ORIENTATION CONTROL OF AN ARTIFICIAL SATELLITE USING THE MULTIPLEX APPROACH

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Abstract: The interval-multiplex approach was applied to controlling the orientation of an artificial satellite under incomplete information about its state vector. When estimating the state vector, use was made of additional restrictions on the values of inaccuracy of measurements. Computer simulation demonstrates the effectiveness of using additional restrictions.

Keywords: adaptive control, identification, artificial satellites, estimation, chaotic behavior, computer simulation.

1. INTRODUCTION

The first automatic system allowing to control the triaxial orientation of an artificial satellite with respect to the orbital coordinate system (OCS) was the control system including an orbital gyro compass (GC), an infrared builder of the local vertical (BLV) and three sensors measuring the angular velocity of the AS with respect to its center of gravity. GC is used for the purpose of determining the course of the AS, i.e. the angle contained by the vertical plane of its symmetry and the orbital plane. With developing vehicle-borne computer aids, a possibility appeared to exclude the GC (representing a complicated electromechanical device) from the structure of the gaging equipment and to replace direct measurement of the angle of the AS course by calculation of its estimates. In order to calculate these estimates, use is usually made of the known algorithms of estimating the status of dynamic systems (state observers and Kalman filters). One can also effectively use robust algorithms based on the multiple (ellipsoidal) estimates of the state of discrete dynamic systems (Volosov, 1998; Volosov and Tutunnik, 2002).

In the present work, the method of constructing guaranteed multiple estimates of the state vector (SV) of a satellite in the form of the solution of a system of linear inequalities as well as the technique of using the obtained estimates in control algorithms are developed. Moreover, it is suggested that measurement errors represent the values of a certain chaotic process (Lychak, 2004) with known interval characteristics. For constructing the control algorithm, use is made of the method of successive optimal linear-quadratic synthesis with the nonlinear object being described with a linearized system in a certain neighborhood of the current state. Computer simulation is carried out in order to examine the effectiveness of estimation and control algorithms.

2. STATEMENT OF THE ORIENTATION PROBLEM

Consideration is given to the problem of orientation of an artificial satellite (AS) with respect to the orbital coordinate system (OCS) $Ox_0y_0z_0$. The orbit of the AS is supposed to be circular. Moreover, the OCS revolves with respect to the inertial space at an angular velocity $\boldsymbol{\omega}_0$. The projections of the vector $\boldsymbol{\omega}_0$ onto the axes $Ox_0y_0z_0$ are given by the relations $\boldsymbol{\omega}_0^T = (0,0,-e)$, where e is the angular velocity of rotation of the AS around the Earth. The equations of motion of the AS with respect to the center of gravity have a form

$$\dot{\boldsymbol{\Phi}} = \mathbf{A}(\boldsymbol{\Phi})\boldsymbol{\omega} + \mathbf{b} \qquad (1)$$

$$\dot{\boldsymbol{\omega}} = \mathbf{J}^{-1}(\mathbf{M} - \breve{\boldsymbol{\omega}} \times \mathbf{J}\boldsymbol{\omega}) \qquad (2)$$

Here $\boldsymbol{\Phi}^T = (\gamma, \psi, \vartheta)$ is the vector of Krylov angles (angles of bank, yaw, tangage) corresponding to the succession of turns 3-1-2 (Branets and Shmyglevskii, 1992), $\boldsymbol{\omega}$ and $\mathbf{M}$ are the vector of the angular velocity of the AS and that of the control moment, which are determined in the AS-fixed coordinate system $Oxyz$ as

$$\boldsymbol{\omega}^T = (\omega_1, \omega_2, \omega_3),$$

$$\mathbf{M}^T = (M_1, M_2, M_3),$$

$$\mathbf{b}^T = (0, 0, e) = -\boldsymbol{\omega}_0,$$

$\mathbf{J}$ – the matrix of the moments of inertia $\mathbf{J} = diag\{J_1, J_2, J_3\}$, $\mathbf{A}(\boldsymbol{\Phi})$ and $\breve{\boldsymbol{\omega}}$ are 3 by 3 matrices

$$\mathbf{A}(\boldsymbol{\Phi}) = \begin{pmatrix} \cos\psi & 0 & \sin\psi \\ \sin\psi tg\gamma & 1 & -\cos\psi tg\gamma \\ -\dfrac{\sin\psi}{\cos\gamma} & 0 & \dfrac{\cos\psi}{\cos\gamma} \end{pmatrix},$$

$$\det \mathbf{A}(\boldsymbol{\Phi}) = \frac{1}{\cos\gamma} \qquad (3)$$

$$\breve{\boldsymbol{\omega}} = \begin{pmatrix} 0 & -\omega_3 & \omega_2 \\ \omega_3 & 0 & \omega_1 \\ -\omega_2 & \omega_1 & 0 \end{pmatrix} \qquad (4)$$

