

TRACK SEEKING CONTROL OF HARD DISK DRIVES BASED ON NEW TWO-DEGREE-OF-FREEDOM CONTROL SCHEME WITH VIBRATION MINIMIZED TRAJECTORIES

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Abstract: This paper proposes a new TDOF control scheme to minimize the tracking error caused by the neglected vibration modes of the plant. The design parameter is optimized from experimental data. The effectiveness is shown by simulation using a precise model of hard disk drive. *Copyright©2005 IFAC.*

Keywords: hard disk drives, track seeking control, two-degree-of-freedom control, vibration minimized trajectories, residual stiffness

1. INTRODUCTION

In track-seeking control of hard disk drives (HDD), a read/write head must be driven from one track to another as fast as possible. Since the system has resonance modes at high frequencies, vibration minimized trajectories can reduce residual vibration to improve the seeking time. Thus, the design methods of these trajectories such as the minimum jerk input have been proposed (Y.Mizoshita *et al.* 1996, Hara *et al.* 2000), and our research group has also been developed a simple method to calculate them for sampled-data system by using final-state control (FSC) method (Hirata *et al.* 2002).

These trajectories should be used with feedback controller because actual system has various disturbances, and two-degree-of-freedom (TDOF) control scheme is effective (Hirata *et al.* 1992, Ishikawa *et al.* 1996, Yi and Tomizuka 1999). However, in (Hirata and Nonami 2004), it has been shown that TDOF controller may increase the positioning error at the target position under some conditions. For these reasons, a new TDOF control system has been proposed to improve the positioning error at the end of seeking by introducing a simple scheme into the conventional TDOF control system (Hirata and Nonami 2004). The method requires the model of high frequency resonance modes. However, in general, it might be difficult to obtain an accurate model of these resonance modes, and it will be fluctuated by piece-to-piece variations of the products.

This paper proposes a practical method to optimize the design parameter of the new TDOF control system from experimental data. It is shown that the tracking error is an affine function of the design parameter

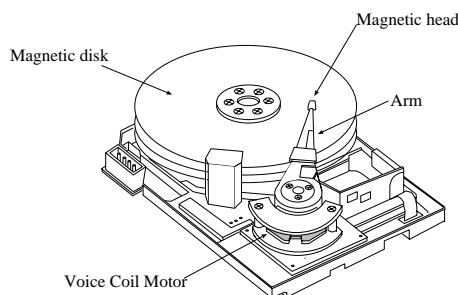


Fig. 1. Hard disk drive.

under a simple approximation, and the parameter is optimized from a measured tracking error profile based on the least-square method. The case that the gain of the plant has a perturbation is also considered. The effectiveness of the proposed method is shown by simulation using a precise model of HDDs.

2. HARD DISK DRIVES AND PLANT MODEL

Fig. 1 shows a typical hard disk drive, and it has read/write heads, a voice coil motor (VCM) and magnetic disks which are driven by a spindle motor. The control input u is a voltage [V] to the current amplifier for VCM and the measurement output y is a head position [track]. In this study, a 2.5inch hard disk drive is used and the frequency response, which was obtained by a servo analyzer, is shown in Fig. 2 by dotted line. The transfer function including high frequency vibration modes is assumed to be given as:

$$P(s) = k_p \left(\frac{1}{s^2} + \sum_{i=1}^{\ell} \frac{k_i \omega_i^2}{s^2 + 2\zeta_i \omega_i s + \omega_i^2} \right), \quad (1)$$

where ℓ is the number of vibration modes. By fitting the parameters of (1) to the measured frequency re-

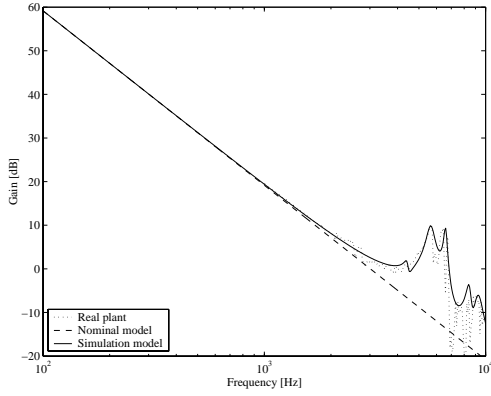


Fig. 2. Frequency response of the plant.

