# PASSIVITY-BASED DYNAMIC VISUAL FEEDBACK CONTROL WITH A MOVABLE CAMERA 

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#### Abstract

This paper deals with the dynamic visual feedback control with a movable camera instead of a fixed camera in the fixed camera configuration. Firstly the brief summary of the visual feedback system with a fixed camera is given with the fundamental representation of a relative rigid body motion. Secondly we construct the new error system in order to consider the camera field of view. Next, we derive the passivity of the dynamic visual feedback system by combining the manipulator dynamics and the visual feedback system. Based on the passivity, stability and $L_{2}$-gain performance analysis are discussed. Finally the validity of the proposed control law can be confirmed by comparing the simulation results. Copyright ${ }^{\circ} 2005$ IFAC


Keywords: Robot Control, Visual Servoing, Lyapunov Stability, Mechanical Systems, Passivity

## 1. INTRODUCTION

The combination of mechanical control with visual information, so-called visual feedback control or visual servoing, should become extremely important, when we consider a mechanical system working under dynamical environments (Hutchinson et al., 1996), (Christensen and Corke, 2003). Classical visual servoing algorithms assume that the manipulator dynamics is negligible and do not interact with the visual feedback loop. However, this assumption is invalid for high speed tasks, while it holds for kinematic control problems. Kelly considered the set-point problems with a static target for the dynamic visual feedback system which includes the manipulator dynamics (Kelly, 1996). Bishop et al. proposed an inverse dynamics based control law for the position tracking and the camera calibration problems of the dynamic visual
feedback system (Bishop and Spong, 1999). Recently, Zergeroglu et al. developed an adaptive control law for the position tracking and the camera calibration problems of the dynamic visual feedback system with parametric uncertainties (Zergeroglu et al., 2001). More recently, the authors proposed the passivity-based dynamic visual feedback control for the 3D target tracking (Kawai and Fujita, 2004), (Kawai et al., 2004). Although these control laws guarantee the stability for the dynamic visual feedback system, the problems of the camera field of view (Chaumette, 1998) are not dealt with in these issues. However, it is important to consider the problems of the camera field of view in order to enlarge the available workspace for the robot hand with the configuration separated a camera frame and a hand (end-


Fig. 1. Visual feedback system in the fixed camera configuration.
effector) one, we call the fixed camera configuration, as shown in Fig. 1.

In this paper, we discuss the dynamic visual feedback control with a movable camera instead of a fixed camera in the fixed camera configuration in order to increase the available workspace for the robot hand. This can be regarded as a solution for one of the problems of the camera field of view in the proposed framework which is based on our previous works (Fujita et al., 2002), (Kawai and Fujita, 2004), (Kawai et al., 2004). Moreover, we can derive that the dynamic visual feedback system preserves the passivity of the visual feedback system which is obtained in our previous works.
Throughout this paper, we use the notation $e^{\hat{\xi} \theta_{a b}} \in \mathcal{R}^{3 \times 3}$ to represent the change of the principle axes of a frame $\Sigma_{b}$ relative to a frame $\Sigma_{a}$. The notation ' $\wedge$ ' (wedge) is the skew-symmetric operator such that $\hat{\xi} \theta=\xi \times \theta$ for the vector crossproduct $\times$ and any vector $\theta \in \mathcal{R}^{3}$. The notation ' $V$ ' (vee) denotes the inverse operator to ' $\wedge$ ': i.e., so(3) $\rightarrow \mathcal{R}^{3}$. $\xi_{a b} \in \mathcal{R}^{3}$ specifies the direction of rotation and $\theta_{a b} \in \mathcal{R}$ is the angle of rotation. Here $\hat{\xi} \theta_{a b}$ denotes $\hat{\xi}_{a b} \theta_{a b}$ for the simplicity of notation. We use the $4 \times 4$ matrix

$$
g_{a b}=\left[\begin{array}{cc}
e^{\hat{\xi} \theta_{a b}} & p_{a b}  \tag{1}\\
0 & 1
\end{array}\right]
$$