It is supposed (Raushenbach and Tokar', 1979), that angular velocity sensors (AVS) measure the projections ω_i, i=1,2,3 of the angular of the AS onto the axes $0xyz$ while the builder of the local vertical (BLV) determines deviations of the axis $0y$ from the current position of the local vertical – $0y_0$ axis. The BLV measures the angles γ and ϑ, i.e. the vector $\mathbf{y}^T = (\gamma, \vartheta)$. In this case, it is supposed that the vector of the phase coordinates initially Φ_i, i=1,2,3 belongs to a certain *a priori* set Ω_0, which is determined by the following system of inequalities

$$l_i^0 \le \Phi_i^0 \le u_i^0, i = 1,2,3. \qquad (5)$$

The problem of orientation of the AS lies in determining the values of control moments under whose action the object is displaced from the initial position $\boldsymbol{\Phi}^0 \in \Omega_0$ to the specified orientation mode. Let this specified orientation mode correspond to $\boldsymbol{\omega}(t) \equiv \boldsymbol{\omega}_0$ while angular coordinates are equal to zero $\boldsymbol{\Phi}(t) \equiv 0$. It is natural that these requirements can be realized only under the condition of $\mathbf{M}(t) \equiv 0$.

It is supposed that measurements are carried out at discrete instants of time $t_n (n = 0,1,2,...)$ with a certain constant step T, that is, $\boldsymbol{\Phi}(t_n) = \boldsymbol{\Phi}_n$, $\boldsymbol{\omega}(t_n) = \boldsymbol{\omega}_n$, $t_n = nT$ and the vector of angular velocities $\boldsymbol{\omega}_n$ is measured accurately. As for the vector $\boldsymbol{\Phi}_n$, only two of its components are measured and there exists a measurement error $\mathbf{f}_n$. Thus, the measurement process is described with a vector-matrix equation in the form of

$$\mathbf{y}_{n+1} = \mathbf{h}^T \boldsymbol{\Phi}_{n+1} + \mathbf{f}_n, \; n = 0,1,2,... \qquad (6)$$

where $\mathbf{y}_{n+1}$ is the measured object output, $\mathbf{h}^T = \begin{pmatrix} 1\,0\,0 \\ 0\,0\,1 \end{pmatrix}$, $\mathbf{f}_n$ - a two-dimensional column vector whose components represent the errors of the indicated measurement $(f_{j,n},\; j = 1,2)$ at the $n-$th step which is considered as the values of some chaotic process.

In the general case, the components of $f_{j,n}$ can be specified by different chaotic processes with known characteristics

$$\hat{m}_{j,l}(N) \le \frac{1}{N}\sum_{i=0}^{N-1} f_{j,k+i} \le \hat{m}_{j,u}(N) \qquad (7)$$

$$j = 1,2, \qquad k = 0,1,2,...,n, \qquad N = 1,2,...,n, \quad N \le n - k$$

where $\hat{m}_{j,l}(N)$ and $\hat{m}_{j,u}(N)$ are specified functions of N.

For controlling the object, the control moments

$$\mathbf{M}(t) = \mathbf{M}_n, \; t_n \le t < t_{n+1} \qquad (8)$$

will be realized discretely, where $n-$ the step number.

