ITERATIVE OPTIMIZATION METHOD OF GOB-VOLTERRA FILTERS

Alain Y. Kibangou [*,**] **, Gérard Favier** [*]
Moha M. Hassani [**]

[*] *Laboratoire I3S, CNRS-UNSA,*
B.P. 121, 06903 Sophia Antipolis cedex, France
[**] *Laboratoire d'Electronique et Instrumentation, FSSM-UCAM,*
B.P. 2390, 40000 Marrakesh, Morocco

Abstract: An iterative method is proposed to optimize the Volterra kernels expansions on Generalized Orthonormal Bases (GOB). Each kernel is expanded on an independent GOB. The expansion coefficients, also called Fourier coefficients, are estimated in using an orthogonal formulation of the Least Squares (LS) algorithm. The proposed method allows optimization of both the Fourier coefficients and the GOBs poles. It can be seen as a good compromise between the exhaustive method for GOB poles optimization, costly by nature, and the analytical solution to Laguerre poles optimization that generally furnishes worse performances for system approximation than expansions on GOBs.

Keywords: Volterra filters; Generalized orthonormal basis; Laguerre basis; Basis selection; Identification.

1. INTRODUCTION

Truncated Volterra filters constitute a class of nonrecursive polynomial models, i.e. models without output feedback which guarantees their stability. Unfortunately, they are characterized by a huge number of parameters. During the last decade, the issue of Volterra model complexity reduction has been addressed following different approaches (Korenberg, 1991; Panicker and Mathews, 1998; Hacioglu and Williamson, 2001; Kibangou *et al.*, 2003; Campello *et al.*, 2004; Khouaja *et al.*, 2004). Among them, expansions of Volterra kernels on discrete orthonormal bases of functions (OBF) are of great interest. The class of OBFs generally used for modelling purposes is that of rational orthonormal bases such as Laguerre basis (Wahlberg, 1991) and Generalized orthonormal basis (GOB)(Ninness and Gustafsson, 1997). These bases are characterized by a set of poles whose the choice strongly influences the parsimony of the expansion.

Although expansion of Volterra kernels on OBFs was firstly suggested by Wiener in the 60's (Schetzen, 1980), in the best of our knowledge (Campello *et al.*, 2004) were the first to derive an analytical solution to the basis selection problem in the special case of Laguerre basis that is characterized by a single pole. Recently the authors have derived an analytical solution with more relaxed conditions by using the estimated Laguerre spectra (Kibangou *et al.*, 2004).

Optimization of GOBs for Volterra kernels expansion is more complex. Two main solutions have been proposed. Firstly, (Hacioglu and Williamson, 2001) have proposed the use of the gradient descent technique to minimize the mean square error (MSE) criterion. However, since the cost function is strongly nonlinear with respect to the GOBs poles, the convergence can only be guaranteed towards local minima. Secondly, the authors (Kibangou *et al.*, 2003) have proposed an exhaustive search method on a set of *a priori* fixed candidate poles obtained by using the knowledge on the system's dynamics or by discretizing the segment $]-1;1[$. The poles are chosen such as a LS criterion be minimized. At each iteration the cost function is evaluated for each candidate pole. Thus, the local minima issue does not arise. However the complexity of this approach grows with the dimension of the candidate poles set.

In order to circumvent this drawback, an alternative method is suggested in this paper. It is iterative as in the exhaustive method (Kibangou *et al.*, 2003) but does not require a set of candidate poles. At each iteration the poles are chosen in using the analytical solution for Laguerre poles given in (Kibangou *et al.*, 2004). Consequently the local minima issue does not arise and the computation complexity is reduced. Note that the parallel-cascade structure of the resulting method allows to iteratively estimate both the Fourier coefficients and the poles.

The organization of the paper is as follows. In the next section, the principle of Volterra kernels expansion on OBFs is recalled and the orthogonal formulation of the least-squares method for estimating the Fourier coefficients is described. This formulation allows to get a multichannel structure which will be exploited for the derivation of the proposed iterative method. In section 3, the analytical solution to the Laguerre-Volterra optimization is recalled. Then the overall procedure is described in section 4 before illustrating it by means of simulation results in section 5 and concluding the paper in section 6.

