ROTOR RESISTANCE ESTIMATION FOR CURRENT-FED INDUCTION MOTORS

Dimitrios Karagiannis, Alessandro Astolfi

Department of Electrical and Electronic Engineering
Imperial College, London SW7 2BT, United Kingdom
Tel: +44 20 7594 6289, Fax: +44 20 7594 6282
E-mail: {d.karagiannis, a.astolfi}@imperial.ac.uk

Abstract: In this paper a new algorithm for estimating the rotor resistance of current-fed induction motors is presented. The proposed method does not require persistent excitation and achieves asymptotic convergence even in the case of zero rotor speed and/or low torque. However, it does require measurement of the rotor speed and flux. To make the scheme practically feasible, a method for estimating the rotor flux is devised. The proposed estimator is combined with an indirect field-oriented control law to achieve torque/speed regulation in the absence of rotor resistance and load torque information. The efficacy of the resulting adaptive output feedback control scheme is tested via simulations.

Keywords: Nonlinear systems, parameter estimation, induction motors

1. INTRODUCTION

The problem of estimating the rotor winding resistance of induction motors has received a lot of attention recently due to its significance in improving the performance of control algorithms, as well as its application in fault detection, see *e.g.* (Marino *et al.*, 1995; Marino *et al.*, 1998; Pavlov and Zaremba, 2001) and references therein. It is well-known that the variation of the rotor resistance, which may be significant during the operation of the induction motor, can affect the performance of the controller and even lead to instability. This is particularly true for *indirect field-oriented control* (IFOC) schemes, which are widely considered as the industry standard, see *e.g.* (Leonhard, 1985; De Wit *et al.*, 1996; Bodson and Chiasson, 1998; Peresada *et al.*, 1999).

In this paper we develop a new method for estimating the rotor resistance of *current-fed* induction motors (Leonhard, 1985; Dawson *et al.*, 1998) using measurements of the rotor speed, the electromagnetic torque and the rotor flux norm. The proposed method does not require persistence of excitation and achieves asymptotic convergence even in the case of zero rotor speed and/or low torque. Moreover, it provides an asymptotic estimate of the unknown load torque. Further, the problem of torque/speed regulation by means of output feedback is addressed by combining the proposed estimator with the standard IFOC scheme.

The paper is organized as follows. Section 2 describes the dynamical model of an IFOC-driven current-fed induction motor. In Section 3 an estimator for the unknown rotor resistance and the unknown load torque is designed based on measurements of the rotor flux norm and the generated electromagnetic torque. Section 4 combines the proposed estimator with the standard IFOC to obtain an adaptive output feedback controller. Simulations of the resulting scheme are carried out in Section 5 and some conclusions are given in Section 6.

2. PROBLEM FORMULATION

The dynamical model of an induction motor in the stator reference frame (also known as *a-b* or two-phase equivalent model) is given by the equations (Leonhard, 1985)

$$\dot{\lambda}_{ab} = -\left(\frac{R_r}{L_r}I - n_p\omega J\right)\lambda_{ab} + \frac{MR_r}{L_r}i_{ab} \quad (1)$$

$$\dot{\omega} = \frac{n_p M}{mL_r} i_{ab}^T J\lambda_{ab} - \frac{\tau_L}{m} \quad (2)$$

$$\dot{i}_{ab} = \frac{M}{\sigma L_s L_r}\left(\frac{R_r}{L_r}I - n_p\omega J\right)\lambda_{ab} - \left(\frac{R_s}{\sigma L_s} + \frac{M^2 R_r}{\sigma L_s L_r^2}\right)i_{ab} + \frac{1}{\sigma L_s}v_{ab}\,, \quad (3)$$

where $\lambda_{ab} = [\lambda_a, \lambda_b]^T \in \mathbb{R}^2$ is the rotor flux vector, $i_{ab} = [i_a, i_b]^T \in \mathbb{R}^2$ is the stator current vector, $v_{ab} = [v_a, v_b]^T \in \mathbb{R}^2$ is the stator input voltage, ω is the rotor speed, R_r, L_r, M, n_p, m, R_s and L_s are positive constants representing the rotor resistance, rotor inductance, mutual inductance, number of pole pairs, moment of inertia, stator resistance and stator inductance, respectively, $\sigma = 1 - M^2/(L_s L_r)$ is the leakage parameter, τ_L is the load torque and

$$I = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}, \qquad J = \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}.$$

