TERRAIN BASED NAVIGATION TOOLS FOR UNDERWATER VEHICLES USING EIGEN ANALYSIS

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Abstract: The development of tools for Terrain Based Navigation of Underwater Vehicles rooted on Principal Component Analysis are proposed and discussed in detail. Resorting to a nonlinear Lyapunov transformation, the synthesis and analysis of a nonlinear multirate $\mathcal{H}_2$ estimator is presented with guaranteed stability and optimal performance on equilibrium trajectories. Post-processing techniques using a fixed interval non-causal smoother are outlined to improve the performance of the overall framework. Results from Monte Carlo simulation techniques to assess the performance of the proposed tools are included.

Keywords: Navigation Systems, Time-varying Systems, Kalman Filters, Robot Navigation, Smoothing filters.

1. INTRODUCTION

Navigation systems for long duration missions of underwater vehicles (UVs) in unstructured environments, without resorting to external sensors, and with bounded estimation errors, have been a major challenge in underwater robotics (Leonard *et al.*, 1998). However, unmodeled dynamics, time-varying phenomena, and the noise present in the sensor measurements continuously degrade the navigation system accuracy along time, precluding its use on a number of important long range missions. To overcome this limitation external positioning systems have been proposed in the past (Vickery, 1998) and successfully integrated in navigation systems for underwater applications (Alcocer *et al.*, 2004; Whitcomb *et al.*, 1999). The tedious deployment and the demanding calibration procedure of the positioning systems, strongly constrain the area where the missions can take place and ultimately the use of UVs.

In the case where the missions take place in areas where detailed bathymetric data are available, one alternative has been exploited in the past: the terrain information can be used as an aiding positioning sensor to bound the error estimates on the navigation systems leading to the so called Terrain Based or Terrain Aided Navigation Systems. Applications on air (Baker and Clem., 1977; Hostetler and Andreas, 1983), land (Crowley *et al.*, 1998) and underwater vehicles (Karlsson and Gustafsson, 2003; Sistiaga *et al.*, 1998) were reported in the last decades.

Extended Kalman Filtering has been the most commonly used synthesis technique to address the terrain based navigation design problem (Hostetler and Andreas, 1983; Sistiaga *et al.*,

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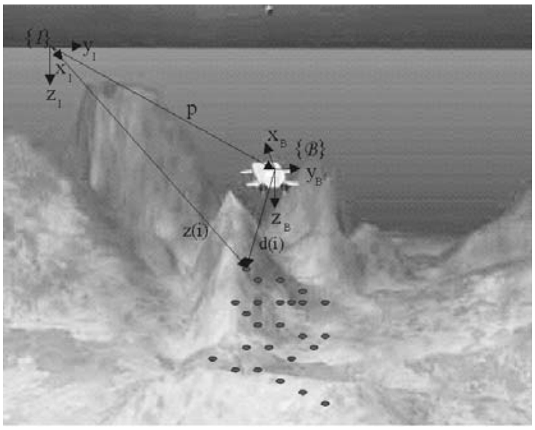

Fig. 1. UV inertial and local coordinate frames. Mechanical scanning sonar range measurements.

1998). However, several authors provide examples on instability and severe performance degradation of the proposed solutions, precluding their use in general. Correlation techniques (Baker and Clem., 1977; Nygren and Jansson, 2003) and particle filters (Karlsson and Gustafsson, 2003; Pham *et al.*, 2003) have also been proposed, requiring a high computational burden.

This paper tries to endow the underwater robotics community with tools for terrain based navigation but departs considerably from the approaches previously described. The methodology proposed is rooted on optimal processing techniques of random signals, namely Principal Component Analysis (PCA) based on the Karhunen-Loève Transform (Mertins, 1999; Jolliffe, 2002).

The paper is organized as follows: in section 2 the notation is introduced and the sensor package installed onboard is described. Section 3 reviews the background on the Karhunen-Loève transform, basis for the principal component analysis of stochastic signals. The approach for the bathymetric data decomposition will also be detailed. In section 4 the estimator structure to be used on UV missions is introduced and some properties are presented. Section 5 outlines the use of an optimal fixed interval smoother for data post-processing and section 6 addresses the implementation issues. Monte Carlo results obtained with a simulated model of an UV in a synthesized terrain are also presented. Finally, some conclusions are drawn in section 7.