2. BACKGROUND

A discrete-time P-th order Volterra filter is described by the following input-output relation:

$$y(n)=\sum_{p=1}^{P}\sum_{n_1=0}^{\infty}\cdots\sum_{n_p=0}^{\infty}h_p(n_1,\cdots,n_p)\prod_{j=1}^{p}u(n-n_j) \tag{1}$$

where u, y and h_p are respectively the input, the output and the p-th order Volterra kernel. (Boyd and Chua, 1985) showed that any time invariant, causal, nonlinear system with fading memory can be represented by a finite expansion in Volterra series.

Expanding the kernel h_p on a set $\mathfrak{B}_p=\{b_{k,p}\}_{k=0}^{\infty}$ of OBFs, where $b_{k,p}$ is the $(k+1)$-th basis function associated with the p-th order kernel, yields:

$$h_p(n_1,\cdots,n_p)=\sum_{k_1=0}^{\infty}\cdots\sum_{k_p=0}^{\infty}g_{k_1,\cdots,k_p}\prod_{j=1}^{p}b_{k_j,p}(n_j)$$

where the coefficients

$$g_{k_1,\cdots,k_p}=\sum_{n_1=0}^{\infty}\cdots\sum_{n_p=0}^{\infty}h_p(n_1,\cdots,n_p)\prod_{j=1}^{p}b_{k_j,p}(n_j)$$

are called the Fourier coefficients relative to the p-th order kernel. When the used OBFs are Laguerre functions, the set of the Fourier coefficients $\{g_{k_1,\cdots,k_p}\}$ constitutes the Laguerre spectrum of the p-th order kernel.

Note that the Fourier coefficients can always be ranged in such a way that a triangular representation be obtained. The input-output relation (1) can be rewritten as:

$$\begin{aligned}y(n)&=\sum_{p=1}^{P}\sum_{k_1=0}^{\infty}\cdots\sum_{k_p=k_{p-1}}^{\infty}g_{k_1,\cdots,k_p}\prod_{j=1}^{p}s_{k_j,p}(n)\\&=\sum_{p=1}^{P}\sum_{k_1=0}^{\infty}\cdots\sum_{k_p=k_{p-1}}^{\infty}g_{k_1,\cdots,k_p}s_{k_1,\cdots,k_p}(n)\end{aligned} \tag{2}$$

where

$$s_{k_j,p}(n)=\sum_{i=0}^{\infty}b_{k_j,p}(i)u(n-i) \tag{3}$$

If the desired representation is stable, then the kernels expansions can be truncated to an arbitrary order K and the input-output relation becomes:

$$y(n)=\sum_{p=1}^{P}\sum_{k_1=0}^{K-1}\cdots\sum_{k_p=k_{p-1}}^{K-1}g_{k_1,\ldots,k_p}s_{k_1,\cdots,k_p}(n) \tag{4}$$

The resulting p^{th}-order kernel has $\frac{(K+p-1)!}{(K-1)!p!}$ parameters while, by considering the triangular form, the original p^{th}-order kernel with memory M has $\frac{(M+p-1)!}{(M-1)!p!}$ parameters. When $K<<M$, the parametric complexity is significantly reduced. The choice of the truncation order K depends on the basis selection. When the poles characterizing the bases are well chosen, K can take a small value.

In this paper two kinds of OBFs are used: the discrete-time Laguerre functions and the GOB functions respectively defined by their z-transforms as follows:

$$L_{k,p}(z)=\mathscr{Z}\{l_{k,p}(i)\}=\sqrt{1-\xi_p^2}\frac{z}{z-\xi_p}\left(\frac{1-\xi_p z}{z-\xi_p}\right)^k \tag{5}$$

$$B_{k,p}(z)=\mathscr{Z}\{b_{k,p}(i)\}=\sqrt{1-\tau_{k,p}^2}\frac{z}{z-\tau_{k,p}}\prod_{i=0}^{k-1}\frac{1-\tau_{i,p}z}{z-\tau_{i,p}} \tag{6}$$

One can note that only the Laguerre pole ξ_p characterizes the set of Laguerre functions $\{l_{k,p}(i)\}$ while the GOB functions are characterized by a set of poles. Note again that the Laguerre basis is a particular case of GOB obtained by setting in (6) all the $\tau_{i,p}$ poles equal to a same value, i.e. $\tau_{i,p}=\xi_p, \forall i$. The Volterra model whose kernels are expanded on OBFs has the filter bank structure depicted on figure 1.