as the homogeneous representation of $g_{a b}=$ $\left(p_{a b}, e^{\hat{\theta} \theta_{a b}}\right) \in S E(3)$ which is the description of the configuration of a frame $\Sigma_{b}$ relative to a frame $\Sigma_{a}$. The adjoint transformation associated with $g_{a b}$ is denoted by $\operatorname{Ad}_{\left(g_{a b}\right)}$ (Murray et al., 1994). Let us define the vector form of the rotation matrix as $e_{R}\left(e^{\hat{\xi} \theta_{a b}}\right):=\operatorname{sk}\left(e^{\hat{\xi} \theta_{a b}}\right)^{\vee}$ where $\operatorname{sk}\left(e^{\hat{\xi} \theta_{a b}}\right)$ denotes $\frac{1}{2}\left(e^{\hat{\xi} \theta_{a b}}-e^{-\hat{\xi} \theta_{a b}}\right)$.

## 2. PASSIVITY-BASED VISUAL FEEDBACK SYSTEM IN THE FIXED CAMERA CONFIGURATION

### 2.1 Fundamental Representation for Visual Feedback System

Visual feedback systems typically use four coordinate frames which consist of a world frame $\Sigma_{w}$, a
target object frame $\Sigma_{o}$, a camera frame $\Sigma_{c}$ and a hand (end-effector) frame $\Sigma_{h}$ as in Fig. 1. Then, $g_{w h}, g_{w c}$ and $g_{w o}$ denote the rigid body motions from $\Sigma_{w}$ to $\Sigma_{h}$, from $\Sigma_{w}$ to $\Sigma_{c}$ and from $\Sigma_{w}$ to $\Sigma_{o}$, respectively. Similarly, the relative rigid body motions from $\Sigma_{c}$ to $\Sigma_{h}$, from $\Sigma_{c}$ to $\Sigma_{o}$ and from $\Sigma_{h}$ to $\Sigma_{o}$ can be represented by $g_{c h}, g_{c o}$ and $g_{h o}$, respectively, as shown in Fig. 1.

The relative rigid body motion from $\Sigma_{c}$ to $\Sigma_{o}$ can be led by using the composition rule for rigid body transformations ((Murray et al., 1994), Chap. 2, pp. 37, eq. (2.24)) as follows

$$
\begin{equation*}
g_{c o}=g_{w c}^{-1} g_{w o} \tag{2}
\end{equation*}
$$

The fundamental representation of the relative rigid body motion involves the velocity of each rigid body. To this aid, let us consider the velocity of a rigid body as described in (Murray et al., 1994). Now, we define the body velocity of the camera relative to the world frame $\Sigma_{w}$ as

$$
\hat{V}_{w c}^{b}=g_{w c}^{-1} \dot{g}_{w c}=\left[\begin{array}{cc}
\hat{\omega}_{w c} & v_{w c} \\
0 & 0
\end{array}\right] \quad V_{w c}^{b}=\left[\begin{array}{l}
v_{w c} \\
\omega_{w c}
\end{array}\right](3
$$

where $v_{w c}$ and $\omega_{w c}$ represent the velocity of the origin and the angular velocity from $\Sigma_{w}$ to $\Sigma_{c}$, respectively ((Murray et al., 1994) Chap. 2, eq. (2.55)).
Then, the fundamental representation of the relative rigid body motion $g_{c o}$ is described as follows (Kawai and Fujita, 2004).

$$
\begin{equation*}
V_{c o}^{b}=-\operatorname{Ad}_{\left(g_{c o}^{-1}\right)} V_{w c}^{b}+V_{w o}^{b} \tag{4}
\end{equation*}
$$

where $V_{w o}^{b}$ is the body velocity of the target object relative to $\Sigma_{w}$. Roughly speaking, the relative rigid body motion $g_{c o}$ will be derived from the difference between the camera velocity $V_{w c}^{b}$ and the target object velocity $V_{w o}^{b}$.