2. NOTATION, SENSOR PACKAGE AND DESIGN MODEL

2.1 Notation

Let $\{\mathcal{I}\}$ be an inertial reference frame located at the pre-specified mission scenario with North, East, and Down axes, and origin at mean sea level, and let $\{\mathcal{B}\}$ denote a body-fixed frame that moves with the UV, as depicted in fig. 1. The following notation is required:

$\mathbf{p} := [x\ \ y\ \ z]^T$ - position of the origin of $\{B\}$ in $\{\mathcal{I}\}$;

${}^{\mathcal{B}}({}^{\mathcal{I}}\mathbf{v}_{\mathcal{B}})$ - linear velocity of the origin of $\{\mathcal{B}\}$ in $\{\mathcal{I}\}$, expressed in $\{\mathcal{B}\}$, i.e. body-fixed linear velocity;

$\boldsymbol{\lambda} := [\phi\ \ \theta\ \ \psi]^T$ - vector of roll, pitch, and yaw angles that parameterize locally the orientation of frame $\{\mathcal{B}\}$ with respect to $\{\mathcal{I}\}$;

${}^{\mathcal{B}}({}^{\mathcal{I}}\omega_{\mathcal{B}})$ - angular velocity of $\{\mathcal{B}\}$ relative to $\{\mathcal{I}\}$, expressed in $\{\mathcal{B}\}$, i.e. body-fixed angular velocity;

Given two frames $\{\mathcal{A}\}$ and $\{\mathcal{B}\}$, ${}^{\mathcal{A}}_{\mathcal{B}}\mathcal{R}$ denotes the rotation matrix from $\{\mathcal{B}\}$ to $\{\mathcal{A}\}$. In particular, ${}^{\mathcal{I}}_{\mathcal{B}}\mathcal{R}(\boldsymbol{\lambda})$ is the rotation matrix from $\{\mathcal{B}\}$ to $\{\mathcal{I}\}$, parameterized locally by $\boldsymbol{\lambda}$. Since $\mathcal{R}$ is a rotation matrix, it satisfies $\mathcal{R}^T = \mathcal{R}^{-1}$ that is, $\mathcal{R}^T\mathcal{R} = I$.

2.2 Sensor package and data geo-referencing

Consider an UV equipped with an Attitude and Heading Reference System (AHRS) providing measurements on the attitude $\boldsymbol{\lambda}$ and on the angular velocities in body frame ${}^{\mathcal{B}}({}^{\mathcal{I}}\omega_{\mathcal{B}})$. Two rotation matrices $\mathcal{R}_Z(\psi)$ (or $\mathcal{R}_\psi$ in compact form) and $\mathcal{R}_{\theta,\phi} = \mathcal{R}_Y(\theta)\mathcal{R}_X(\phi)$ will be used, verifying ${}^{\mathcal{I}}_{\mathcal{B}}\mathcal{R}(\boldsymbol{\lambda}) = \mathcal{R}_\psi\mathcal{R}_{\theta,\phi}$. To complement the information on position, a Doppler velocity log will be installed onboard the UV, providing measurements of ${}^{\mathcal{B}}({}^{\mathcal{I}}\mathbf{v}_{\mathcal{B}})$. The body fixed velocity expressed in the horizontal plane, using $\mathbf{v}_H = \mathcal{R}(\theta,\phi){}^{\mathcal{B}}({}^{\mathcal{I}}\mathbf{v}_{\mathcal{B}})$, i.e. corrected with the attitude information in $\mathcal{R}(\theta,\phi)$, will be an input to the design model, as detailed next. A depth cell will also be considered to provide measurements on the vertical.