In the sequel, we first present a LS estimation of the Fourier coefficients when the OBFs are assumed to be *a priori* fixed. This estimation method will constitute the basis of the proposed iterative procedure of GOBs poles estimation.

Let us assume that a record of N couples of input-desired output signals is available. We define

- $\varphi_{k_1,\cdots,k_p}$, $p=1,\cdots,P$, the vectors of p-th order products of the filtered inputs:

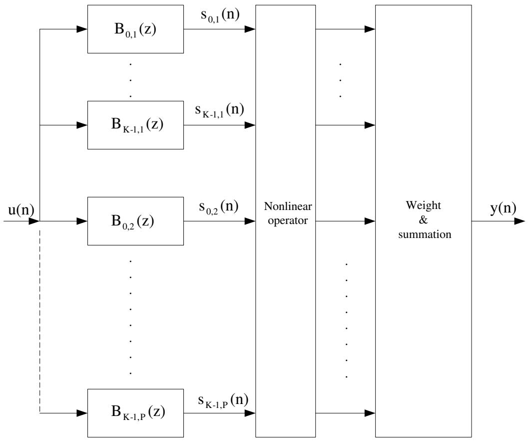

Fig. 1. Filter bank structure of a Volterra model expanded on OBFs

$$\varphi_{k_1,\cdots,k_p} = \left(s_{k_1,\cdots,k_p}(0),\cdots,s_{k_1,\cdots,k_p}(N-1)\right)^T$$

- $\mathbf{\Phi}_k^{(p)}$ the matrix constituted by the columns vectors $\varphi_{j_1,\cdots,j_{p-1},k}$, for $0 \le j_1 \le \cdots \le j_{p-1} \le k$ and $2 \le p \le P$.
- $\mathbf{\Phi}_k = \left(\varphi_k \quad \mathbf{\Phi}_k^{(2)} \cdots \mathbf{\Phi}_k^{(P)}\right)$
- G_k, the Fourier coefficients vectors whose entries are the Fourier coefficients associated with the filtered inputs contained in $\mathbf{\Phi}_k$.

One can note that the data contained in $\mathbf{\Phi}_k$ depend only on the input u and on the basis functions $b_{i,p}$, $0 \le i \le k$, $1 \le p \le P$. The matrix formulation of the input-output relation can then be written as:

$$Y = (y(0)\ y(1)\cdots y(N-1))^T = \mathbf{\Phi}G$$

where

$$\mathbf{\Phi} = (\mathbf{\Phi}_0 \cdots \mathbf{\Phi}_{K-1}), \quad G = \left(G_0^T \cdots G_{K-1}^T\right)^T$$

The optimal Fourier coefficients vector G is obtained by solving the following optimization problem:

$$\hat{G} = \arg\min_G \|d - \mathbf{\Phi}G\|^2$$

where $d = (d(0)\ d(1)\cdots d(N-1))^T$ is the vector of desired outputs. Then the LS estimator of the Fourier coefficients is:

$$\hat{G} = \left(\mathbf{\Phi}^T\mathbf{\Phi}\right)^{-1}\mathbf{\Phi}^T d$$

The QR factorization of $\mathbf{\Phi}$ yields $\mathbf{\Phi} = \bar{\mathbf{\Phi}}\mathbf{U}$, where $\bar{\mathbf{\Phi}}$ is a column orthonormal matrix and $\mathbf{U}$ an upper triangular matrix. Then the optimal Fourier coefficients are given by:

$$\hat{G} = U^{-1}\bar{\mathbf{\Phi}}^T d \tag{7}$$

Note that the $\bar{\mathbf{\Phi}}$ matrix has also a block structure:

$$\bar{\mathbf{\Phi}} = \left(\bar{\mathbf{\Phi}}_0 \cdots \bar{\mathbf{\Phi}}_{K-1}\right) \tag{8}$$

where the $\bar{\mathbf{\Phi}}_k$ matrices have the same structure than $\mathbf{\Phi}_k$ and satisfy the following orthogonality property:

$$\bar{\mathbf{\Phi}}_l^T \bar{\mathbf{\Phi}}_k = \delta_{l,k}\mathbf{I}$$

δ being the Kronecker symbol and $\mathbf{I}$ the identity matrix.

