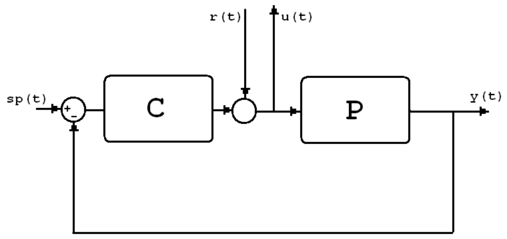

Fig. 2. Measurement configuration

4. The measurements will be taken during an integer number of periods to avoid leakage effects that would introduce errors in the estimation of the frequency domain Fourier coefficients.

5. Via the automated script function in Matlab using the real time control system of the simulator the measurements will be synchronized and no further estimation errors due to an inaccurate trigger on the measurements is going to be considered (Guillaume *et al.*, 1991). The measurement configuration will be given by figure (2). The measured signals are u(t) and y(t) generated by excitation signal in the force level r(t). The excitation signal is a multi-sine thus in the frequency domain we will have a collection of dirac functions representing it.

4. DATA PROCESSING

The approach taken for data processing is the least squares projection of each of the measured signals in a basis composed by the frequency components known to be present in the excitation signal. The basis will be composed additionally by a first column element that takes into account the mean value that could be present on the data. The general form of the basis used is given by (5), where f_i is the i_{th} frequency component of the exciting n-component signal and by linear mapping also the frequency components of the measured signals.

$$H = \begin{bmatrix} 1 & cos(f_1 t) & sin(f_1 t) & cos(f_2 t) & \cdots & sin(f_n t) \end{bmatrix} \quad (5)$$

The same basis is also used to test the level of non-linearities present in the measurements.

The illustrative experiment to be considered spans a frequency range from 0.2 to 24 Hz with a resolution (basic harmonic) of 0.2 Hz. This frequency

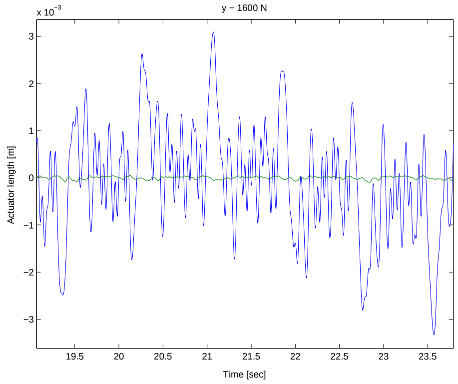

Fig. 3. Measured Output and Estimated Residuals (Time Domain)

range corresponds to one of the high resolution experiments.

Using the basis previously defined the Fourier coefficients (Y_F) that are main components of the measurement ($y(t)$) are estimated.

$$y(t) = HY_F + e(t) \tag{6}$$

$$Y_F = (H^T H)^{-1} H^T y(t) \tag{7}$$

e are the residuals considered to be a quasi-stationary signal orthogonal to Y_F. The following matrix computations are then of interest:

$$(H^T H)^{-1} H^T (\text{Projector}) \tag{8}$$

$$e = (I - H(H^T H)^{-1} H^T) y(t) (\text{Residuals}) \tag{9}$$

Using this least squares projection in place of a Fast Fourier Transform the effects of random noise and other disturbances are diminished.

Applying the projector (8), composed of the frequency points contained in the excitation signal, to all outputs we estimate the residuals as given by (9). As it can be seen in Figure(3), for the average case of the diagonal terms, the level of noise in the measurement signal amounts to less than 1% of the total signal amplitude, this gives a good measure of the signal to noise ratio present in the experiments.

5. LINEAR APPROXIMATION OF NON-LINEAR BEHAVIOUR

The mildly non-linear real plant is approximated by a linear mapping. In order to analyze the degree of non-linearity, the residuals of the first base frequency projection are again projected into a basis defined by second and third order harmonics of the base frequency components. These components will be used to evaluate the level of non-linearity of the system for a given working point and amplitude gain. Non-linearities like friction can be reduced with a higher amplitude but

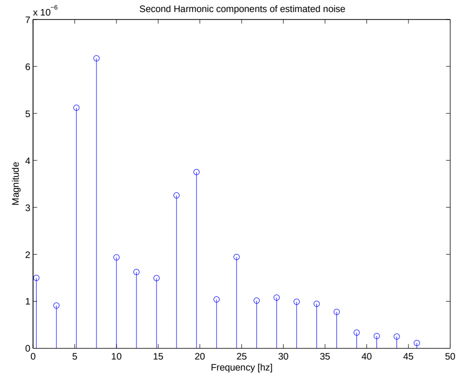

Fig. 4. Fourier Coefficients Second Order Harmonics (1600 N gain)