3. THE SYNTHESIS OF THE CONTROL MOMENTS

In order to find the control moments $\mathbf{M}$, let's carry out the linearization of the right-hand sides of Eqs. (1), (2) at point $\hat{\boldsymbol{\Phi}}_n$ being the point estimate of the state vector, while the column vector of the angular velocities is equal to $\boldsymbol{\omega} = (\omega_{1,n}, \omega_{2,n}, \omega_{3,n})$, $n = 0,1...$. At the n-th step, the following linearized system is obtained

$$\dot{\boldsymbol{\Phi}} = \mathbf{A}_c \boldsymbol{\Phi} + \mathbf{A}_\omega \boldsymbol{\varepsilon} + \mathbf{C}_\Phi, \; \boldsymbol{\varepsilon} = \boldsymbol{\omega} - \boldsymbol{\omega}_0 \qquad (9)$$

$$\dot{\boldsymbol{\varepsilon}} = \mathbf{A}_d \boldsymbol{\varepsilon} + \mathbf{J}^{-1}\mathbf{M} + \mathbf{C}_\varepsilon \qquad (8)$$

where

$$\mathbf{A}_c = \begin{pmatrix} \dfrac{\partial(\mathbf{A}\boldsymbol{\omega})}{\partial\gamma} & \dfrac{\partial(\mathbf{A}\boldsymbol{\omega})}{\partial\psi} & \dfrac{\partial(\mathbf{A}\boldsymbol{\omega})}{\partial\vartheta} \end{pmatrix}$$

$$\mathbf{A}_\omega = \begin{pmatrix} \dfrac{\partial(\mathbf{A}\boldsymbol{\omega})}{\partial\omega_1} & \dfrac{\partial(\mathbf{A}\boldsymbol{\omega})}{\partial\omega_2} & \dfrac{\partial(\mathbf{A}\boldsymbol{\omega})}{\partial\omega_3} \end{pmatrix}$$

$$\mathbf{C}_\Phi = \mathbf{A}\boldsymbol{\omega} - \mathbf{A}_c \boldsymbol{\omega}_n + \mathbf{A}_\omega \boldsymbol{\omega}_0 - \mathbf{A}_\omega \boldsymbol{\omega}_n,$$

$$\mathbf{A}_{\mathrm{d}} = \mathbf{J}^{-1}\begin{pmatrix} \dfrac{\partial(\breve{\boldsymbol{\omega}}\mathbf{J}\boldsymbol{\omega})}{\partial\omega_1} & \dfrac{\partial(\breve{\boldsymbol{\omega}}\mathbf{J}\boldsymbol{\omega})}{\partial\omega_2} & \dfrac{\partial(\breve{\boldsymbol{\omega}}\mathbf{J}\boldsymbol{\omega})}{\partial\omega_3} \end{pmatrix} = \begin{pmatrix} 0 & d_1 & d_1 \\ d_2 & 0 & d_2 \\ d_3 & d_3 & 0 \end{pmatrix}\boldsymbol{\omega}_n ,$$

where

$d_1 = J_1^{-1}(J_3 - J_2)$, $d_2 = J_2^{-1}(J_1 - J_3)$,

$d_3 = J_3^{-1}(J_2 - J_1)$;

$$\mathbf{C}_\varepsilon = (d_1\omega_{2,n}\omega_{3,n} \quad d_2\omega_{1,n}\omega_{3,n} \quad d_3\omega_{1,n}\omega_{2,n})^T + \\ + \mathbf{A}_{\mathrm{d}}\boldsymbol{\omega}_n + \mathbf{A}\boldsymbol{\omega}_0 .$$

When determining $\mathbf{A}_C, \mathbf{A}_{\boldsymbol{\omega}}, \mathbf{C}_\Phi$, the derivative is calculated at $(\hat{\boldsymbol{\Phi}}_n, \boldsymbol{\omega}_n)$, but in $\mathbf{A}_d$ it should be calculated at $\boldsymbol{\omega}_n$. Eqs. (9) and (10) can be rewritten as

$$\begin{pmatrix} \dot{\boldsymbol{\Phi}} \\ \dot{\boldsymbol{\varepsilon}} \end{pmatrix} = \overline{\mathbf{A}}\begin{pmatrix} \boldsymbol{\Phi} \\ \boldsymbol{\varepsilon} \end{pmatrix} + \overline{\mathbf{B}}\mathbf{M} + \overline{\mathbf{C}}, \quad (11)$$

where $\overline{\mathbf{A}} = \begin{pmatrix} \mathbf{A}_c & \mathbf{A}_\omega \\ \mathbf{0} & \mathbf{A}_d \end{pmatrix}$ is 6 by 6 matrix, $\overline{\mathbf{B}} = \begin{pmatrix} \mathbf{0} \\ \mathbf{J}^{-1} \end{pmatrix}$ is 6 by 3 matrix, $\overline{\mathbf{C}} = \begin{pmatrix} \mathbf{C}_\Phi \\ \mathbf{C}_\varepsilon \end{pmatrix}$ is 6 by 1 column vector.