Let us define $\bar{Y}_k = \bar{\mathbf{\Phi}}_k\bar{G}_k$ and $\bar{G} = \mathbf{U}G = \left(\bar{G}_0^T \cdots \bar{G}_{K-1}^T\right)^T$. The outputs vector is then given by:

$$Y = \bar{\mathbf{\Phi}}\bar{G} = \sum_{k=0}^{K-1}\bar{Y}_k = \sum_{k=0}^{K-1}\bar{\mathbf{\Phi}}_k\bar{G}_k$$

and:

$$y(n) = \sum_{k=0}^{K-1}\bar{y}_k(n), \quad \bar{y}_k(n) = \bar{S}_k(n)\bar{G}_k \tag{9}$$

where $\bar{S}_k(n)$ is the $(n+1)$-th row of the $\bar{\mathbf{\Phi}}_k$ matrix. It yields the multichannel structure depicted by figure 2.

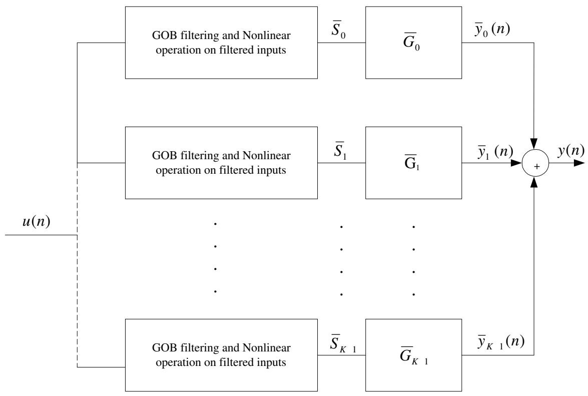

Fig. 2. Equivalent multichannel representation of GOB-Volterra model

The $(k+1)$-th subchannel, associated with $\bar{G}_k$ and $\bar{\mathbf{\Phi}}_k$, depends on the basis functions $b_{i,p}$, $i = 0,\cdots,k$, $p = 1,\cdots,P$. Suppose that the basis functions $b_{i,p}$ have been already optimized up to the k-th function, i.e. the poles $\tau_{i,p}$, $i = 0,\cdots,k-1$, $p = 1,\cdots,P$, have been determined and fixed. Then each GOB function $b_{k,p}$ is characterized by a single parameter: the pole $\tau_{k,p}$ whose optimization can be done by using the input-output data corresponding to the $(k+1)$-th subchannel. To achieve this purpose, since analytical solution exists for Laguerre poles optimization, the $(k+1)$-th subchannel can be modelled using Laguerre functions. The determined optimal Laguerre poles, associated with this subchannel, are used as the poles $\tau_{k,p}$ of the GOB functions $b_{k,p}$. Then the corresponding Fourier coefficients are optimized in the LS sense. In the following section the Laguerre poles optimization procedure proposed by the authors (Kibangou *et al.*, 2004) is briefly recalled.

3. OPTIMIZATION OF LAGUERRE POLES

The nonlinear behavior of the $(k+1)$-th subchannel can be described by a Volterra model expanded on Laguerre bases. An arbitrary truncation order is chosen since the interesting issue, in this section, is the determination of optimal Laguerre poles. The authors have shown that the analytical Laguerre poles obtained with the method described below do not depend on the *a* *priori* fixed truncation order. The larger the truncation order is, the faster the convergence is.