A sonar ranging sensor is required to provide measurements for the PCA based positioning system, described later. Among the several types available, a mechanical scanning sonar, with a scan bearing angle ϵ, will be considered. See fig. 1 in detail, where the seafloor points sensed in consecutive ranging measurements - $z(i)$ - are depicted in red. Assuming, without loss of generality, that the sonar is installed pointing down at the origin of the reference frame $\mathcal{B}$ and the scanning angle lies in the transversal plane (containing the (y_B, z_B) axes), the i^{th} range measurement $d(i)$ can be geo-referenced in the inertial reference frame $\mathcal{I}$ using the relation $z(i) = \mathbf{p} + {}^{\mathcal{I}}_{\mathcal{B}}\mathcal{R}(\boldsymbol{\lambda})\mathcal{R}(\epsilon)[0\ \ 0\ \ d(i)]^T$, where $\mathcal{R}(\epsilon)$ is the rotation matrix from the instantaneous sonar bearing to the UV reference frame $\mathcal{B}$. No support from other external systems/devices will be required.

2.3 Design model

The underlying design model $\mathcal{G}$, that plays a central role in the design of the estimator is based on a simplified discrete time version of the UV kinematics and has the realization

$$\Sigma_{\mathcal{G}} = \begin{cases} \mathbf{p}(k+1) = \mathbf{p}(k) + h\mathcal{R}_\psi(k)(\mathbf{v}_H(k) + \mathbf{b}(k)) + \eta_p \\ \mathbf{b}(k+1) = \mathbf{b}(k) + \mathcal{R}_\psi^T(k)\eta_b \end{cases} \quad (1)$$

where h is the sampling period, k describes in compact form the time instant $t_k = kh$ for $k = 0, 1, \ldots, T$ (the final mission time), $\mathbf{b}$ captures the bias terms due to velocity sensor installation and calibration mismatches, assumed constant or slowly varying, and η_p and η_b are auxiliary inputs to be used in the stochastic $\mathcal{H}_2$ problem considered later. Note that the UV velocity $\mathbf{v}_H$ is also considered as an input to the model. The overall model structure is depicted as part of the block diagram on fig. 2.

3. PRINCIPAL COMPONENT ANALYSIS

Considering all linear transformations, the Karhunen-Loève (KL) transform allows for the optimal approximation to a stochastic signal, in the least squares sense. Furthermore, it is a well known signal expansion technique with uncorrelated coefficients for dimensionality reduction. These features make the KL transform interesting for many signal processing applications such as data compression, image and voice processing, data mining, exploratory data analysis, pattern recognition and time series prediction (Mertins, 1999; Jolliffe, 2002).

3.1 PCA background

Consider a set of M stochastic signals $\mathbf{x}_i \in \mathcal{R}^N, i = 1, \ldots, M$, each represented as a column vector, with mean $m_x = 1/M \sum_{i=1}^{M} \mathbf{x}_i$. The purpose of the KL transform is to find an orthogonal basis to decompose a stochastic signal $\mathbf{x}$, from the same original space, to be computed as $\mathbf{x} = \mathbf{U}\mathbf{v} + m_x$, where the vector $\mathbf{v} \in \mathcal{R}^N$ is the projection of $\mathbf{x}$ in the basis, i.e., $\mathbf{v} = \mathbf{U}^T(\mathbf{x} - m_x)$. The matrix $\mathbf{U} = [\mathbf{u}_1\, \mathbf{u}_2 \ldots \mathbf{u}_N\,]$ should be composed by the N orthogonal column vectors of the basis, verifying the eigenvalue problem

$$\mathbf{R}_{xx}\mathbf{u}_j = \lambda_j \mathbf{u}_j,\ \ j = 1, ..., N, \quad (2)$$

where $\mathbf{R}_{xx}$ is the covariance matrix that can be computed from the set of M experiments using

$$\mathbf{R}_{xx} = \frac{1}{M-1}\sum_{i=1}^{M}(\mathbf{x}_i - m_x)(\mathbf{x}_i - m_x)^T. \quad (3)$$

Assuming that the eigenvalues are ordered, i.e. $\lambda_1 \geq \lambda_2 \geq \cdots \geq \lambda_N$, the choice of the first $n << N$ principal components, leads to an approximation to the stochastic signals given by the ratio on the covariances associated with the components, i.e. $\sum_n \lambda_n / \sum_N \lambda_N$. In many applications, where stochastic multidimensional signals are the key to overcome the problem at hand, this approximation can constitute a large dimensional reduction and thus a computational complexity reduction. The advantages of PCA are threefold: i) it is an optimal (in terms of mean squared error) linear scheme for compressing a set of high dimensional vectors into a set of lower dimensional vectors; ii) the model parameters can be computed directly from the data (by diagonalizing the ensemble covariance); iii) given the model parameters, projection into and from the bases are computationally inexpensive operations $\mathcal{O}(nN)$.