It is supposed that measurements are taken at discrete moments at constant intervals T. Correction of control moments is fulfilled at the same intervals. Such a discrete system controlling a continuous object (9), (10) can be described with the help of the system of difference equations

$$\begin{pmatrix} \boldsymbol{\Phi}_{n+1} \\ \boldsymbol{\varepsilon}_{n+1} \end{pmatrix} = \widetilde{\mathbf{A}}\begin{pmatrix} \boldsymbol{\Phi}_n \\ \boldsymbol{\varepsilon}_n \end{pmatrix} + \widetilde{\mathbf{B}}\mathbf{M}_n + \widetilde{\mathbf{C}} \quad (12)$$

where $\widetilde{\mathbf{A}} = e^{\overline{\mathbf{A}}T}$, $\widetilde{\mathbf{B}} = \widetilde{\mathbf{A}} \cdot \int_0^T e^{-\overline{\mathbf{A}}T} dt \cdot \overline{\mathbf{B}}$, $\widetilde{\mathbf{C}} = \widetilde{\mathbf{A}} \cdot \int_0^T e^{-\overline{\mathbf{A}}T} dt \cdot \overline{\mathbf{C}}$, T is the duration of one control stroke.

Let's find the control vector of moments $\mathbf{M}_n = \mathbf{M}(nT)$ which minimizes the following quality functional at any initial conditions

$$\widehat{J} = \sum_{n=0}^{\infty} \underline{\omega}(\boldsymbol{\Phi}_{n+1}, \boldsymbol{\varepsilon}_{n+1}, \mathbf{M}_n),$$

$$\underline{\omega}(\cdot) = \begin{pmatrix} \boldsymbol{\Phi}_{n+1} \\ \boldsymbol{\varepsilon}_{n+1} \end{pmatrix}^T \underline{\mathbf{Q}}\begin{pmatrix} \boldsymbol{\Phi}_{n+1} \\ \boldsymbol{\varepsilon}_{n+1} \end{pmatrix} + \mathbf{M}_n^T \underline{\mathbf{R}}\mathbf{M}_n . \quad (13)$$

Here $\underline{\mathbf{Q}}$ is the specified numerical 6 by 6 matrix, $\underline{\mathbf{R}}$ is the numerical 3 by 3 matrix. In view of Eq.10, the functional (13) can be rewritten as

$$\widehat{J} = \sum_{n=0}^{\infty} \underline{\omega}(\boldsymbol{\Phi}_n, \boldsymbol{\varepsilon}_n, \mathbf{M}_n),$$

$$\underline{\omega}(\cdot) = \begin{pmatrix} \boldsymbol{\Phi}_{n+1} \\ \boldsymbol{\varepsilon}_{n+1} \end{pmatrix}^T \mathbf{Q}\begin{pmatrix} \boldsymbol{\Phi}_{n+1} \\ \boldsymbol{\varepsilon}_{n+1} \end{pmatrix} + \\ + 2\begin{pmatrix} \boldsymbol{\Phi}_{n+1} \\ \boldsymbol{\varepsilon}_{n+1} \end{pmatrix}^T \mathbf{H}\mathbf{M}_n + \mathbf{M}_n^T \mathbf{R}\mathbf{M}_n \quad (14)$$

where $\mathbf{Q} = \widetilde{\mathbf{A}}^T \underline{\mathbf{Q}}\widetilde{\mathbf{A}}$, $\mathbf{H} = \widetilde{\mathbf{A}}^T \underline{\mathbf{Q}}\widetilde{\mathbf{B}}$, $\mathbf{R} = \underline{\mathbf{R}} + \widetilde{\mathbf{B}}^T \underline{\mathbf{Q}}\widetilde{\mathbf{B}}$ and $\underline{\mathbf{Q}} > 0$, $\underline{\mathbf{R}} \geq 0$. Without losing generality, one can assume that

$$\lambda_j(\widetilde{\mathbf{A}}) < 1, \; j = 1,2,...6 , \quad (15)$$

where $\lambda_j(\widetilde{\mathbf{A}})$ are the proper numbers of $\widetilde{\mathbf{A}}$ matrix. If inequalities (13) are not valid for the initial equation, the control $\mathbf{M}_n$ can be taken as