### 2.2 Estimation Error and Control Error Systems

Here the brief summary of our prior works in (Kawai and Fujita, 2004) and (Kawai et al., 2004) are given. The visual information $f$ which includes the relative rigid body motion can be exploited, while the relative rigid body motion $g_{c o}$ can not be obtained directly in the visual feedback system. In order to bring the actual relative rigid body motion $g_{h o}$ to a given reference $g_{d}$ in Fig. 1, we consider the control and estimation problems in the visual feedback system. Firstly, we shall consider the following model which just comes from the fundamental representation (4).

$$
\begin{equation*}
\bar{V}_{c o}^{b}=-\operatorname{Ad}_{\left(\bar{g}_{c o}^{-1}\right)} V_{w c}^{b}+u_{e} \tag{5}
\end{equation*}
$$

where $\bar{g}_{c o}$ is the estimated relative rigid body motion from $\Sigma_{c}$ to $\Sigma_{o}$ and $u_{e}$ is the input in order to converge the estimated value to the actual relative rigid body motion. Next, the estimation error of the relative rigid body motion from $\Sigma_{c}$ to
$\Sigma_{o}$, i.e. the error between $\bar{g}_{c o}$ and $g_{c o}$, is defined as

$$
\begin{equation*}
g_{e e}=\bar{g}_{c o}^{-1} g_{c o}, \tag{6}
\end{equation*}
$$

which is called the estimation error. Using the notation $e_{R}\left(e^{\hat{\xi} \theta}\right)$, the vector of the estimation error is given by $e_{e}:=\left[p_{e e}^{T} e_{R}^{T}\left(e^{\hat{\xi} \theta_{e e}}\right)\right]^{T}$. Then, the estimation error vector $e_{e}$ can be obtained by using image information $f$. The estimation error system is represented by

$$
\begin{equation*}
V_{e e}^{b}=-\operatorname{Ad}_{\left(g_{e e}\right)}^{-1} u_{e}+V_{w o}^{b} . \tag{7}
\end{equation*}
$$

Similarly, we define the error between $g_{d}$ and $\bar{g}_{h o}$, which is called the control error, as follows

$$
\begin{equation*}
g_{e c}=g_{d}^{-1} \bar{g}_{h o}, \tag{8}
\end{equation*}
$$

where $\bar{g}_{h o}$ is the estimated relative rigid body motion from $\Sigma_{h}$ to $\Sigma_{o}$ and obtained from $\bar{g}_{h o}=$ $g_{c h}^{-1} \bar{g}_{c o}$. Here, we assume that $g_{c h}$ is calculated by using the known motion, i.e. $g_{w c}$ and $g_{w h}$, exactly. The vector of the control error is defined as $e_{c}:=\left[p_{e c}^{T} e_{R}^{T}\left(e^{\hat{e} \theta_{e c}}\right)\right]^{T}$. The control error system is described by

$$
\begin{equation*}
V_{e c}^{b}=-\operatorname{Ad}_{\left(\bar{g}_{h o}^{-1}\right)} V_{w h}^{b}+u_{e}-\operatorname{Ad}_{\left(g_{e c}^{-1}\right)} V_{d}^{b} \tag{9}
\end{equation*}
$$

where $V_{w h}^{b}$ and $V_{d}^{b}$ are the body velocity of the hand relative to $\Sigma_{w}$ and the desired body velocity of the relative rigid body motion $g_{h o}$, respectively.

Combining (7) and (9), the visual feedback system in the fixed camera configuration is constructed as follows

$$
\left[\begin{array}{c}
V_{e c}^{b} \\
V_{e e}^{b}
\end{array}\right]=\left[\begin{array}{cc}
-\operatorname{Ad}_{\left(\bar{g}_{h o}^{-1}\right)} & I \\
0 & -\operatorname{Ad}_{\left(g_{e e}^{-1}\right)}
\end{array}\right] u_{c e}+\left[\begin{array}{l}
0 \\
I
\end{array}\right] V_{w o}^{b}(10)
$$

where

$$
u_{c e}:=\left[\begin{array}{c}
V_{w h}^{b}+\operatorname{Ad}_{\left(g_{d}\right)} V_{d}^{b}  \tag{11}\\
u_{e}
\end{array}\right]
$$

denotes the input for the visual feedback system.
Let us define the error vector of the visual feedback system as $e_{c e}:=\left[\begin{array}{ll}e_{c}^{T} & e_{e}^{T}\end{array}\right]^{T}$ which contains of the control error vector $e_{c}$ and the estimation error vector $e_{e}$. Here, we define the output of the visual feedback system (10) as