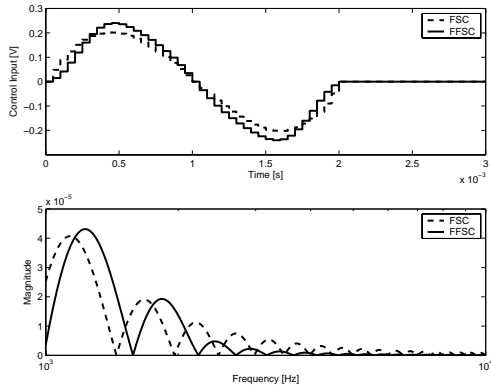


Fig. 3. FSC and FFSC inputs.

sponse shown in Fig. 2 by dotted line, the 12th order model with five vibration modes is obtained, e.g., $\ell = 5$ in (1). On the other hand, for simplicity, the nominal model of $P(s)$ is defined as a double-integrator plant as follows:

$$P_n(s) = \frac{k_p}{s^2}. \quad (2)$$

The nominal model (2) can be used to design a vibration minimized trajectory and a feedback controller. The frequency responses of $P(s)$ and $P_n(s)$ are shown in Fig. 2 by solid line and dashed line respectively.

The discrete-time model of the plant is obtained by the zero-order hold method. The sampling frequency of the plant is 10kHz and the Nyquist frequency is 5kHz. Therefore almost all the resonance modes reside above the Nyquist frequency. From this point of view, it might also be appropriate to define the nominal model as the double-integrator plant.

3. VIBRATION MINIMIZED TRAJECTORY AND TDOF CONTROL

3.1 Vibration minimized trajectory

In this paper, two kinds of vibration minimized trajectory are used as the input to the new TDOF control system.

FSC input In (Y.Mizoshita *et al.* 1996), a vibration minimized control input was obtained so that the square integral of the differentiated acceleration (jerk) is minimum. This input can be easily obtained by using final-state control in discrete-time domain

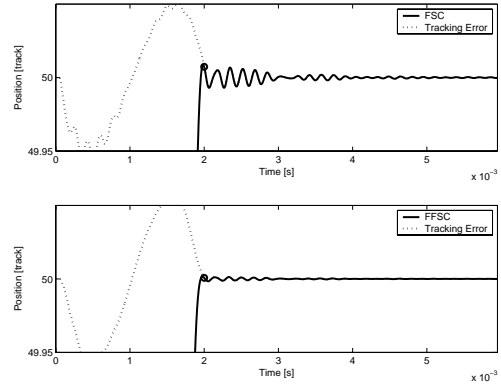


Fig. 4. The open-loop response with FSC and FFSC inputs. The tracking errors are shown by shifted to 50track.

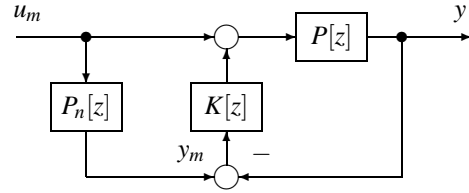


Fig. 5. The block diagram of the conventional TDOF control system.

(T.Totani and H.Nishimura 1994). The minimum jerk input will be referred to as *FSC input* in this paper.

FFSC input If the frequency components of FSC input can be minimized at a certain frequency region, the residual vibration is reduced more. In this case, a frequency shaped final-state control (FFSC) method is effective (Hirata *et al.* 2002). This method can calculate a feedforward input so that the gain of the Fourier transform of the control input, which is generated though zero-order hold, is minimum at desired frequency points. The frequency shaped final state control input will be referred to as *FFSC input* in this paper.

Fig. 3 shows the time responses and the Fourier transform (FT) of FSC and FFSC inputs of 50 tracks seeking with step number $N = 40$. FFSC input is obtained so that the frequency component of the control input is reduced from 2kHz to 10kHz. Fig. 4 shows the simulation results of the open-loop response when FSC and FFSC inputs are applied to the high order model $P(s)$. It shows that FFSC input achieves lower residual vibration after track seeking than FSC input, and the seeking time might be shorten.

3.2 Two-degree-of-freedom control

FSC and FFSC inputs should be implemented with feedback controller because actual systems have various disturbances and plant uncertainties. In this case, a two-degree-of-freedom (TDOF) control is effective and commonly used. The block diagram of TDOF control system is shown in Fig. 5, where P , P_n and K are an actual plant, a nominal model and a feedback controller respectively, and K works to minimize the tracking error $y_m - y$. If $P_n = P$ and the tracking error is zero, the transfer function from u_m to y becomes P_n .

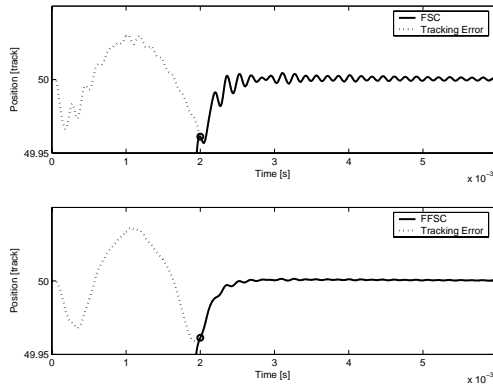


Fig. 6. The time response of the conventional TDOF control system with FSC and FFSC inputs. The tracking errors are shown by shifted to 50track.