3.2 PCA based Positioning System

Assume a mission scenario where bathymetric data are available and that a terrain based navigation system should be designed. The steps to implemented a PCA based positioning sensor using this bathymetric data will be outlined.

Prior to the mission, the bathymetric data of the area under consideration should be partitioned in *mosaics* with fixed dimensions N_x by N_y. After reorganizing this two-dimensional data in vector form, e.g. stacking the columns, a set of M stochastic signals $\mathbf{x}_i \in \mathcal{R}^N$, $N = N_x N_y$, results. The number of signals M to be considered depends on the mission scenario and on the *mosaic* overlapping. The KL transform can be computed, using (2) and (3), the eigenvalues must be ordered, and the number n of the principal components to be used should be selected, according with the required level of approximation.

The following data should be recorded for latter use: i) the data ensemble mean m_x; ii) the matrix transformation with n eigenvectors $\mathbf{U}_n = [\mathbf{u}_1 \ldots \mathbf{u}_n]$; iii) the projection on the selected basis of all the *mosaics*, computed using $\mathbf{v}_i = \mathbf{U}_n^T(x_i - m_x), i = 1, \ldots, M$; iv) the coordinates of the center of the *mosaics*, $(x_i, y_i),\ i = 1, \ldots, M$.

During the mission, at the time instants $t_k = Kk$, where K is an integer greater than 1, the georeferenced range measurements from the present *mosaic*, are packed and will constitute the input signal $\mathbf{x}$ to the PCA positioning system. The following tasks should be performed: i) compute the projection of the signal $\mathbf{x}$ into the basis, using $\mathbf{v} = \mathbf{U}_n^T(\mathbf{x} - m_x)$; ii) given an estimate on the actual horizontal coordinates of the UV position $\hat{x}$ and $\hat{y}$, provided by the navigation system, search on a given neighborhood δ the *mosaic* that verifies $\forall_i \|[\hat{x}\,\hat{y}]^T - [x_i\, y_i]^T\|_2 < \delta$, $r_{PCA} = \min_i \|\mathbf{v} - \mathbf{v}_i\|_2$; iii) given the *mosaic* i that is the closest to the present input, its center coordinates (x_i, y_i) will be selected as the x_m and y_m measurements.

Note that the bathymetric data based PCA positioning system described above, can be straightforward extended to use multidimensional geophysical data that can be measured with other sensors installed onboard UVs such as magnetometers and gradiometers (Leonard *et al.*, 1998).

4. NONLINEAR ESTIMATOR DESIGN AND ANALYSIS

Based on the measurements from the set of sensors installed on board and on the underlying design model, derived from the kinematic relations with realization $\mathcal{G}$ in (1), the estimator design will be presented. The navigation system will provide non-biased estimates $\hat{\mathbf{p}}$ and ${}^{\mathcal{I}}\hat{\mathbf{v}}_{\mathcal{B}}$ of the position and velocity of the body fixed frame $\{\mathcal{B}\}$ relative to the inertial frame $\{\mathcal{I}\}$, respectively.

A multi-rate minimization stochastic $\mathcal{H}_2$ problem will be the setup adopted for the navigation system design. First, some algebraic relations will be outlined, a nonlinear transformation will be introduced and applied when the UVs are describing trimming trajectories, i.e. equilibrium trajectories, see (Fryxell *et al.*, 1996) for details.