The dynamical model of the current-fed induction motor is obtained from the above equations by taking the stator current vector i_{ab} as the control input, *i.e.* by neglecting the electromagnetic dynamics in the stator circuit. This is justified in practice by the use of high-gain current control loops. Defining the rotation matrix

$$e^{-Jn_p q} = \begin{bmatrix} \cos(n_p q) & \sin(n_p q) \\ -\sin(n_p q) & \cos(n_p q) \end{bmatrix},$$

where q is the rotor shaft angle, and the transformations

$$\lambda_r = e^{-Jn_p q}\lambda_{ab}, \qquad i_s = e^{-Jn_p q} i_{ab} \quad (4)$$

yields the system

$$\dot{\lambda}_r = -\frac{R_r}{L_r}\lambda_r + \frac{MR_r}{L_r}i_s$$
$$\dot{\omega} = \frac{n_p M}{mL_r} i_s^T J\lambda_r - \frac{\tau_L}{m}\,,$$

where $\lambda_r \in \mathbb{R}^2$ is the transformed rotor flux vector and $i_s \in \mathbb{R}^2$ is the transformed stator current vector. Note that the above system describes the dynamic behavior of the current-fed induction motor in a frame rotating with angular speed $n_p\omega$ (Kim *et al.*, 1997).

In the sequel, to simplify the presentation and without loss of generality, we assume that all constants are equal to one, except for the rotor resistance and the load torque which are considered unknown. Defining the control input as $u = i_s$ yields the simplified model

$$\dot{\lambda}_r = -R_r\lambda_r + R_r u \quad (5)$$
$$\dot{\omega} = u^T J\lambda_r - \tau_L\,. \quad (6)$$

The indirect field-oriented controller (IFOC) is described by the equations (De Wit *et al.*, 1996)

$$u = e^{J\rho_d}\begin{bmatrix} y_d \\ \tau_d/y_d \end{bmatrix} \quad (7)$$

$$\dot{\rho}_d = \frac{\bar{R}_r\tau_d}{y_d^2}\,, \quad (8)$$

where y_d and τ_d are the reference values of the flux norm and the torque, respectively, $\bar{R}_r$ is an estimate of the rotor resistance and

$$e^{J\rho_d} = \begin{bmatrix} \cos\rho_d & -\sin\rho_d \\ \sin\rho_d & \cos\rho_d \end{bmatrix}.$$

In what follows we focus (mainly) on the case of torque regulation, where τ_d is a constant reference, as opposed to speed regulation, where τ_d is the output of a PI controller, *i.e.*

$$\tau_d = -\left(K_P + \frac{K_I}{s}\right)(\omega - \omega_d)\,, \quad (9)$$

where ω_d is the speed reference, s denotes the Laplace operator and K_P, K_I are constant gains. Note, however, that the two cases can be considered as approximately equivalent, if the PI is sufficiently slow.

Consider now the variables

$$\xi_1 = u^T J\lambda_r\,, \qquad \xi_2 = u^T\lambda_r\,, \qquad y = \sqrt{\lambda_r^T\lambda_r}$$

and suppose that the generated electromagnetic torque ξ_1 and the flux norm y are available for measurement. The closed-loop system (5)–(7) can be rewritten in the ξ_1, ξ_2 and ω co-ordinates as

$$\dot{\xi}_1 = -R_r\xi_1 + \dot{\rho}_d\xi_2 + \frac{\dot{\tau}_d}{c}\left(\frac{\tau_d}{y_d^2}\xi_1 + \xi_2\right) \quad (10)$$

$$\dot{\xi}_2 = -\dot{\rho}_d\xi_1 - R_r\xi_2 + R_r c + \frac{\dot{\tau}_d}{c}\left(-\xi_1 + \frac{\tau_d}{y_d^2}\xi_2\right) \quad (11)$$

$$\dot{\omega} = \xi_1 - \tau_L\,, \quad (12)$$

where $c = y_d^2 + (\tau_d/y_d)^2$, while the dynamics of the flux norm are given by

$$\dot{y} = -R_r y + \frac{1}{y}R_r\xi_2\,. \quad (13)$$

Note that, for the case of the torque regulation problem, $\dot{\tau}_d = 0$, hence the equations (10)-(11) reduce to

$$\dot{\xi}_1 = -R_r\xi_1 + \dot{\rho}_d\xi_2$$
$$\dot{\xi}_2 = -\dot{\rho}_d\xi_1 - R_r\xi_2 + R_r c\,.$$

Our objective is to obtain asymptotic estimates of the rotor resistance R_r and the load torque τ_L using measurements of ξ_1, y and ω.