Optimization of Laguerre poles is based on the respective minimization of the following cost functions:

$$J_p = \frac{1}{p \left\| \bar{h}_p \right\|^2} \sum_{k_1=0}^{\infty} \cdots \sum_{k_p=0}^{\infty} (k_1 + \cdots + k_p) \gamma^2_{k_1,\cdots,k_p} \tag{10}$$

where $\bar{h}_p$ is the p-th order kernel of the considered subchannel, $\gamma_{k_1,\cdots,k_p}$ are the Fourier coefficients associated with the $\bar{h}_p$'s expansion on a Laguerre basis, $\left\|\bar{h}_p\right\|^2 = \sum_{n_1=0}^{\infty} \cdots \sum_{n_p=0}^{\infty} \bar{h}^2_p(n_1,\cdots,n_p)$. It was shown in (Campello *et al.*, 2004) that this cost function is an upper bound of the modelling squared error due to the truncation with a finite order of the Laguerre expansion.

Let us define, for $l = 1,\cdots,p$:

$$T_{1,l} = \sum_{k_1=0}^{\infty} \cdots \sum_{k_p=0}^{\infty} (2k_l + 1) \gamma^2_{k_1,\cdots,k_p} \tag{11}$$

$$T_{2,l} = 2 \sum_{k_1=0}^{\infty} \cdots \sum_{k_{l-1}=0}^{\infty} \sum_{k_l=1}^{\infty} \sum_{k_{l+1}=0}^{\infty} \cdots \sum_{k_p=0}^{\infty} k_l \gamma_{k_1,\cdots,k_p} \gamma_{k_1,\cdots,k_{l-1},k_l-1,k_{l+1},\cdots,k_p} \tag{12}$$

$$R_{j,p} = \sum_{l=1}^{p} T_{j,l}, \quad j = 1,2 \tag{13}$$

The optimization method of Laguerre poles is based on the following lemmas and theorem proved in (Kibangou *et al.*, 2004):

Lemma 1. $R_{1,p}$ and $R_{2,p}$ are linked by means of their derivatives with respect to ξ_p as follows:

$$\frac{\partial R_{1,p}}{\partial \xi_p} = \frac{-2}{1-\xi_p^2} R_{2,p} \tag{14}$$

$$\frac{\partial R_{2,p}}{\partial \xi_p} = \frac{-2}{1-\xi_p^2} R_{1,p} \tag{15}$$

Knowing that $\left\|\bar{h}_p\right\|^2 = \sum_{k_1=0}^{\infty} \cdots \sum_{k_p=0}^{\infty} \gamma^2_{k_1,\cdots,k_p}$, from the definitions of $R_{1,p}$ and J_p, a simple calculation yields

$$R_{1,p} = p \left\|\bar{h}_p\right\|^2 (1 + 2J_p)$$

Thus

$$\frac{\partial R_{1,p}}{\partial \xi_p} = 2p \left\|\bar{h}_p\right\|^2 \frac{\partial J_p}{\partial \xi_p} \tag{16}$$

Theorem 1. Given the Laguerre spectrum associated with the expansion of the Volterra kernel $\bar{h}_p$ on any Laguerre basis characterized by ξ_p, the optimal Laguerre pole $\xi_{p,opt}$ is such as:

$$\xi_{p,opt} = \begin{cases} \rho_{o,p} - \sqrt{\rho^2_{o,p} - 1}, & if \quad \rho_{o,p} > 1 \\ \rho_{o,p} + \sqrt{\rho^2_{o,p} - 1}, & if \quad \rho_{o,p} < -1 \end{cases} \tag{17}$$

where:

$$\rho_{o,p} = \frac{(1+\xi_p^2)R_{1,p} + 2\xi_p R_{2,p}}{2\xi_p R_{1,p} + (1+\xi_p^2)R_{2,p}} \tag{18}$$

When the expansion is infinite, $\rho_{o,p}$ is a characteristic of the Volterra kernel $\bar{h}_p$. The theorem stated above is particularly meaningful. Indeed it allows to obtain an optimal pole knowing the Laguerre spectrum associated with an arbitrary pole. In practical case the expansion on a Laguerre basis is truncated to a finite order K. Consequently, $\rho_{o,p}$ is only an approximation of the actual characteristic of the system. An iterative procedure allows to reach its optimal value.