On the horizontal plane the rotation of the UV admits the first order discrete time approximation $\psi(k+1) = \psi(k) + h\omega_z(k)$, where ω_z is the z component of the projection of the angular velocity in body axis to the horizontal plane, i.e. $[\omega_x\ \omega_y\ \omega_z]^T =^{\mathcal{I}}_{\mathcal{B}} \mathcal{R}(\boldsymbol{\lambda})^{\mathcal{B}}({}^{\mathcal{I}}\omega_{\mathcal{B}})$. Moreover, $\mathcal{R}_\psi(k+1) = \mathcal{R}_\psi(k)\mathcal{R}_Z(h\omega_z(k)) = \mathcal{R}_Z(h\omega_z(k))\mathcal{R}_\psi(k)$.

Lemma 4.1. Let $\mathbf{T}(k) \in \mathcal{R}^{6\times 6}$ be a nonlinear time varying Lyapunov transformation, parameterized by $\psi(k)$

$$\mathbf{T}(k) = \begin{bmatrix} I_{3\times3} & 0_{3\times3} \\ 0_{3\times3} & \mathcal{R}_\psi(k) \end{bmatrix},$$

verifying $\mathbf{T}^{-1}(k) = \mathbf{T}^T(k)$, $\|\mathbf{T}(k)\|_2 \leq 1$, and $\|\mathbf{T}(k)\|_\infty \leq 1$.

Lemma 4.2. The estimation problem associated with the UV kinematics, with realization in $\mathcal{G}$, is linear and time invariant over any trimming trajectory, using the transformation $\mathbf{z}(k) = \mathbf{T}(k)[\mathbf{p}^T(k)\ \mathbf{b}^T(k)]^T$, introduced above.

PROOF. Using the transformation $\mathbf{T}(k)$ the dynamics of the new state variables can be written as

$$\begin{aligned} \mathbf{z}(k+1) &= \overline{\mathbf{A}}(k)\mathbf{z}(k) + \overline{\mathbf{B}}_1(k)(\mathbf{v}_H(k) + \eta_{\mathbf{p}}(\mathbf{k})) \\ &\qquad + \overline{\mathbf{B}}_2(k)\eta_{\mathbf{b}}(k), \\ \mathbf{y}(k) &= \overline{\mathbf{C}}(k)\mathbf{z}(k) \end{aligned} \tag{4}$$

where

$$\overline{\mathbf{A}}(k) = \begin{bmatrix} I_{3\times3} & hI_{3\times3} \\ 0_{3\times3} & \mathcal{R}_Z(h\omega_z(k)) \end{bmatrix}, \overline{\mathbf{B}}_1(k) = \begin{bmatrix} h\mathcal{R}_\psi(k) \\ 0_{3\times3} \end{bmatrix},$$

$\overline{\mathbf{B}}_2(k) = [0_{3\times3}\ \mathcal{R}_Z^T(h\omega_z(k))]^T$, and $\overline{\mathbf{C}}(k) = [I_{3\times3}\ 0_{3\times3}]^T$, from the relations in (1). The key idea is that the equilibrium trajectories for UVs (also known as trimming trajectories) are circles in the horizontal plane, parameterized by constant vehicle's body yaw rate $\omega_z(k)$ (Fryxell *et al.*, 1996). Excluding as usually the deterministic inputs (nulled when computing the error estimates), the resulting system dynamics, described

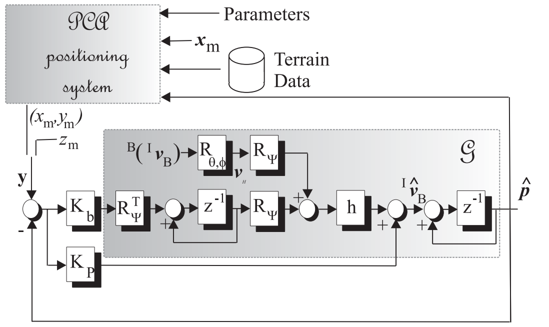

Fig. 2. Block diagram of the nonlinear estimator proposed.

by (4) on trimming trajectories is linear and time invariant.