In this paper, it is assumed that the feedback controller has been designed in advance by using the discrete-time H_∞ control theory to have a good disturbance rejection and a robustness to the unmodeled dynamics of the plant (Takiguchi *et al.* 2002, Hirata *et al.* 2003).

Fig. 6 shows the result when FSC and FFSC inputs are applied to the conventional TDOF control system. It shows that the FFSC input can improve the residual vibration after track seeking. However, the tracking error at $t = 2$ [ms] is degraded in both cases compared with the result of the open-loop control as shown in Fig. 4. This is a problem because the TDOF control is introduced to improve the performance but it fails. If the tracking error $y_m - y$ is small enough, the TDOF control system acts as feedforward control and the seeking error at $t = 2$ [ms] will be improved. One possible way is to use a high order model for the nominal model $P_n(s)$. However, it might be difficult to obtain a precise model of high frequency resonance modes with a good accuracy. It is also undesirable to have vibration modes in the nominal model because the reference input y_m might have oscillation.

If the frequencies of resonance modes of the plant are higher than that of control bandwidth, these vibration modes can be approximated by a static gain, which is referred to as *residual stiffness* (Nagamatsu 1985). Since vibration minimized trajectories do not excite high frequency vibration modes, the introduction of the static gain in the nominal model is effective to include the property of high frequency resonance modes in the model. Thus a new TDOF control system with a static gain α as shown in Fig. 7 has been proposed to improve the tracking error $y_m - y$ (Hirata and Nonami 2004).

Remarks: From the structure of Fig. 7, the feedforward input $u_m[k]$ must satisfy $u_m[N] = 0$ so that $y[k] = y_m[k]$ holds at $k = N$ for a nonzero α .

It is straightforward way to determine α from the residual stiffness itself (Hirata and Nonami 2004). The residual stiffness is defined by substituting $s = 0$ into the transfer function of the neglected vibration modes as follows (Nagamatsu 1985):

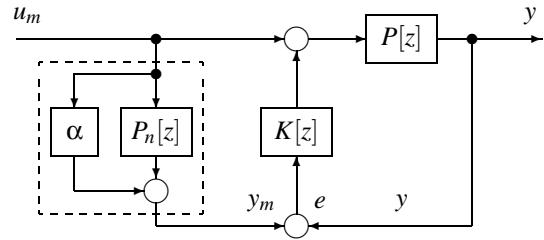


Fig. 7. The block diagram of the new TDOF control system with α .

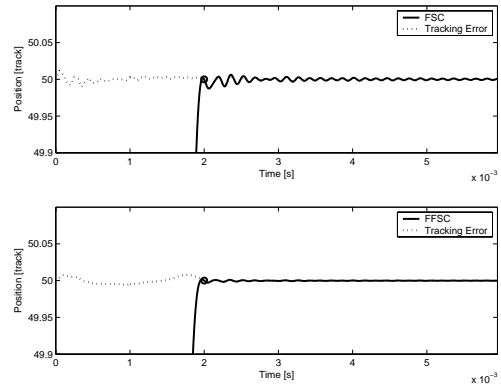


Fig. 8. The time responses of the new TDOF control system with FSC and FFSC inputs for $\alpha = k_{res}$.

$$k_{res} = k_p \sum_{i=1}^{\ell} k_i. \quad (3)$$

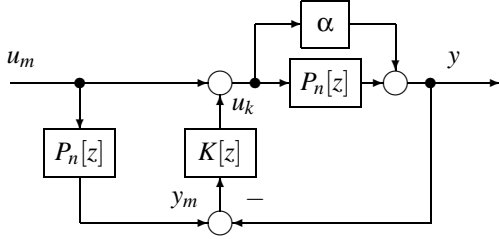
k_{res} corresponds to the gain of the neglected vibration modes at low frequency, and it was obtained as $k_{res} = -0.244$ using the high order model $P(s)$. Fig. 8 shows the simulation results of the new TDOF control with $\alpha = k_{res}$. Comparing with Fig. 6, the error at the end of seeking is improved in both FSC and FFSC inputs. Especially for the result with FFSC inputs, a good seeking performance without residual vibration is achieved.