Let us consider the Laguerre-Volterra filter described as follows:

$$w(n) = \sum_{p=1}^{P} \sum_{k_1=0}^{K-1} \cdots \sum_{k_p=k_{p-1}}^{K-1} \gamma_{k_1,\cdots,k_p} \prod_{j=1}^{p} s_{k_j,p}(n)$$

Similarly to (2), with the appropriate matrices and vectors, and by considering the output v of the subchannel to be modelled, the orthogonal LS estimator (7) can be used to estimate the Laguerre spectra.
In order to derive an iterative procedure for Laguerre pole estimation, note that the combination of (16) with (14), yields:

$$2p \left\|\bar{h}_p\right\|^2 \frac{\partial J_p}{\partial \xi_p} = \frac{-2}{1-\xi_p^2} R_{2,p}$$

Therefore when J_p is minimal, $R_{2,p}$ is equal to zero. Reciprocally, as J_p admits a single minimum for $|\xi_p| < 1$, when $R_{2,p}$ takes values close to zero, then J_p is close to its minimal value. Consequently the pole ξ_p is close to its optimal value. This property, due to the pseudo-convex nature of J_p, allows to derive the following batch estimation method :

(1) Select arbitrary poles in the segment $]-1,1[$ and construct the corresponding Laguerre bases, i.e. each kernel is expanded on an independent basis.
(2) Estimate the Laguerre spectrum associated with each kernel;
(3) For each basis, i.e. $p = 1,\cdots,P$, evaluate $T_{j,l}$, $l = 1,\cdots,p$, and $R_{j,p}$, $j = 1,2$, by using truncated versions of (11)-(13).
(4) If $R_{2,p}$ is close to zero, stop; else
   (a) Evaluate $\rho_{o,p}$ by using (18).
   (b) Determine a new pole ξ_p by using (17).
   (c) Construct a Laguerre basis associated with ξ_p and return to the step (2).

Note that this method does not require particular initial conditions to provide the solution.

4. THE ITERATIVE METHOD FOR GOB OPTIMIZATION

The multichannel structure of the GOB-Volterra model, as described in section 1, allows the iterative construction of GOBs. As stated in section 2, the input-output of the $(k+1)$-th subchannel can be used to optimize the pole $\tau_{k,p}$ associated with the GOB functions $b_{k,p}$. Particularly, $\tau_{k,p}$ can be chosen as the optimal Laguerre pole ξ_p associated with the expansion of the $(k+1)$-th subchannel's Volterra model on a Laguerre basis. Let us define the following signals:

- $\hat{y}_k(n)$, the cumulated outputs up to the $(k+1)$-th subchannel:

$$\hat{y}_k(n) = \sum_{i=0}^{k} \bar{y}_i(n)$$

- $v_k(n)$, the residual signal:

$$v_k(n) = d(n) - \hat{y}_{k-1}(n)$$

  where $v_0(n) = d(n)$.

The residual signals v_k satisfy the following recursive relation:

$$v_k(n) = v_{k-1}(n) - \bar{y}_{k-1}(n), \quad k > 0 \qquad (19)$$

The principle of the proposed method is that of well-known identification methods for parallel-cascade models (Korenberg, 1991). It consists, at each step $k \geq 0$, on the identification of the subchannel driven by the input signal u and the residual signal v_k. In our case, the input/output data (u, v_k) will be used, firstly for the estimation of Laguerre poles associated with the corresponding subchannel and secondly for the estimation of Fourier coefficients associated with the expansion of the overall model on GOBs. The proposed procedure is as follows:

- For $k \geq 0$:
  (1) Determine the optimal Laguerre poles $\xi_{k,p}$, $p = 1, \cdots, P$ by using the input-output data u and v_k.
  (2) Set $\tau_{k,p} = \xi_{k,p}$ and construct the $b_{k,p}$ functions.
  (3) Generate the filtered inputs contained in $\bar{S}_k(n)$ and construct the $\bar{\Phi}_k$ matrix.
  (4) Estimate $\bar{G}_k$ as:

$$\bar{G}_k = \bar{\Phi}_k^T v_k$$

  where v_k is the column vector of residual signals $v_k(n)$ $n = 0, \cdots, N-1$.
  (5) Calculate $v_{k+1}(n)$, $n = 0, \cdots, N-1$ using (19) and return to the step 1 until a stop criterion is reached.