Remark that the dynamics described in (4), will be used on the $\mathcal{H}_2$ estimator synthesis problem, where $\eta = [\eta_p^T\ \eta_b^T]^T$ is zero mean white noise with uncorrelated covariance $E[\eta(k)\eta^T(k)] = \mathbf{Q}(k)$. The multi-rate problem will be solved resorting to the usual nonlinear recursions for the Kalman filter:

$$\begin{aligned} \hat{\mathbf{z}}^-(k+1) &= \overline{\mathbf{A}}(k)\hat{\mathbf{z}}^+(k) + \overline{\mathbf{B}}_1(k)\mathbf{v}_H(k) \\ \mathbf{P}^-(k+1) &= \overline{\mathbf{A}}(k)\mathbf{P}^+(k)\overline{\mathbf{A}}^T(k) + \mathbf{Q}(k) \\ &\text{for } k = 0, 1, \ldots \end{aligned} \tag{5}$$

where $\hat{\mathbf{z}}^-(k+1)$ is the predicted state variable estimate and $\mathbf{P}^-(k+1)$ is the covariance of the prediction estimation error, respectively.

In the time instants multiple of K, the PCA positioning system, described in subsection 3.2, and the depth cell provide measurements $\mathbf{y} = [x_m\ y_m\ z_m]^T$, with covariance

$$\mathbf{R}(k) = diag(f\ r_{PCA}^{1/2}, f\ r_{PCA}^{1/2}, r_m),$$

where f is a proportion factor depending on the terrain (Oliveira, 2005).

The Kalman filter state and error covariance updates, $\hat{\mathbf{z}}^+(k)$ and $P^+(k)$, respectively, can be obtained according with

$$\begin{aligned} \hat{\mathbf{z}}^+(k) &= \hat{\mathbf{z}}^-(k) + \mathbf{K}(k)(y - \overline{\mathbf{C}}(k)\hat{\mathbf{z}}^-(k)) \\ \mathbf{P}^+(k) &= \mathbf{P}^-(k) - \mathbf{P}^-(k)\overline{\mathbf{C}}^T(k) \\ &(\overline{\mathbf{C}}(k)\mathbf{P}^-(k)\overline{\mathbf{C}}^T(k) + \mathbf{R}(k))^{-1}\overline{\mathbf{C}}^T(k)\mathbf{P}^-(k) \end{aligned} \tag{6}$$

where $\mathbf{K}(k) = \mathbf{P}^-(k)\overline{\mathbf{C}}^T(k)(\overline{\mathbf{C}}(k)\mathbf{P}^-(k)\overline{\mathbf{C}}^T(k) + \mathbf{R}(k))^{-1} = [\mathbf{K}_p^T\ \mathbf{K}_b^T]^T$ is the Kalman gain, with two diagonal blocks. For the time instants $mod(k, K) \neq 0$ $\mathbf{P}^+(k) = \mathbf{P}^-(k)$ and $\hat{\mathbf{z}}^+(k) = \hat{\mathbf{z}}^-(k)$. The resulting estimator is represented in fig. 2, with some abuse of notation.

Under the assumption of homogeneous space properties, note that the covariances in both x and y directions of the initial error covariance $\mathbf{P}^-(0)$, the state noise covariance $\mathbf{Q}$, and the observation covariances from the PCA positioning system are identical. It is important to remark that is this case the probability density functions are symmetric and under a rotation they are preserved (the level curves are circles). This fact supports the use of a linear Kalman filter for the nonlinear system and explains the fact that over any trimming trajectory the evolution of the estimate covariances are correctly described, which is clearly not true in the general case.

The proposed structure is a Complementary Filter, see (Oliveira, 2002) for a discussion on properties of complementary filters. The low-pass characteristics from the PCA position measurement to the position estimate are of utmost importance to reject the high frequency noise due to finite space resolution imposed by the dimensions of the *mosaics* chosen. The bias present in the Doppler velocity log measurements is fully compensated by the bias terms of the filter.