$$
\nu_{c e}:=\left[\begin{array}{cc}
-\operatorname{Ad}_{\left(g_{d}^{-1}\right)}^{T} & 0 \\
\operatorname{Ad}_{\left(e^{-\xi} \theta_{e c}\right)} & -I
\end{array}\right] e_{c e},
$$

then the visual feedback system (10) satisfies $\int_{0}^{T} u_{c e}^{T} \nu_{c e} d \tau \geq-\beta_{c e}$ where $\beta_{c e}$ is a positive scalar (Kawai et al., 2004). This would suggest that the visual feedback system (10) is passive from the input $u_{c e}$ to the output $\nu_{c e}$ just formally as in the definition in (van der Schaft, 2000).

## 3. VISUAL FEEDBACK SYSTEM WITH A MOVABLE CAMERA

### 3.1 Camera Field Error System

In this section, we construct the error system of the movable camera in the fixed camera config-
uration, we call the camera field error system, in order to increase the available workspace for the robot hand. Here we define the camera field error between the estimated value $\bar{g}_{c o}$ and a given reference $g_{c d}$ for the camera motion as

$$
\begin{equation*}
g_{e v}=g_{c d}^{-1} \bar{g}_{c o} \tag{12}
\end{equation*}
$$

If $\bar{g}_{c o}$ is equal to $g_{c d}$, then the target object can be kept in the center of the camera field of view. Using the notation $e_{R}\left(e^{\hat{\xi} \theta}\right)$, the vector of the camera field error is defined as $e_{v}:=$ $\left[p_{e v}^{T} e_{R}^{T}\left(e^{\hat{\xi} \theta_{e v}}\right)\right]^{T}$. Note that $e_{v}=0$ iff $p_{e v}=0$ and $e^{\hat{\xi} \theta_{e v}}=I_{3}$.

Similarly to (7) and (9), the camera field error system can be obtained as

$$
\begin{equation*}
V_{e v}^{b}=u_{e}-\operatorname{Ad}_{\left(\bar{g}_{c o}^{-1}\right)} V_{w c}^{b}-\operatorname{Ad}_{\left(g_{e v}^{-1}\right)} V_{c d}^{b} \tag{13}
\end{equation*}
$$

where $V_{c d}^{b}$ is the desired body velocity of the relative rigid body motion $g_{c o}$.

### 3.2 Property of Visual Feedback System

Combining (7), (9) and (13), we construct the visual feedback system with a movable camera in the fixed camera configuration as follows

$$
\left[\begin{array}{c}
V_{e c}^{b}  \tag{14}\\
V_{e e}^{b} \\
V_{e v}^{b}
\end{array}\right]=\left[\begin{array}{ccc}
-\operatorname{Ad}_{\left(\bar{g}_{h o}^{-1}\right)} & I & 0 \\
0 & -\operatorname{Ad}_{\left(g_{e e}^{-1}\right)} & 0 \\
0 & I & -\operatorname{Ad}_{\left(\bar{g}_{c o}^{-1}\right)}^{-1}
\end{array}\right] u_{c e v}+\left[\begin{array}{l}
0 \\
I \\
0
\end{array}\right] V_{w o}^{b}
$$

where

$$
u_{c e v}:=\left[\begin{array}{c}
V_{w h}^{b}+\operatorname{Ad}_{\left(g_{d}\right)} V_{d}^{b}  \tag{15}\\
u_{e} \\
V_{w c}^{b}+\operatorname{Ad}_{\left(g_{c d}\right)} V_{c d}^{b}
\end{array}\right]
$$

denotes the input for the visual feedback system. Let us define the error vector of the visual feedback system (14) as $e:=\left[\begin{array}{lll}e_{c}^{T} & e_{e}^{T} & e_{v}^{T}\end{array}\right]^{T}$. It should be noted that if the vector of the estimation error is equal to zero, not only $\bar{g}_{c o}$ equals $g_{c o}$ but also $\bar{g}_{h o}$ equals $g_{h o}$. Moreover, if the vectors of the control error and the camera field error are equal to zero, then $\bar{g}_{h o}$ and $\bar{g}_{c o}$ equal $g_{d}$ and $g_{c d}$, respectively. Thus, when $e \rightarrow 0, g_{h o}$ and $g_{c o}$ tend to $g_{d}$ and $g_{c d}$, respectively. This states that the control objective can be achieved, in addition, the available workspace for the robot hand will be increased by moving of the camera field of view.