The iterative procedure can be stopped in using a model selection criterion as in (Kibangou *et al.*, 2003) or by evaluating the power of the residual signal. Note that from a certain iteration, the residual signal will be mainly constituted by the additive noise. If this noise is supposed to be white, (Korenberg, 1991) suggests to apply a whiteness test on the residual signal. In our case the iterative procedure is stopped when the power of the residual signal becomes relatively constant.

5. SIMULATION RESULTS

In this section the performances of the proposed identification method are illustrated by means of simulations. The identification of the second order Volterra system used in (Kibangou *et al.*, 2004) and described below is considered:

First order kernel

$$H_1(z) = \frac{z(z+0.5)}{(z-0.3)(z-0.2)}$$

Second order kernel

$$h_2(i,j) = 0.25h(i)h(j)$$
$$where \quad h(i) = \mathscr{Z}^{-1}\left\{\frac{z(z+1)}{(z-0.8)(z+0.8)}\right\}$$

This system was simulated as a quadratic Volterra system with memory $M = 20$. In using a triangular representation of the quadratic kernel this filter has 230 parameters to estimate. The input signal is white, Gaussian, centered and has an unit variance. A white gaussian noise is added to the system output and the signal to noise ratio is equal to 30 dB. $N = 5000$ input/output data are simulated. Laguerre poles are initialized to 0.001. The Monte Carlo method is used for simulations with 50 independent noise sequences.

Figure 3 shows the variation of the residual signal power that is the output mean square error. One can note that this power becomes nearly constant for $K = 7$. Then the iterative procedure for both GOBs construction and Fourier coefficients estimation can be stopped.

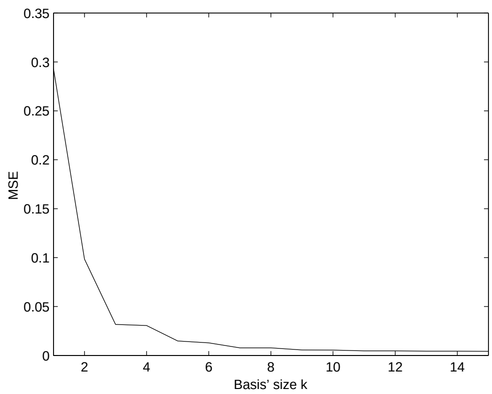

Fig. 3. Power of the residual signal in function of the basis size

Now let us compare the proposed procedure with the methods called Laguerre-Volterra and Exhaustive method respectively described in (Kibangou *et al.*, 2004) and (Kibangou *et al.*, 2003). In the first method, optimization of a Laguerre-Volterra filter is considered. The optimal Laguerre poles obtained are $\xi_1 = 0.518$ for the linear kernel and $\xi_2 = 0.820$ for the quadratic one. One can see that the Laguerre poles converge towards their optimal values in relatively few iterations (figure 4).

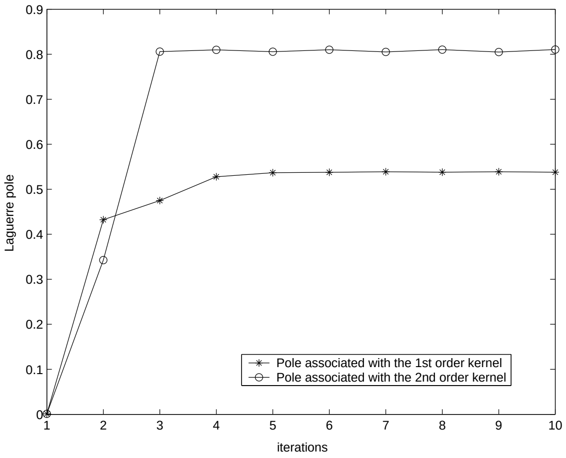

Fig. 4. Laguerre poles estimation

In the second method the poles are chosen among a set of candidate poles obtained in discretizing the $]-1;1[$ segment with a fixed step-size taken equal to 0.1. Note that the complexity and the precision of the exhaustive method is linked to the chosen step-size. The cost function must be evaluated for each of the candidate poles.