5. OPTIMAL SMOOTHER

To obtain the optimal state and covariance estimates, a noncausal fixed interval smoother can be used for post-processing the data acquired during the mission. As the underlying system obtained along equilibrium trajectories is linear (4), the proposed solution is close to the classical solution (Gelb, 1975), with minor changes due to the multirate nature of the estimation problem at hand. The smoother implemented is based on the backward recursion

$$\left\{\begin{array}{lcl} \mathbf{G}(k) & = & \mathbf{P}^-(k)\overline{\mathbf{A}}_{per}^T(k)(\mathbf{P}^+(k))^{-1} \\ \hat{\mathbf{z}}(k|T) & = & \hat{\mathbf{z}}^-(k) + \mathbf{G}(k)(\hat{\mathbf{z}}^+(k|T) - \hat{\mathbf{z}}^+(k)) \\ \mathbf{P}^+(k|T) & = & \mathbf{P}^-(k) + \mathbf{G}(k)(\mathbf{P}^+(k+1|T) - \mathbf{P}^+(k|T)) \end{array}\right. \tag{7}$$

where $\hat{\mathbf{z}}(k|T)$ and $\mathbf{P}^+(k|T)$ are the a posteriori state and error covariance estimates for instant k, respectively. The initial conditions for the recursion are $\hat{\mathbf{z}}(T|T) = \hat{\mathbf{z}}(T)$ and $\mathbf{P}^+(T|T) = \mathbf{P}^+(T)$, the backwards iterations should be computed at the time instants $mod(k, K) = 0$, and $\overline{\mathbf{A}}_{per}(k) = \overline{\mathbf{A}}(k*H-1)\ldots\overline{\mathbf{A}}((k-1)*H)$ is the monodromy of the periodic system.

6. SIMULATION RESULTS AND PERFORMANCE ASSESSMENT

Table 1. Simulation parameters

Parameters and Values

Vehicle: $T = 1800\ s$, $h = 1\ s$, ${}^{\mathcal{B}}({}^{\mathcal{I}}\mathbf{v}_{\mathcal{B}}) = [2\ \ 0\ \ 0]^T\ m.s^{-1}$, $\mathbf{p}(0) = [30\ \ 180\ \ 10]^T\ m$, $\mathbf{b} = [0.1\ \ 0.2\ \ 0]^T\ m.s^{-1}$, and $\psi(0) = 1.65\, rad$.

Sensors: $\sigma_{u,v} = 6 \times 10^{-1}\ m.s^{-1}$, $\sigma_{d(i)} = 10^{-1}\ m$, and $\sigma_\psi = pi/180\, rad$.

PCA: $H = 10\, s$, $N = N_x * N_y = 20*20$, $M = 42*21$, $n = 10$, and $\delta = 60\, m$.

Estimator
$P(0) = diag(10^2, 10^2, 1, 10^{-2}, 10^{-2}, 10^{-3})$,
$Q = diag(10^{-3}, 10^{-3}, 10^{-3}, 10^{-6}, 10^{-6}, 10^{-7})$,
$\hat{\mathbf{p}}(0) = [10\ 200\ 10]^T\, m$, and $\hat{\mathbf{b}}(0) = [0\ 0\ 0]^T\, m.s^{-1}$.

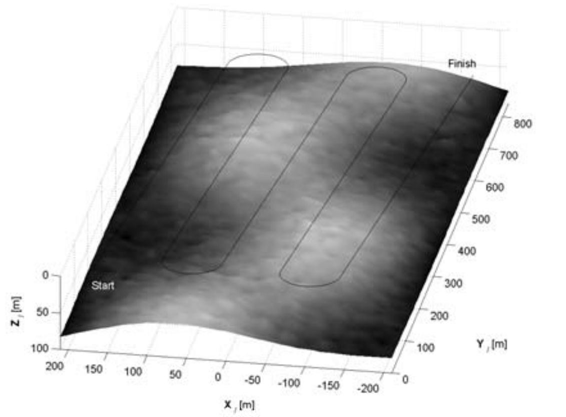

Fig. 3. Mission scenario for UV Terrain Based Navigation System assessment. Ideal trajectory.