3. ESTIMATOR DESIGN

Motivated by the adaptive control tools developed in (Astolfi and Ortega, 2003; Karagiannis *et al.*, 2003) we define the error variables

$$z_1 = \hat{\tau}_L - \tau_L + \beta_1(\omega) \tag{14}$$
$$z_2 = \hat{R}_r - R_r + \beta_2(\xi_1)\,, \tag{15}$$

where $\beta_1(\cdot)$ and $\beta_2(\cdot)$ are continuous functions yet to be specified, and the update laws

$$\dot{\hat{\tau}}_L = -\frac{\partial\beta_1}{\partial\omega}\left(\xi_1 - \hat{\tau}_L - \beta_1(\omega)\right) \tag{16}$$
$$\dot{\hat{R}}_r = -\frac{\partial\beta_2}{\partial\xi_1}\Bigg[\left(-\hat{R}_r - \beta_2(\xi_1)\right)\xi_1 + \dot{\rho}_d\xi_2 + \frac{\dot{\tau}_d}{c}\left(\frac{\tau_d}{y_d^2}\xi_1 + \xi_2\right)\Bigg]\,, \tag{17}$$

where ξ_2 is obtained from the identity [1]

$$\xi_1^2 + \xi_2^2 = cy^2\,. \tag{18}$$

The resulting error dynamics are described by the equations

$$\dot{z}_1 = \frac{\partial\beta_1}{\partial\omega}z_1 \tag{19}$$
$$\dot{z}_2 = \frac{\partial\beta_2}{\partial\xi_1}\xi_1 z_2\,. \tag{20}$$

Selecting the function $\beta_1(\cdot)$ as

$$\beta_1(\omega) = -k_1\omega \tag{21}$$

with $k_1 > 0$ yields the error system

$$\dot{z}_1 = -k_1 z_1\,, \tag{22}$$

which has a globally exponentially stable equilibrium at the origin, hence z_1 converges to zero. As a result, from (14) an asymptotic estimate of the load τ_L is given by

$$\bar{\tau}_L = \hat{\tau}_L + \beta_1(\omega)\,. \tag{23}$$

Note, moreover, that the constant k_1 which corresponds to the convergence rate of the estimation error can be arbitrarily assigned.

Consider now the problem of finding a function $\beta_2(\cdot)$ such that the system (20) has an asymptotically stable equilibrium at $z_2 = 0$.[2] A possible selection is

$$\beta_2(\xi_1) = \frac{k_2}{2}\frac{1}{1 + k_3\xi_1^2} \tag{24}$$

with $k_2 > 0$, $k_3 > 0$ constants, yielding the stable (uniformly in ξ_1) error dynamics

$$\dot{z}_2 = -\frac{k_2k_3\xi_1^2}{\left(1 + k_3\xi_1^2\right)^2}z_2\,. \tag{25}$$

Remark 1. The advantage of (24) over the more obvious selection $\beta_2(\xi_1) = \frac{k_2}{2}\xi_1^2$ is that it ensures boundedness of $\hat{R}_r$ for any ξ_1. This will be particularly useful in the following section to prove stability of the adaptive closed-loop system.

Remark 2. A guideline for tuning the parameters k_2 and k_3 in (25) is to select k_3 to maximize the function $k_3\xi_1^2/\left(1 + k_3\xi_1^2\right)^2$ for the nominal value of ξ_1 and then select k_2 according to the desired convergence rate.

From (15), assuming that $\xi_1/\left(1 + k_3\xi_1^2\right)$ is not in $\mathcal{L}_2$, an asymptotic estimate of the rotor resistance R_r is given by

$$\bar{R}_r = \hat{R}_r + \beta_2(\xi_1)\,. \tag{26}$$

Summarizing, the proposed (second-order) estimator is given by the equations (16)-(17), (21), (24) and (26).

4. ADAPTIVE OUTPUT FEEDBACK CONTROL

Consider again the closed-loop system (5)–(8), where $\bar{R}_r$ is given by (26). It was shown in the previous section that the estimate $\bar{R}_r$ remains bounded and asymptotically converges to the true value R_r, provided that ξ_1 is not identically equal to zero. We will now show that, for the torque regulation case, the rotor flux and the generated

[1] Solving (18) for ξ_2 we obtain two solutions from which we select the positive one, *i.e.* $\xi_2 = \sqrt{cy^2 - \xi_1^2}$. This is justified by the fact that the dynamics of the square of the flux norm are given by $\dot{y}^2 = -2R_r y^2 + 2R_r\xi_2$, hence y^2 is a filtered version of ξ_2.