$$\mathbf{M}_n = \underline{\mathbf{M}}_n + \mathbf{C}_1^T\begin{pmatrix} \boldsymbol{\Phi}_n \\ \boldsymbol{\varepsilon}_n \end{pmatrix}, \quad (16)$$

where $\mathbf{C}_1$ is a numerical 6 by 3 matrix, $\underline{\mathbf{M}}_n$ is a three-dimensional vector denoting the unknown part of control actions that should be determined from the condition of minimization of functional (14). Then Eqs.10 assume the form

$$\begin{pmatrix} \boldsymbol{\Phi}_{n+1} \\ \boldsymbol{\varepsilon}_{n+1} \end{pmatrix} = \underline{\mathbf{A}}\begin{pmatrix} \boldsymbol{\Phi}_n \\ \boldsymbol{\varepsilon}_n \end{pmatrix} + \widetilde{\mathbf{B}}\mathbf{M}_n + \widetilde{\mathbf{C}}, \quad (17)$$

where $\underline{\mathbf{A}} = \widetilde{\mathbf{A}} + \widetilde{\mathbf{B}}\mathbf{C}_1^T$ and the matrix $\mathbf{C}_1$ should be chosen in such a way that the proper number of $\underline{\mathbf{A}}$ matrix satisfy the condition (15). It is worth noting that the required matrix can be always chosen in view of the assumption concerning the controllability of the pair $\widetilde{\mathbf{A}}, \widetilde{\mathbf{B}}$. A possible algorithm of choosing $\mathbf{C}_1$ at specified proper numbers of $\widetilde{\mathbf{A}}$ matrix is proposed in (Kuntzevich and Lychak, 1977).

In view of the substitution (14), the functional (12) takes the form

$$\widehat{J} = \sum_{n=0}^{\infty}\left(\begin{pmatrix} \boldsymbol{\Phi}_{n+1} \\ \boldsymbol{\varepsilon}_{n+1} \end{pmatrix}^T \mathbf{Q}_1\begin{pmatrix} \boldsymbol{\Phi}_{n+1} \\ \boldsymbol{\varepsilon}_{n+1} \end{pmatrix} + \\ + 2\begin{pmatrix} \boldsymbol{\Phi}_{n+1} \\ \boldsymbol{\varepsilon}_{n+1} \end{pmatrix}^T \mathbf{H}_1\mathbf{M}_n + \mathbf{M}_n^T \mathbf{R}_1\mathbf{M}_n \right) \quad (18)$$

where $\mathbf{Q}_1 = \mathbf{Q} + \mathbf{C}_1\mathbf{R}\mathbf{C}_1^T + \mathbf{C}_1\mathbf{H}^T + \mathbf{H}\mathbf{C}_1^T$, $\mathbf{H}_1 = \mathbf{H} + \mathbf{C}_1^T\mathbf{R}$, $\mathbf{R}_1 = \mathbf{R}$.

Thus, the problem is reduced to the initial one but formulated for matrix $\underline{\mathbf{A}}$ meeting the condition (15). In this case, the control is linear and has a form

$$\mathbf{M}_n = \mathbf{M}_n(\mathbf{\Phi}_n, \boldsymbol{\varepsilon}_n) = = -(\mathbf{R}_1 + \mathbf{B}^T \mathbf{P} \mathbf{B})^{-1} (\mathbf{B}^T \mathbf{P} \mathbf{A} + \mathbf{H}^T) \begin{pmatrix} \mathbf{\Phi}_n \\ \boldsymbol{\varepsilon}_n \end{pmatrix} \quad (19)$$

where matrix $\mathbf{P}$ satisfies the following matrix equation

$$\underline{\mathbf{A}}^T \mathbf{P} \underline{\mathbf{A}} - \mathbf{P} + \mathbf{Q}_1 - (\underline{\mathbf{A}}^T \mathbf{P} \mathbf{B} + \mathbf{H}_1)^T \cdot \cdot (\mathbf{R} + \mathbf{B}^T \mathbf{P} \mathbf{B})^{-1} (\underline{\mathbf{A}}^T \mathbf{P} \mathbf{B} + \mathbf{H}_1)^T = 0 \quad (20)$$

which represents a discrete algebraic Riccati equation for linear discrete systems (Kuntzevich and Lychak, 1977).