Lemma 1. If $V_{w o}^{b}=0$, then the visual feedback system (14) satisfies

$$
\begin{equation*}
\int_{0}^{T} u_{c e v}^{T} \nu_{c e v} d \tau \geq-\beta_{c e v}, \quad \forall T>0 \tag{16}
\end{equation*}
$$

where $\nu_{\text {cev }}$ is defined as

$$
\nu_{c e v}:=\left[\begin{array}{ccc}
-\operatorname{Ad}_{\left(g_{d}^{-1}\right)}^{T} & 0 & 0  \tag{17}\\
\operatorname{Ad}_{\left(e^{\left.-\hat{\xi} \theta_{e c}\right)}\right.} & -I & \operatorname{Ad}_{\left(e^{-\hat{\xi}} \theta_{e v}\right)} \\
0 & 0 & -\operatorname{Ad}_{\left(g_{c d}^{-1}\right)}^{T}
\end{array}\right] e
$$

and $\beta_{\text {cev }}$ is a positive scalar.

PROOF. Consider the following positive definite function

$$
\begin{equation*}
V_{c e v}=E\left(g_{e c}\right)+E\left(g_{e e}\right)+E\left(g_{e v}\right) \tag{18}
\end{equation*}
$$

where $E(g):=\frac{1}{2}\|p\|^{2}+\phi\left(e^{\hat{\xi} \theta}\right)$ and $\phi\left(e^{\hat{\xi} \theta}\right):=$ $\frac{1}{2} \operatorname{tr}\left(I-e^{\hat{\xi} \theta}\right)$ which is the error function of the rotation matrix (see e.g. (Bullo and Murray, 1999)). Differentiating (18) with respect to time yields

$$
\dot{V}_{c e v}=e^{T}\left[\begin{array}{ccc}
\operatorname{Ad}_{\left(e^{\hat{\xi}} \theta_{e c}\right)} & 0 & 0 \\
0 & \operatorname{Ad}_{\left(e^{\hat{\xi}} \theta_{e e}\right)} & 0 \\
0 & 0 & \operatorname{Ad}_{\left(e^{\hat{\xi}} \theta_{e v}\right)}
\end{array}\right]\left[\begin{array}{c}
V_{e c}^{b} \\
V_{e e}^{b} \\
V_{e v}^{b}
\end{array}\right](19)
$$

where we use the property $\dot{\phi}\left(e^{\hat{\xi} \theta}\right):=e^{\hat{\xi} \theta} \omega$. Observing the skew-symmetry of the matrices $\hat{p}_{e c}, \hat{p}_{e e}$ and $\hat{p}_{e v}$, the above equation along the trajectories of the system (14) can be transformed into

$$
\begin{align*}
\dot{V}_{c e v} & =e^{T}\left[\begin{array}{ccc}
-\operatorname{Ad}_{\left(g_{d}^{-1}\right)} & \operatorname{Ad}_{\left(e^{\hat{\xi}} \theta_{e c}\right)} & 0 \\
0 & -I & 0 \\
0 & \operatorname{Ad}_{\left(e^{\hat{\xi} \theta} e v\right)} & -\operatorname{Ad}_{\left(g_{c d}^{-1}\right)}
\end{array}\right] u_{c e v} \\
& =u_{\text {cev }}^{T} \nu_{c e v} . \tag{20}
\end{align*}
$$

Integrating (20) from 0 to $T$, we can obtain

$$
\begin{equation*}
\int_{0}^{T} u_{c e v}^{T} \nu_{c e v} d \tau \geq-V_{c e v}(0):=-\beta_{c e v} \tag{21}
\end{equation*}
$$

where $\beta_{c e v}$ is the positive scalar which only depends on the initial states of $g_{e c}, g_{e e}$ and $g_{e v}$.