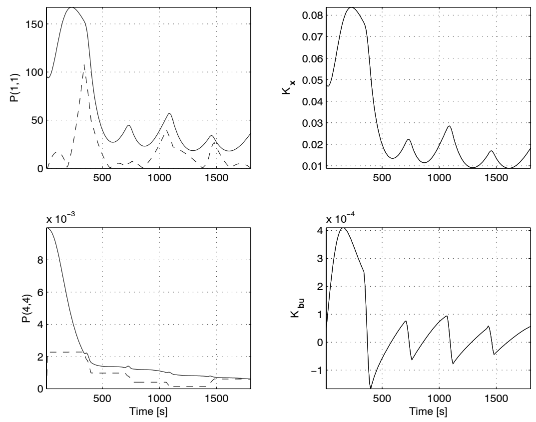

Fig. 4. Estimator covariances for horizontal position axes ($x = y$) and horizontal bias, upper right and left pictures, respectively (dashed-smoothed). Position and bias estimator gains, upper and lower right pictures, respectively.

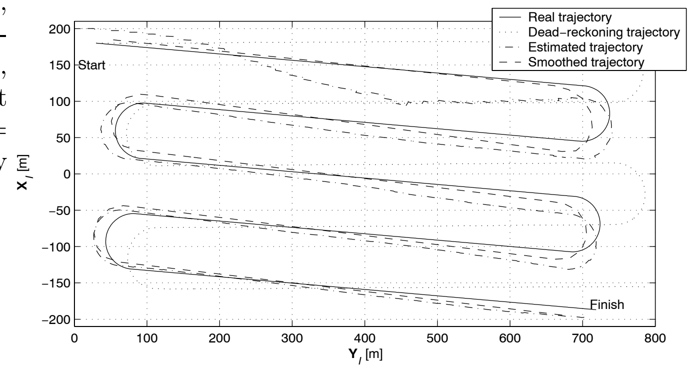

Fig. 5. Trajectory in the horizontal plane.

The performance of the proposed PCA based positioning, estimator and smoother systems are assessed on a synthetic world, through extensive Monte Carlo simulations (based on 100 runs). This mission area is constrained to $x \in]-210, 210]\ m$, $y \in]0, 840]\ m$, and the depth is given by

$$\begin{array}{rcl} z(x,y) & = & 80 - 10sin(2\pi/400x)cos(2\pi/600y) \\ & - & 20cos(2\pi/800x) + \eta_z, \end{array}$$

where η_z is correlated white noise, depicted in fig. 3. The set of parameters used are listed on table 1 as well as the initial parameters for the ideal vehicle and for the estimator. Note that the choice of δ can be critical: in the case of a large estimation error a small value can difficult to obtain the correct neighbor; the choice of a large δ augments the probability of misidentifying an incorrect neighbor. The optimal choice of the *mosaics* dimensions N_x and N_y are not completely clear: if they are too small not enough bathymetric data will be available, given the resolution of the bathymetric data available (according with the survey quality and accuracy); if they are too big a large number of returns is required to characterize a *mosaic* and therefore the period of the resulting multirate system and the accuracy of the PCA based positioning systems increases. This problems will be addressed in the near future, resorting to Kalman filter multi-model adaptive estimators.

The measurements of the simulated sensors are corrupted by white noise with characteristics similar to the sensors commercially available and commonly installed on UVs. The mission is composed by a sequence of trimming trajectories similar to the ones used in survey missions. In the first *leg*, in fig. 5, the estimation error is larger due to the initial position and bias estimate mismatches, to illustrate the performance of the estimator (depicted in figure 4). The advantages relative to the dead reckoning open loop integration are evident. This error is obviously reduced in the (off-line) smoothed trajectory estimate. The advantages of the proposed framework can hardly be overemphasized.

7. CONCLUSIONS AND FUTURE WORK

In this work tools for Terrain Based Navigation rooted on the Principal Component Analysis are proposed, discussed in detail, and validated in simulation. The results obtained validate the methodology of the overall system and pave the way for a more intense effort in the near future in a series of concurrent issues: i) use of sub-pixel techniques to overcome the resolution limitations due to the PCA approach; ii) application of multi-model adaptive estimators to address the issues of robustness and parameters' sensitivity for the proposed methodology; iii) assessment on the application of recent results on incremental PCA to a Simultaneous Localization and Mapping problem; iv) development of a tightly coupled version, exploiting the linear relations on the KL transform.

The possibility of application to Autonomous Underwater Vehicles and Remotely Operated Vehicles should be pursued with the aim of impacting on overcoming the limitations that the UVs have today.

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