4. ESTIMATION OF THE STATE VECTOR

Calculation of the point estimate of the phase vector $\hat{\mathbf{\Phi}}_n$ describing the state of the object represents a separate problem. In the foregoing considerations, this estimate is supposed to be known. At the initial moment, it can be chosen as a certain middle point of the *a priori* set Ω_0. In particular, if such a set represents a hyperparallelepiped specified by inequalities (5), the point estimate can be chosen as middle points of the corresponding intervals. On the basis of this estimate, one can find the vector of control moments $\mathbf{M}_0$, under whose influence the object described by Eqs. (1) and (2) moves in the phase space from the actual position $\mathbf{\Phi}_0 \in \Omega_0$ to the point $\mathbf{\Phi}_1 \in \Omega_1$ which is also unknown. On the basis of the new point estimate of the object position $\hat{\mathbf{\Phi}}_1 \in \Omega_1$, one can determine the vector of control moments $\mathbf{M}_1$ and repeat the described procedure. In such a way, one can construct the following sequence of sets Ω_n

$$\mathbf{\Phi}_n \in \Omega_n \quad (21)$$

Now let's consider obtaining the sets Ω_n in more detail. At the initial moment, Ω_0 represents a set described by a certain system of inequalities. At the following step, Ω_0 is transformed into the set $\Omega_0^{(1)}$ representing the set Ω_0 shifted and rotated according to the linear relation (14) where angular velocities can be replaced by their values as they are accurately measured at each iteration. The ideal value of the state vector satisfies the relation $\mathbf{\Phi}_1 \in \Omega_0^{(1)}$, that is the set $\Omega_0^{(1)}$ represents the guaranteed estimate of the state vector at the first step ($n = 0$). Two components of the state vector of the object are also measured at the first step. Thus, according to (6) and (7), one can obtain two bilateral inequalities for estimating two components of the state vector which determine a hyperzone in the phase space $\hat{\mathbf{m}}_l(1) \le \mathbf{y}_1 - \mathbf{h}\mathbf{\Phi}_1 \le \hat{\mathbf{m}}_u(1)$. Here $\hat{\mathbf{m}}_l(1)$, $\hat{\mathbf{m}}_u(1)$ are two-dimensional vectors composed from the components $\hat{m}_{j,l}(1)$ and $\hat{m}_{j,u}(1)$ $(j = 1,2)$, respectively.

Intersection of the obtained hyperzone with the set $\Omega_0^{(1)}$ gives a multiple estimate Ω_1 of the state vector at the first step in the form of a polyhedron

$$\Omega_1 = \Omega_0^{(1)} \cap \cap \{\mathbf{\Phi}_1 : \hat{\mathbf{m}}_l(1) \le \mathbf{y}_1 - \mathbf{h}\mathbf{\Phi}_1 \le \hat{\mathbf{m}}_u(1)\}. \quad (22)$$

At the following steps, due to transformation of the multiple estimate Ω_n obtained at the previous step into $\Omega_n^{(n+1)}$ and combination of the hyperzone obtained after $n+1$ measurements with the preceding ones, one can conclude that the multiple estimate of the state vector at the n-th step is determined by (Zelyk *et al,* 2003)

$$\Omega_{n+1} = \Omega_n^{(n+1)} \cap \cap \left\{ \begin{array}{c} \mathbf{\Phi}_{n+1} : \hat{\mathbf{m}}_l(N) \le \sum_{i=k}^{N} (\mathbf{y}_i + \mathbf{h}\mathbf{\Phi}_i(\mathbf{\Phi}_{n+1})) \le \hat{\mathbf{m}}_u(N), \\ N = n+1, n, \ldots, 1, \quad k = 1, \ldots, n+1 \end{array} \right\}$$

It is worth noting that constructing the system of inequalities is accompanied with the abrupt increase of their number, that's why those of them which became spurious at this stage are suppressed.

5. COMPUTER SIMULATION

When carrying out computer simulation, the object was described by Eqs.(1), (2), the object output was measured with an error taken as "white noise". The control moments were obtained according to (19), (20).