Remark 2. In the visual feedback system, $p_{e c}^{T}\left(e^{-\hat{\xi} \theta_{d}}\right.$ $\left.\omega_{e c}\right)^{\wedge} p_{e c}=0, p_{e e}^{T} \hat{\omega}_{e e} p_{e e}=0, p_{e v}^{T}\left(e^{-\hat{\xi} \theta_{c d}} \omega_{e v}\right)^{\wedge} p_{e v}=$ 0 hold. This skew-symmetric property is analogous to the one of the robot dynamics, i.e. $x^{T}(\dot{M}-$ $2 C$ ) $x=0, \forall x \in \mathcal{R}^{n}$ (where $M \in \mathcal{R}^{n \times n}$ is the manipulator inertia matrix and $C \in \mathcal{R}^{n \times n}$ is the Coriolis matrix (Murray et al., 1994)). Thus, Lemma 1 suggests that the visual feedback system (14) is passive from the input $u_{\text {cev }}$ to the output $\nu_{\text {cev }}$ as in the definition in (van der Schaft, 2000).

## 4. DYNAMIC VISUAL FEEDBACK CONTROL

### 4.1 Dynamic Visual Feedback System

The manipulator dynamics can be written as

$$
\begin{equation*}
M(q) \ddot{q}+C(q, \dot{q}) \dot{q}+g(q)=\tau+\tau_{d} \tag{22}
\end{equation*}
$$

where $q, \dot{q}$ and $\ddot{q}$ are the joint angles, velocities and accelerations, respectively. $\tau$ is the vector of the input torques and $\tau_{d}$ represents a disturbance input. The body velocity of the hand is given by

$$
\begin{equation*}
V_{w h}^{b}=J_{b}(q) \dot{q} \tag{23}
\end{equation*}
$$

where $J_{b}(q)$ is the manipulator body Jacobian (Murray et al., 1994). We define the reference of the joint velocities as $\dot{q}_{d}:=J_{b}^{\dagger}(q) u_{d}$ where $u_{d}$ represents the desired body velocity of the hand. Thus, $V_{w h}^{b}$ in (15) should be replaced by $u_{d}$.

Let us define the error vector with respect to the joint velocities of the manipulator dynamics as $\xi:=\dot{q}-\dot{q}_{d}$. Now, we consider the passivity-based dynamic visual feedback control law as follows

$$
\begin{align*}
\tau=M(q) \ddot{q}_{d}+ & C(q, \dot{q}) \dot{q}_{d}+g(q) \\
& +J_{b}^{T}(q) \operatorname{Ad}_{\left(g_{d}^{-1}\right)}^{T} e_{c}+u_{\xi} \tag{24}
\end{align*}
$$

The new input $u_{\xi}$ is to be determined in order to achieve the control objectives.
Using (14), (22) and (24), the visual feedback system with manipulator dynamics (we call the dynamic visual feedback system) can be derived as follows

$$
\begin{align*}
& {\left[\begin{array}{c}
\dot{\xi} \\
V_{e c}^{b} \\
V_{e e}^{b} \\
V_{e v}^{b}
\end{array}\right]=\left[\begin{array}{c}
-M^{-1} C \xi+M^{-1} J_{b}^{T} \operatorname{Ad}_{\left(g_{d}^{-1}\right)}^{T} e_{c} \\
-\mathrm{Ad}_{\left(\bar{g}_{h o}^{-1}\right)} J_{b} \xi \\
0 \\
0
\end{array}\right]} \\
& +\left[\begin{array}{ccc}
M^{-1} & 0 & 0 \\
0 & -\mathrm{Ad}_{\left(\bar{g}_{h o}^{-1}\right)} & I \\
0 & 0 & 0 \\
0 & 0 & I \mathrm{Ad}_{\left(g_{e e}^{-1}\right)} \\
0 & 0 \\
0 & \operatorname{Ad}_{\left(\bar{g}_{c o}^{-1}\right)}
\end{array}\right] u+\left[\begin{array}{cc}
M^{-1} & 0 \\
0 & 0 \\
0 & I \\
0 & 0
\end{array}\right] w \tag{25}
\end{align*}
$$

where

$$
u:=\left[\begin{array}{c}
u_{\xi}  \tag{26}\\
u_{d}+\operatorname{Ad}_{\left(g_{d}\right)} V_{d}^{b} \\
u_{e} \\
V_{w c}^{b}+\operatorname{Ad}_{\left(g_{c d}\right)} V_{c d}^{b}
\end{array}\right]
$$

and $x:=\left[\xi^{T} e^{T}\right]^{T}$. We define the disturbance of dynamic visual feedback system as $w:=$ $\left[\tau_{d}^{T}\left(V_{w o}^{b}\right)^{T}\right]^{T}$. Before constructing the dynamic visual feedback control law, we derive an important lemma.