[2] It is interesting to note that, for any function $\beta_2(\cdot)$, when $\xi_1 = 0$ the system (20) has a stable equilibrium manifold given by the z_2-axis. This implies that it is not possible to estimate the rotor resistance when the torque is identically equal to zero.

torque remain bounded and asymptotically converge to the reference values.

Proposition 1. Consider the IFOC-driven current-fed induction motor described by the equations (10)–(13), where $\dot{\rho}_d$ is given by (8) and y_d and τ_d are constant references, in closed-loop with the estimator given by the equations (16)-(17), (21), (24) and

$$\bar{R}_r = \max(\hat{R}_r + \beta_2(\xi_1), R_{min}) \tag{27}$$

with $R_{min} > 0$ an arbitrarily small lower bound on R_r. Then, for all initial conditions, the states ξ_1 and ξ_2 remain bounded and, moreover,

$$\lim_{t\to\infty} \xi_1(t) = \tau_d, \qquad \lim_{t\to\infty} \xi_2(t) = y_d^2$$

and

$$\lim_{t\to\infty} y(t) = y_d\,.$$

Proof: Define the error variables

$$x_1 = \xi_1 - \tau_d, \qquad x_2 = \xi_2 - y_d^2$$

and note that the system (10)–(12) can be rewritten in the x_1, x_2 and ω co-ordinates as

$$\dot{x}_1 = -R_r x_1 + \frac{\bar{R}_r \tau_d}{y_d^2} x_2 + \left(\bar{R}_r - R_r\right)\tau_d \tag{28}$$

$$\dot{x}_2 = -\frac{\bar{R}_r \tau_d}{y_d^2} x_1 - R_r x_2 - \left(\bar{R}_r - R_r\right)\frac{\tau_d^2}{y_d^2} \tag{29}$$

$$\dot{\omega} = x_1 + \tau_d - \tau_L\,. \tag{30}$$

Consider now the Lyapunov function

$$V(x_1, x_2) = \frac{1}{2}x_1^2 + \frac{1}{2}x_2^2\,,$$

whose time-derivative along the trajectories of (28)-(29) satisfies

$$\dot{V}(x_1,x_2) = -R_r\left(x_1^2 + x_2^2\right) + \tau_d\left(\bar{R}_r - R_r\right)\left(x_1 - \frac{\tau_d}{y_d^2}x_2\right)\,.$$

A simple application of Young's inequality shows that there exist constants $\epsilon > 0$ and $\delta > 0$ such that

$$\dot{V}(x_1,x_2) \le -\epsilon V(x_1,x_2) + \delta\left(\bar{R}_r - R_r\right)^2\,,$$

hence the system (28)-(29) is input-to-state stable with respect to $\bar{R}_r - R_r$. It remains to prove that the error $\bar{R}_r - R_r$ is bounded and asymptotically converges to zero. To this end, recall first that the dynamics of the error variable z_2 defined in (15) are described by (25), hence $z_2 \in \mathcal{L}_\infty$ and

$$\frac{\xi_1 z_2}{1 + k_3\xi_1^2} \in \mathcal{L}_2\,.$$

Since z_2 is bounded, $\bar{R}_r$ is also bounded, hence $\xi_1, \xi_2 \in \mathcal{L}_\infty$ and, from (10)-(11) and (25), the time-derivatives of ξ_1, ξ_2 and z_2 are also bounded. From Barbalat's lemma, this implies that

$$\lim_{t\to\infty} \frac{\xi_1(t) z_2(t)}{1 + k_3\xi_1(t)^2} = 0\,,$$

hence either z_2 converges to zero or ξ_1 converges to zero and z_2 converges to a nonzero constant. Due to the dynamics (10)-(11) the latter is only possible if $\dot{\rho}_d = 0$. But from (8) and (27) we have

$$|\dot{\rho}_d| \ge \frac{R_{min}|\tau_d|}{y_d^2} > 0\,.$$

Hence z_2 and therefore $\bar{R}_r - R_r$ converge to zero. As a result, x_1 and x_2 are bounded and asymptotically converge to zero, hence ξ_1 converges to τ_d and ξ_2 converges to y_d^2. This, from (13), implies that the flux norm y converges to the reference value y_d. ◁