The systems (1), (2), (9), (10) were integrated by means of Runge-Kutta method for the following set of the initial values of the angles $\gamma_0 = 25^o$, $\psi_0 = 15^o$, $\vartheta_0 = 30^o$ at $\boldsymbol{\varepsilon}_0 = 0$. The values of the parameters for the equation of the AS motion, control and estimate algorithms were equal to $e = \pi/2700$ s^{-1}, $\mathbf{J} = diag(50,100,80)$, $\underline{\mathbf{R}} = diag(50,50,50)$ and $\underline{\mathbf{Q}} = diag(5,5,5,70,70,70)$. Quantization interval T of the control is equal to $0,1$ s. The level of measurement noise is equal to $|\mathbf{f}_n| \le 0.5^o$. The results of simulation are presented in Fig.1-3. Fig.1–2 demonstrate temporal evolution of the angles γ, ψ, ϑ and angular velocities $\omega_1, \omega_2, \omega_3$ which shows that the control object appears in the orientation mode $\mathbf{\Phi}(t) \equiv 0$ and $\boldsymbol{\omega}(t) \equiv \boldsymbol{\omega}_0$ with an accuracy of

$$\Delta\mathbf{\Phi} = \mathbf{\Phi}\,(60) = =(-0.0179^o, -0.0368^o, 0.0036^o)^T,$$

$$\Delta\boldsymbol{\omega} = \boldsymbol{\omega}(t) - \boldsymbol{\omega}_0 = =(-0.0001, -0.0017, -0.0008)^T \text{ deg/s.}$$

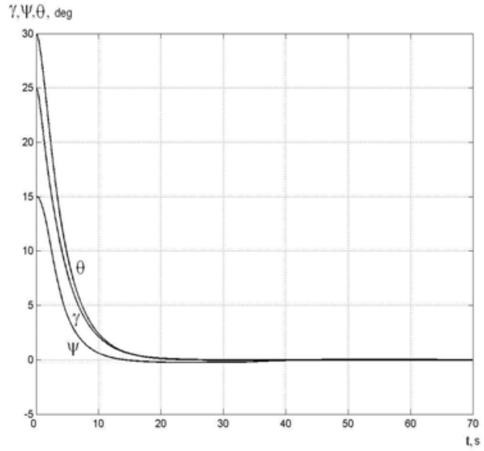

Fig.1. Temporal evolution of the angles γ, ψ, ϑ.

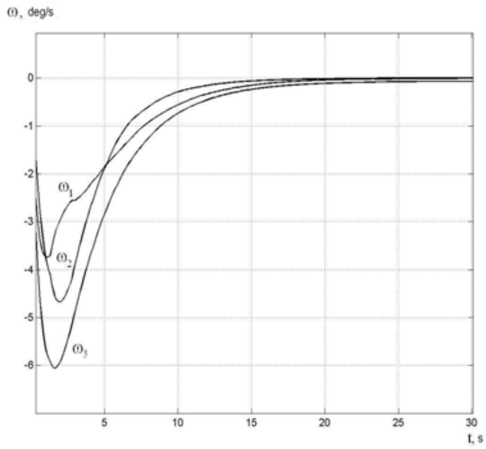

Fig.2. Temporal evolution of the angular velocities $\omega_1, \omega_2, \omega_3$

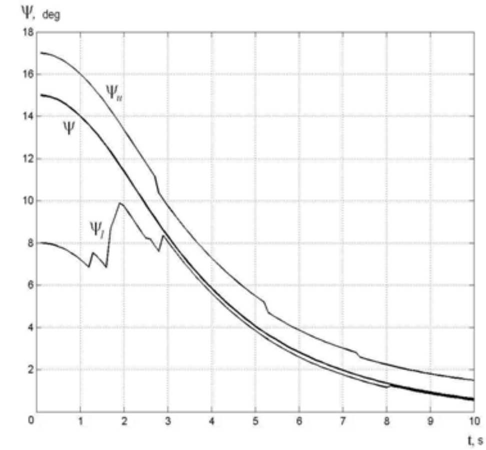

Fig.3.Temporal evolution of the angle ψ and its upper and lower estimates.

6. CONCLUSIONS

The constructed algorithm for calculating control moments, in contrast to the one described in (Volosov and Tutunnik, 2002), is insensitive to the increase of the initial values of angles γ, ψ, ϑ. It is caused by the fact that the proposed algorithm is not based on the assumption about their smallness ($\gamma, \psi, \vartheta < 10^\circ$). Instead of linearizing the nonlinear system at the single point corresponding to the orientation mode, the linearization is carried out at each step, at the points representing the estimates of the state vector. The estimation of the object state on the basis of
the created programs has demonstrated the effectiveness and expediency of using them in practical applications.
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