Lemma 3. If $w=0$, then the dynamic visual feedback system (25) satisfies

$$
\begin{equation*}
\int_{0}^{T} u^{T} \nu d \tau \geq-\beta, \quad \forall T>0 \tag{27}
\end{equation*}
$$

where

$$
\nu:=N x, N:=\left[\begin{array}{cccc}
I & 0 & 0 & 0 \\
0 & -\operatorname{Ad}_{\left(g_{d}^{-1}\right)}^{T} & 0 & 0 \\
0 & \operatorname{Ad}_{\left(e^{\left.-\hat{\xi} \theta_{e c}\right)}\right.} & -I & \operatorname{Ad}_{\left(e^{-\hat{\xi} \theta} e v\right)} \\
0 & 0 & 0 & -\operatorname{Ad}_{\left(g_{c d}^{T-1}\right)}^{T}
\end{array}\right]
$$

Due to space limitations, the proof is only sketched. By using the following positive definite function, the proof can be completed.

$$
\begin{equation*}
V=\frac{1}{2} \xi^{T} M \xi+E\left(g_{e c}\right)+E\left(g_{e e}\right)+E\left(g_{e v}\right) \tag{28}
\end{equation*}
$$

Remark 4. Similarly to Lemma 1, Lemma 3 would suggest that the dynamic visual feedback system is passive from the input $u$ to the output $\nu$ just formally. From Lemma 3, we can state that the dynamic visual feedback system (25) preserves the passivity of the visual feedback system (14). This is one of main contributions of this work.

### 4.2 Stability Analysis for Dynamic Visual Feedback System

It is well known that there is a direct link between passivity and Lyapunov stability. Thus, we propose the following control input.

$$
u=-K \nu=-K N x, K:=\left[\begin{array}{cccc}
K_{\xi} & 0 & 0 & 0  \tag{29}\\
0 & K_{c} & 0 & 0 \\
0 & 0 & K_{e} & 0 \\
0 & 0 & 0 & K_{v}
\end{array}\right]
$$

where $K_{\xi}:=\operatorname{diag}\left\{k_{\xi 1}, \cdots, k_{\xi n}\right\}$ denotes the positive gain matrix for each joint axis. $K_{c}:=$ $\operatorname{diag}\left\{k_{c 1}, \cdots, k_{c 6}\right\}, K_{e}:=\operatorname{diag}\left\{k_{e 1}, \cdots, k_{e 6}\right\}$ and $K_{v}:=\operatorname{diag}\left\{k_{v 1}, \cdots, k_{v 6}\right\}$ are the positive gain matrices of $x, y$ and $z$ axes of the translation and the rotation for the control error, the estimation one and the camera field one, respectively. The result with respect to asymptotic stability of the proposed control input (29) can be established as follows.

Theorem 5. If $w=0$, then the equilibrium point $x=0$ for the closed-loop system (25) and (29) is asymptotic stable.

Theorem 5 can be proved using the energy function (28) as a Lyapunov function. The proof is omitted here due to space limitations. Considering the manipulator dynamics, Theorem 5 shows the stability via Lyapunov method for the full 3D dynamic visual feedback system. It is interesting to note that stability analysis is based on the passivity as described in (27).

### 4.3 L L 2 -gain Performance Analysis for Dynamic Visual Feedback System

Based on the dissipative systems theory, we consider $L_{2}$-gain performance analysis for the dynamic visual feedback system (25) in one of the typical problems, i.e. the disturbance attenuation problem. Now, let us define

$$
\begin{equation*}
P:=N^{T} K N-\frac{1}{2 \gamma^{2}} W-\frac{1}{2} I \tag{30}
\end{equation*}
$$

where $\gamma \in \mathcal{R}$ is positive and $W:=\operatorname{diag}\{I, 0, I, 0\}$. Then we have the following theorem.