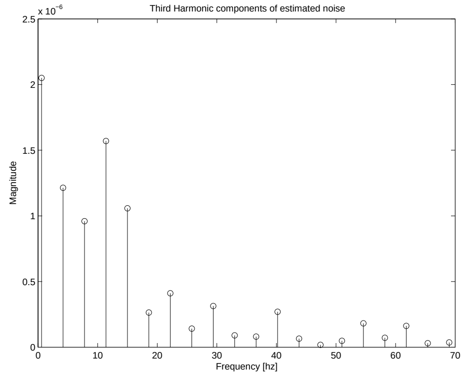

Fig. 5. Fourier Coefficients Third Order Harmonics (1600 N gain)

then the position dependent non-linear effects also increase. An optimal balance between these two effects is to be found.

An additional basis defined by the second order harmonics from the first basis is then created. Then the components of the estimated residuals (Figure 3) corresponding to the second harmonics are estimated. The Fourier coefficients for the second basis are comparatively small as it can be seen on figure 4. In this figure, the projection corresponding to the first diagonal element output is shown, as it represents the average case for all diagonal terms.

Further the residuals are projected into a basis defined by third harmonics frequencies. The third harmonic components of the residuals are even smaller than the second order harmonic components and amount to less than 1% of the signal energy. In figure 5 the projection corresponding to the first diagonal element output is shown.

Moreover the components are in the same amplitude level as the remaining residuals. There also may be nonlinear effects in the main frequency components. For the purpose of analyzing this effect additional measurements were taken using half the gain in the exciting signals and

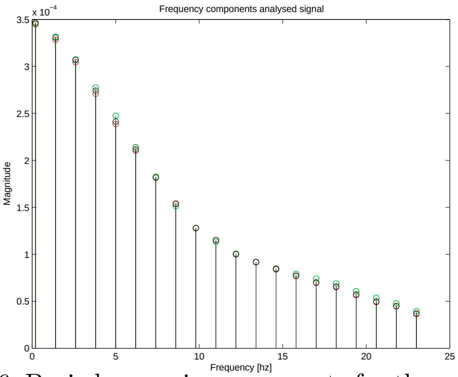

Fig. 6. Basic harmonic components for the output measurement at different amplitudes (scaled to basic amplitude)

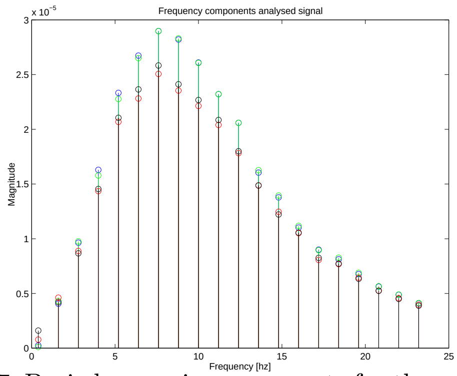

Fig. 7. Basic harmonic components for the output measurement at different amplitudes (scaled to basic amplitude)

also a negative gain. In Figure (6) the responses to all different gain experiments are scaled to base amplitude and superposed. It is clear that even though the gain is varying the superposition principle of linear systems is still an accurate description. Additionally, when using different gains the amplitude of the noise changes accordingly to the gain which makes us think of small non-linear components. There are small differences for the output description in terms of base frequency components. This would indicate the presence of slight noise plus non-linearities. However the transfer function between output and inputs is consistent for all experimented gains.

In the case of non-diagonal elements the noise component and non-linearities are as low as in the diagonal case, however the magnitude of the basic components is in the level of the noise and more dissimilarities are found when comparing the experiment for different amplitudes, due to the lower signal to noise ratio achieved. The average case for the non-diagonal element projection can be found on Figure (7).