Remark 3. A practical limitation of the proposed scheme is that it relies on measurements of the generated torque ξ_1 and the flux norm y, which, in turn, require knowledge of the rotor flux. To overcome this problem it is necessary to devise a method for estimating the flux vector λ_{ab}. To this end, note that from (1) and (3) we obtain

$$\dot{\lambda}_{ab} = -\frac{\sigma L_s L_r}{M}\dot{i}_{ab} - \frac{R_s L_r}{M} i_{ab} + \frac{L_r}{M} v_{ab}\,,$$

where $\dot{i}_{ab}$ can be computed using (4) and (7). Hence, the (open-loop) observer

$$\dot{\hat{\lambda}}_{ab} = -\frac{\sigma L_s L_r}{M}\dot{i}_{ab} - \frac{R_s L_r}{M} i_{ab} + \frac{L_r}{M} v_{ab}$$

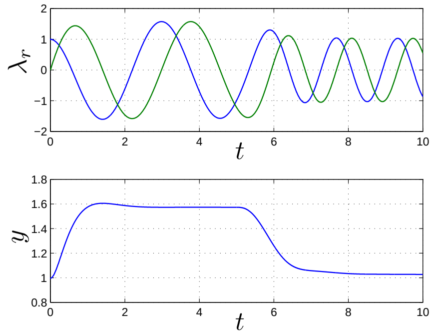
is such that $\hat{\lambda}_{ab} - \lambda_{ab} = \text{const}$, *i.e.* $\hat{\lambda}_{ab}$ is an exact estimate of λ_{ab} up to a constant error term. Since under normal operation the flux has zero mean, this error can be practically removed by filtering out the DC component of $\hat{\lambda}_{ab}$.

5. SIMULATION RESULTS

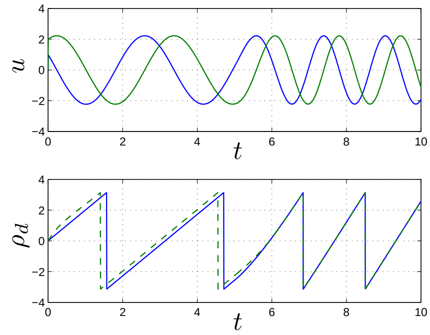
The model of the induction motor (1)–(3) has been simulated using the parameters $R_r = 2$, $\tau_L = 2$ and assuming all other constants are equal to one. A high-gain current control loop has been implemented as

$$v_{ab} = K_c\left(e^{Jn_p q}u - i_{ab}\right)$$

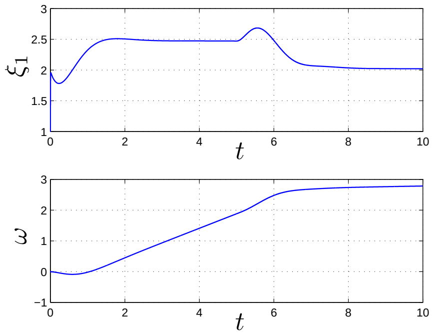
with $K_c = 500$, where u is given by (7)-(8) and $\bar{R}_r$ is given by (27). The initial conditions are defined as $\lambda_{ab}(0) = [1,0]^T$, $\omega(0) = 0$ and $i_{ab}(0) = [0,0]^T$. The reference of the flux norm has been set to



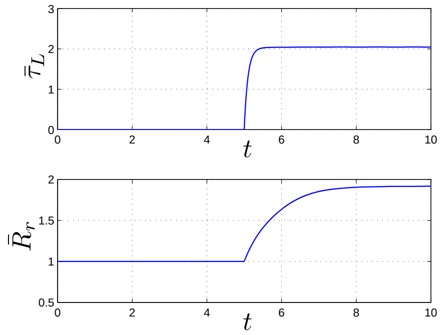
(a) Top: Rotor flux vector λ_r. Bottom: Flux norm y.



(b) Top: Stator current vector u. Bottom: Controller state ρ_d (solid line) and flux angle (dashed line).



(c) Top: Generated torque ξ_1. Bottom: Rotor speed ω.



(d) Top: Load torque estimate $\bar{\tau}_L$. Bottom: Rotor resistance estimate $\bar{R}_r$.

Fig. 1. Time histories for the torque regulation case.

$y_d = 1$. The adaptive gains have been set to $k_1 = k_2 = 10$ and $k_3 = 1$. In order to compare the response with the non-adaptive case, during the first 5s we replace $\bar{R}_r$ in (8) with a fixed estimate. In this case we have taken $\bar{R}_r = R_r/2$.