4. PROPOSED METHODS TO DETERMINE α FROM EXPERIMENTAL DATA

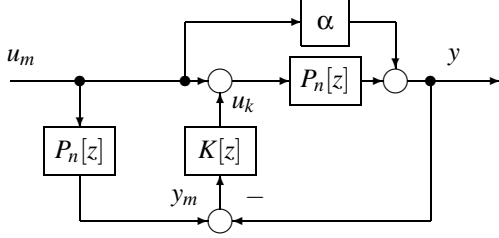
In the previous section, it has been shown that the parameter α can be obtained from the residual stiffness, which is the sum of the residue of each vibration mode. However, in actual systems, it is difficult to identify the correct value of residue from the measured frequency response of the plant. Thus, in this section, methods to obtain the optimum α from the measured tracking error profile during track seeking are proposed.

4.1 Method A

This section shows a basic idea to obtain the optimum α corresponding to the residual stiffness from experimental data. It is assumed that the high frequency vibration modes of the actual plant can be modeled as a constant gain because vibration minimized trajectory does not excite these vibration modes. Thus, the actual plant can be represented by the nominal model with an additive gain α corresponding to the residual stiffness



(a) TDOF control system with α .



(b) An approximation of (a).

Fig. 9. Method A

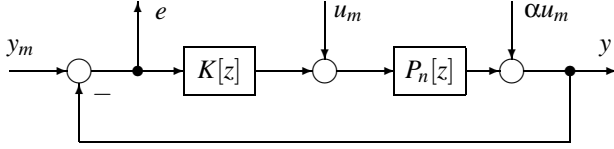


Fig. 10. ODOF control system with feedforward inputs

as shown in Fig. 9(a). To identify the parameter α from measured tracking error signal, the relation between the parameter α and the tracking error profile $e = y_m - y$ should be obtained.

In Fig. 9(a), the tracking error e becomes a nonlinear function of α for a given feedforward input u_m because the feedback property is affected by the change of α . However, the tracking error is small, e.g., smaller than 1track as shown in Fig. 6, and the compensation input u_k generated by the feedback controller is also very small compared with the feedforward input u_m . Therefore Fig. 9(a) can be approximated by Fig. 9(b). Moreover Fig. 9(b) is equivalent to an one-degree-of-freedom (ODOF) control system with feedforward inputs as shown in Fig. 10.

The discrete-time state-space realization of P_n and K are given as follows:

$$\begin{aligned} P_n[z] &:= \{A_n, B_n, C_n, 0\}, \\ K[z] &:= \{A_k, B_k, C_k, D_k\}. \end{aligned}$$

Thus the closed-loop system shown in Fig. 10 can be represented as:

$$\begin{aligned} x[k+1] &= Ax[k] + B_u u_m[k] + B_r (y_m[k] - \alpha u_m), \\ y[k] &= Cx[k] + \alpha u_m[k], \end{aligned} \quad (4)$$

where

$$\begin{aligned} A &:= \begin{bmatrix} A_n - B_n D_k C_n & B_n C_k \\ -B_k C_n & A_k \end{bmatrix}, \\ B_u &:= \begin{bmatrix} B_n \\ 0 \end{bmatrix}, B_r := \begin{bmatrix} B_n D_k \\ B_k \end{bmatrix}, C := [C_n \ 0]. \end{aligned}$$

For simple notation, define

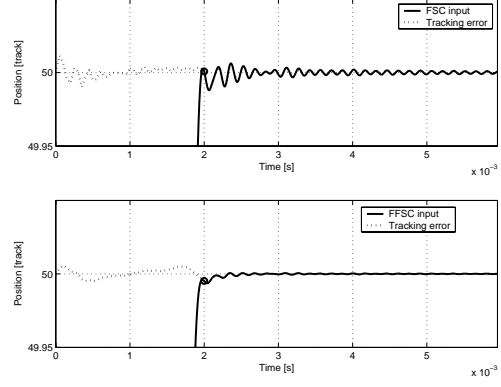


Fig. 11. The results of Method A.

$$U_m := [u_m[0], \dots, u_m[N-1]]^T, \quad (5)$$

$$E := [e[0], \dots, e[N-1]]^T. \quad (6)$$

Here introduce the following notation for Toeplitz matrix:

$$\begin{aligned} T_p\{A, B, C, D, N\} \\ := \begin{bmatrix} D & O & \dots & O \\ t_{21} & D & O & \\ t_{31} & t_{32} & D & \ddots \\ \vdots & & & \ddots \\ t_{N1} & t_{N2} & \dots & D \end{bmatrix} \in \mathbb{R}^{N \times N}, \end{aligned} \quad (7)$$

where $t_{ij} := CA^{i-j-1}B$ ($i > j > 0$).