6. OPEN-LOOP RESPONSE RECOVERY FROM CLOSED-LOOP DATA

The reference, input and output measurements denoted by $\mathcal{R}_0(\omega)$, $\mathcal{U}_0(\omega)$ and $\mathcal{Y}_0(\omega)$ respectively, are defined in a similar manner as (1) for the collection of data in the frequency domain of all necessary experiment realizations.
The estimator from the data contained in 6 linear independent realizations under the specified experiment considerations can be written for the closed loop ($T(\omega)$) and process sensitivity ($S(\omega)$) transfer function matrices as given in (10) and (11).

$$\hat{T}(\omega) = \mathcal{Y}(\omega)\mathcal{R}^{-1}(\omega) \tag{10}$$

$$\hat{S}(\omega) = \mathcal{U}(\omega)\mathcal{R}^{-1}(\omega) \tag{11}$$

Further a non-parametric estimate of the open-loop plant can be found indirectly as (12).

$$\hat{P}(\omega) = \hat{T}(\omega)\hat{S}^{-1}(\omega) \tag{12}$$

In figure (8) all the measurements are combined to generate a non-parametric representation of the 6-degrees-of-freedom Stewart Platform.

7. CONCLUSIONS

The input signal design based on the information matrix provides a good alternative for the selection of experiment combinations relevant for multi-input systems. The advantages of low crest factor signals are further emphasized by the use of an appropriate sequence of experiment realizations that will provide good conditioning for the further estimation of non-parametric or parametric models. Additionally, in this setting the given experiment constraints are taken into account which allow the user to design efficient feasible experiments to be applied in practical servo-mechanical systems in an automated iterative way that has proven to be efficient.

The a priori knowledge of excitation signal frequency contents provides a useful tool for the analysis of the frequency response focused on the excited dynamics and thus reducing the effects of noise. The effects of random noise are further reduced by the use of the least squares projection of the measured signals in an appropriately chosen basis. The frequency content of the measured signals was recovered with the least squares projection taking into account in some measure the effects of random white noise effects, and the effects of means. This gives better results than using the Fast Fourier Transform.

The level of approximated non-linearities for the Stewart Platform motion system is low enough to permit a linear approximation of models for an specific operation point.

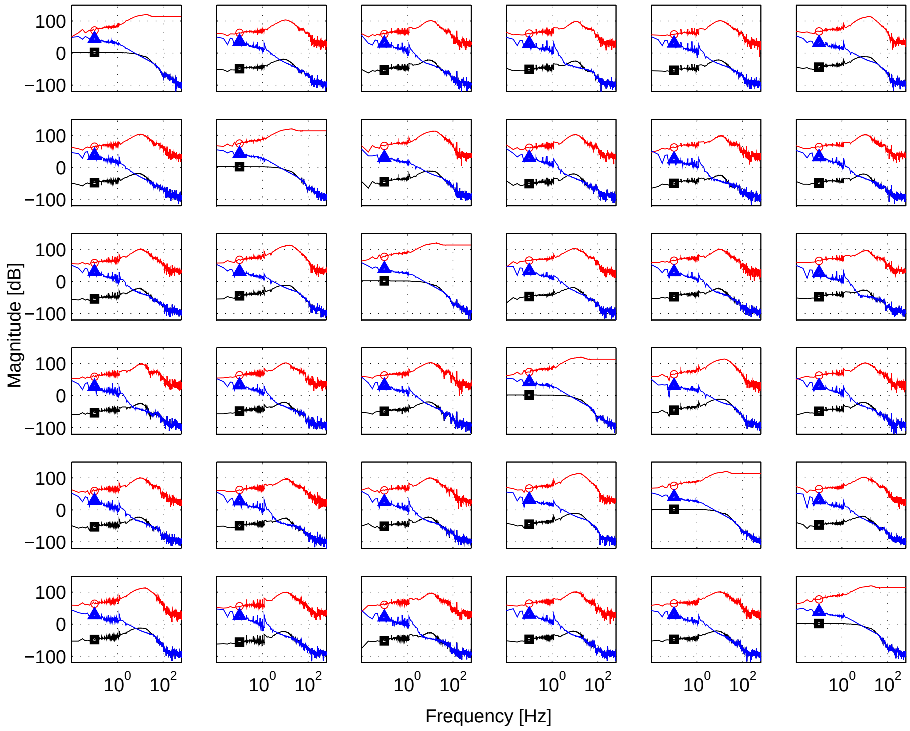

Fig. 8. Non-parametric representation of the Stewart Platform dynamics. Closed-loop (black,□), Process Sensitivity (red,o) and Open-loop recovered response (blue, Δ).

The quality of the signal recovery makes possible the use of straightforward least squares estimators for the recovery of the frequency response of the open loop plant.

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