5.1 Torque regulation

We first consider the torque regulation problem, where the torque reference is fixed at $\tau_d = 2$. Figure 1(a) shows the time histories of the flux vector λ_r and flux norm y, while Figure 1(b) shows the time histories of the control input u, the controller state ρ_d and the flux angle. A plot of the generated torque ξ_1 is shown in Figure 1(c). Notice that, as expected from the results in (De Wit *et al.*, 1996), the mismatch in the estimate of the rotor resistance during the first 5s results in a significant steady-state error both in the flux level and in the generated torque. The convergence of the estimates $\bar{\tau}_L$ and $\bar{R}_r$ to the true values τ_L and R_r is shown in Figure 1(d).

5.2 Speed regulation

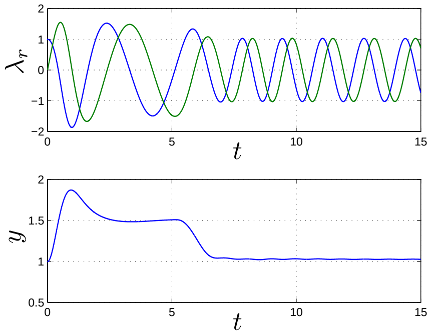
For the speed regulation case, recall that τ_d is given by the PI control law (9). An implementation of this control law, which uses the estimate of the load torque τ_L computed in (23), is given by

$$\dot{\tau}_d = -K_P\left(\xi_1 - \bar{\tau}_L\right) - K_I\left(\omega - \omega_d\right) .$$

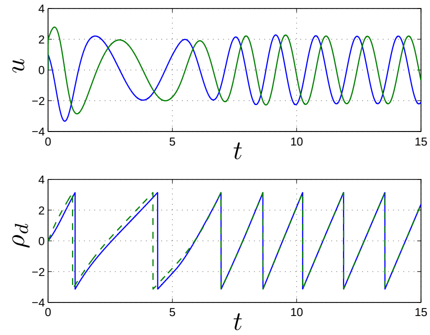
A plot of the flux vector λ_r and the flux norm y is shown in Figure 2(a), while Figure 2(b) shows the time histories of the control input u, the state ρ_d and the flux angle. The time histories of the generated torque ξ_1 and the rotor speed ω are shown in Figure 2(c). Again we see that all signals converge to their respective reference values. Figure 2(d) shows the convergence of the estimates $\bar{\tau}_L$ and $\bar{R}_r$ to the true values τ_L and R_r with a small steady-state error due to the filtering of the flux observations (see Remark 3).

6. CONCLUSIONS

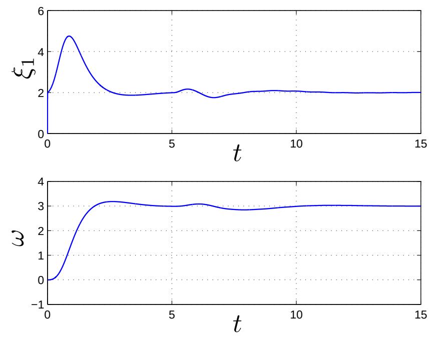
In this paper we have presented a new algorithm for estimating the rotor resistance and load torque of a current-fed induction motor, using measurements of the rotor speed, flux magnitude and electromagnetic torque. It has been shown that the generated estimates converge asymptotically



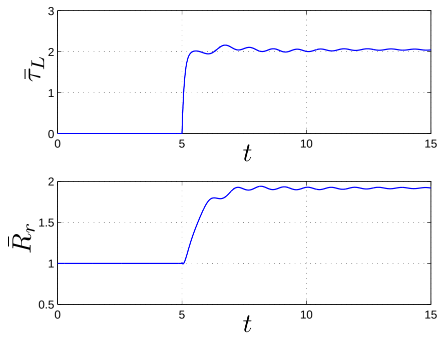
(a) Top: Rotor flux vector λ_r. Bottom: Flux norm y.



(b) Top: Stator current vector u. Bottom: Controller state ρ_d (solid line) and flux angle (dashed line).



(c) Top: Generated torque ξ_1. Bottom: Rotor speed ω.



(d) Top: Load torque estimate $\bar{\tau}_L$. Bottom: Rotor resistance estimate $\bar{R}_r$.

Fig. 2. Time histories for the speed regulation case.

to the true values. The proposed estimator has been combined with an indirect field-oriented control law to achieve torque/speed regulation in the absence of rotor resistance or load torque information.

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