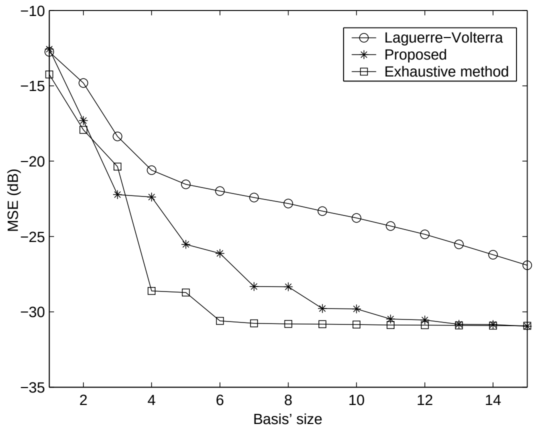

Fig. 5. Comparison of three estimation methods

As shown by figure 5, the exhaustive search method gives better results in the MSE sense. Note that in this approach the poles are selected in order to explicitly decrease the square error, that is not the case of the proposed method. Therefore the proposed iterative method requires less operations than the exhaustive search scheme since the Laguerre poles selection is done in few iterations. Then it can be seen as a good alternative to the exhaustive search method.

6. CONCLUSION

In this paper, the optimization of Volterra kernels expansions on GOBs has been addressed. The proposed method uses some features of the previous methods developed by the authors. Growing method based on an exhaustive search of GOB poles has been proposed in (Kibangou *et al.*, 2003). Unfortunately the computational cost can be very important due to its exhaustive nature. In (Kibangou *et al.*, 2004) the authors have proposed an analytical solution to the Laguerre-Volterra filters optimization problem. This solution is based on the estimated Laguerre spectra and needs only few iterations to converge. In this work the growing nature of the algorithm has been conserved but the exhaustive search method replaced by the analytical determination of Laguerre poles. As shown by means of simulations, the proposed procedure is a good tradeoff between the two previous methods. Its computational cost is less than that of the exhaustive method. An open challenge is to derive a completely analytical solution for GOBs as done for Laguerre bases.

REFERENCES

Boyd, S. and L.O. Chua (1985). Fading memory and the problem of approximating nonlinear operators with Volterra series. *IEEE Trans. on Circuits and Systems* **32**(11), 1150–1161.

Campello, R.J.G.B., G. Favier and W.C. Amaral (2004). Optimal expansions of discrete-time Volterra models using Laguerre functions. *Automatica* **40**, 815–822.

Hacioglu, R. and G.A. Williamson (2001). Reduced complexity Volterra models for nonlinear system identification. *EURASIP Journal on Applied Signal Processing* **4**, 257–265.

Khouaja, A., A.Y. Kibangou and G. Favier (2004). Third-order Volterra kernels complexity reduction using PARAFAC. In: *Proc. of IEEE Int. Symp. on Control, Communications and Signal Processing (ISCCSP'03)*. Hammamet, Tunisia.

Kibangou, A.Y., G. Favier and M.M. Hassani (2003). Generalized orthonormal basis selection for expanding quadratic Volterra filters. In: *Proc. of IFAC SYSID*. Rotterdam, The Netherlands. pp. 1119–1124.

Kibangou, A.Y., G. Favier and M.M. Hassani (2004). Optimization of Laguerre-Volterra filters based on Laguerre spectra. *submitted to EURASIP journal on Applied Signal Processing*.

Korenberg, M.J. (1991). Parallel cascade identification and kernel estimation for nonlinear systems. *Annals of Biomedical Engineering* **19**, 429–455.

Ninness, B. and F. Gustafsson (1997). A unifying construction of orthonormal bases for system identification. *IEEE Trans. on Automatic Control* **42**(4), 515–521.

Panicker, T.M. and V.J. Mathews (1998). Parallel-cascade realizations and approximations of truncated Volterra systems. *IEEE Trans. on Signal Processing* **46**(10), 2829–2831.

Schetzen, M. (1980). *The Volterra and Wiener theories of nonlinear systems*. John Wiley and Sons, Inc., New-York.

Wahlberg, B. (1991). System identification using Laguerre models. *IEEE Trans. on Automatic Control* **36**(5), 551–562.