Theorem 6. Given a positive scalar $\gamma$ and consider the control input (29) with the gains $K_{\xi}, K_{c}, K_{e}$ and $K_{v}$ such that the matrix $P$ is positive semidefinite, then the closed-loop system (25) and (29) has $L_{2}$-gain $\leq \gamma$.

The proof is omitted due to space limitations, Theorem 6 can be proved using the energy function (28) as a storage function for $L_{2}$-gain performance analysis. The $L_{2}$-gain performance analysis of the dynamic visual feedback system is discussed
via the dissipative systems theory. In $H_{\infty}$-type control, we can consider some problems by establishing the adequate generalized plant. This paper has discussed $L_{2}$-gain performance analysis for the disturbance attenuation problem. The proposed strategy can be extended for the other-type of generalized plants of the dynamic visual feedback systems.

## 5. SIMULATION

The simulation results on the two degree-offreedom manipulator as depicted in Fig. 2 are shown in order to understand our proposed method simply, though it is valid for 3D visual feedback systems. The target object has four feature points and moves for $t=4.8$ [s] along a straight line $(0 \leq t<2)$ and a "Figure 8 " motion ( $2 \leq t<4.8$ ). Specifically, we compare the performance in the case of the movable camera system and the fixed camera system discussed in (Kawai et al., 2004). We use the reference of the relative rigid body motion as constant values, i.e. $p_{d}=\left[\begin{array}{lll}0 & 0 & -0.81\end{array}\right]^{T}, e^{\hat{\xi} \theta_{d}}=I, p_{c d}=\left[\begin{array}{lll}0 & 0 & -2\end{array}\right]^{T}$, $e^{\hat{\xi} \theta_{c d}}=I, V_{d}^{b}=0$ and $V_{c d}^{b}=0$, for the tracking problems in the simulation.


Fig. 2. Coordinate frames for dynamic visual feedback system with two degree of freedom manipulator.

Fig. 3 shows the error between $g_{h o}$ and $g_{d}$ which is defined as $g_{e r}:=g_{d}^{-1} g_{h o}$ for the control objective, the left graphs denote the error of the case of the proposed control law, and the right ones denote the error of the case of the previous one (Kawai et al., 2004). In this figure, we focus on the errors of the translations of $x$ and $y$ and the rotation of $z$, because the errors of the translation of $z$ and the rotations $x$ and $y$ are zeros ideally on the defined coordinates in Fig. 2. Clearly, the proposed control law achieves the control objective, not causing the performance deterioration in comparison with the previous one. Omitted due to space limitations, stability and $L_{2}$-gain performance analysis are verified as same as the simulation in (Kawai et al., 2004). Fig. 4


Fig. 3. Error for the control objective. (Left side: Movable camera system, Right side: Fixed camera system)


Fig. 4. One of feature points. (Movable camera system: solid, Fixed camera system: dashed)
presents one of the four feature points. In this figure, the solid lines denote the feature point of the case of the proposed control law, and the dashed lines denote the feature point of the case of the previous one. We can verify that the change of the feature point with the proposed control law is less than with the previous one. This result states that the target object almost exists in the center of the camera, and the camera can move not to miss the moving target object. Thus, we consider that the dynamic visual feedback control with a movable camera can enlarge the camera field of view, not causing the performance deterioration, in comparison with a fixed camera (Kawai et al., 2004).

## 6. CONCLUSIONS

This paper dealt with the dynamic visual feedback control with a movable camera instead of a fixed camera in the fixed camera configuration in order to increase the available workspace for the robot hand. Moreover, we derived that the dynamic visual feedback system preserved the passivity of the visual feedback system by the same strategy
in our previous works (Fujita et al., 2002), (Kawai and Fujita, 2004), (Kawai et al., 2004). Stability and $L_{2}$-gain performance analysis for the dynamic visual feedback system have been discussed based on passivity with the energy function. The validity of the proposed control law was confirmed by comparing the simulation results.

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