Lemma 1. Assume that the initial value of the feedback system (4) is zero, and $|u_m| \gg |u_k|$ holds. The tracking error E in Fig. 7 can be approximated by a linear function of α as:

$$E = \alpha(\Omega_r - I)U_m, \quad (8)$$

where

$$\Omega_r = T_p\{A, B_r, C, 0, N\} \in \mathbb{R}^{N \times N}.$$

The proof is omitted because it is straightforward. \square

Here it is assumed that the tracking error $e[i]$ during track seeking can be measured, and the measured signal is defined as $\bar{e}[i]$. Using $\bar{e}[i]$, let us consider to obtain α so that the least square error of $e[i] - \bar{e}[i]$ for $i = 0, \dots, N-1$ is minimum.

Theorem 1. (Least square solution). The optimum α which minimizes the square error $\sum (e[i] - \bar{e}[i])^2$ for a given U_m can be calculated as follows:

$$\alpha_{opt} = \frac{U_m^T (\Omega_r - I)^T}{U_m^T (\Omega_r - I)^T (\Omega_r - I) U_m} \bar{E}, \quad (9)$$

where \bar{E} is defined as

$$\bar{E} = [\bar{e}[0], \dots, \bar{e}[N-1]]^T.$$

Design procedure (Method A)

Step1 Using the conventional TDOF control as shown in Fig. 5, measure the tracking error $\bar{e}[i]$ during track seeking.

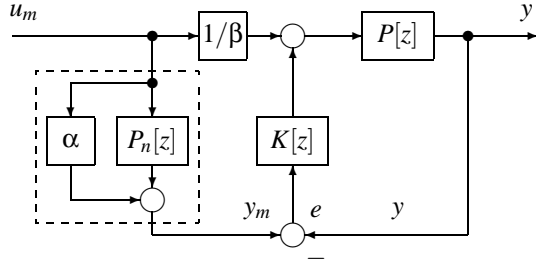


Fig. 14. TDOF control system for method B.

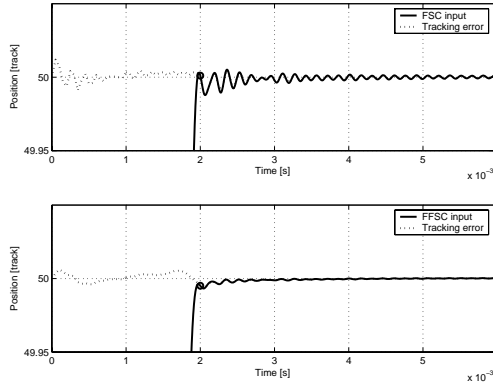


Fig. 15. The results of method B.

Based on the result of Theorem2, the following procedure can be considered.

Design procedure (Method B)

- Step1 Using the conventional TDOF control as shown in Fig. 5, measure the tracking error $\bar{e}[i]$ during track seeking.
- Step2 From the measured data $\bar{e}[i]$, calculate the optimum α_{opt} and β_{opt} from Theorem2.
- Step3 Execute a track seeking by using the new TDOF control system shown in Fig. 14 with α_{opt} and β_{opt} .

In Fig. 14, the compensation gain $1/\beta$ is introduced only in the feedforward input u_m not in the feedback loop because of the following reasons:

- (1) In hard disk drives industry, it is very nervous to introduce an adjustment gain into the feedback loop because it may destabilize the system.
- (2) Since $|u_m| \gg |u_k|$ holds, it is enough to compensate the feedforward input u_m only to improve track seeking performance.

Method B is applied to both FSC and FFSC inputs assuming a gain change of -1 percent. The optimized parameters of α and β in step 2 are shown in Table1. It shows that α_{opt} is close to the value of residual stiffness and β_{opt} reflects the gain change of -1 percent correctly. Fig. 15 shows the track seeking performance obtained by using the parameters in Table1 and the TDOF control system shown in Fig. 14. The figure shows that a good performance can be obtained in the face of the gain change of the plant by Method B.

Table 1. α_{opt} and β_{opt} .

	α_{opt}	β_{opt}
FSC input	-0.246	0.990
FFSC input	-0.249	0.990

5. CONCLUSION

This paper has proposed a new TDOF control scheme which can improved the track error at the end of seeking, and two methods, Method A and Method B, have been proposed to optimize the performance from the measured tracking error profile. Especially for Method B, it can achieve a good performance in the face of a gain perturbation of the plant. The effectiveness of the proposed method has been shown by simulation using the high order model including five vibration modes at high frequencies. Since the proposed method does not depend on reference trajectory and feedback controller, it can apply to many other positioning systems.

ACKNOWLEDGMENT

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