FLATNESS BASED CONTROL OF LINEAR TIME VARYING BOND GRAPHS

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Abstract: A general flatness based control design and parametrization procedure for single input time varying systems modelled by bond graphs is presented. The methodology is introduced in an algebraic framework provided by differential rings and modules theory. The method is applied to a flatness based control design of a separately excited DC motor.

1. INTRODUCTION

The bulk of existing control theory is devoted to time invariant (stationary) systems. The reason is that time invariant models are the simplest. Nevertheless, it is interesting to consider time varying models in many application problems. Time varying models also results from linearization of nonlinear models around trajectories.

Many references devoted to linear time varying systems are available in the open literature. But, most of them presents general theoretical results in a purely abstract mathematical levels. It is worth to develop methods in accord with the usual tools used in engineering applications, particularly graphical tools. For instance, petri nets for discrete systems, signal flow graphs, digraphs and bond graphs for continuous systems. These modelling languages may be powerful for modelling but need an adequate theoretical background to take advantage of all the information contained in these graphical representations. Some results are already available in (Pliam and Lee, 1995) for signal flow graphs, (Reinschke, 1988) for digraphs, (Dauphin-Tanguy *et al.*, 1999) for linear time invariant bond graphs, (Junco, 1993) and (Achir *et al.*, 2004) for nonlinear time invariant bond graphs.

This contribution deals with time varying bond graphs. More precisely; modelling, flat output identification and differential parametrization. The presented results are introduced in the algebraic framework provided by differential rings and modules theory which constitute a very suitable framework for studying structural properties of time varying linear systems (Fliess, 1990; Bourlès and Fliess, 1984). The main results consist in the extension to time varying bond graphs of many graphical tools available for time invariant ones. Particularly, Mason loop rule which is no longer valid for time varying linear bond graphs due the property of non commutativity get a nice extension in terms of Riegle's gain rule.

The paper is organized as follows: section 2 is devoted to some background of differential rings and modules. Section 3 deals with bond graph modelling of time varying linear systems. Flat output of single input bond graphs and flatness parametrization are both presented in sections 4 and 5 respectively. The paper ends with an application of the presented results to a separately excited DC motor model.

2. THEORETICAL BACKGROUND

2.1 Modules and linear time varying systems

An (ordinary) differential ring k is a commutative ring equipped with a derivation $\frac{d}{dt} : k \to k$, satisfying the common properties of a differentiation operator (Fliess, 1989). An (ordinary) differential field is an (ordinary) differential ring which is a field.

An example of differential rings is the ring of differential operators $k[\delta]$ equipped with the derivation operator $\delta = \frac{d}{dt}$ with coefficients in k, (Malgrange, 1963). The elements of $k[\delta]$ are polynomials with indeterminate δ of the form $\sum_{finite} a_i \delta^i$, where $a_i \in k$.

Although this ring is in general non commutative, the multiplication in $k[\delta]$ is defined by the Leibniz rule(1).

$$\delta a = a\delta + \dot{a} \qquad a \in k \tag{1}$$

This operation is commutative if and only if k is a ring of constants.

Definition 1. A differential module Ω over a differential ring $k[\delta]$ is endowed with the common properties of usual modules (Bourlès and Fliess, 1984).

Definition 2. A left $k[\delta]$-module Ω is said to be free if and only if it has a basis :i.e., there exists m elements $z = (z_1, ..., z_m)$ of Ω that are $k[\delta]$-linearly independent and every element $w \in \Omega$ is $k[\delta]$-linear combination of z.

The following properties of modules over principal ideals are well known, (Fliess *et al.*, 1995).

- An element ω of Ω is said to be torsion, if and only if there exists a polynomial $\pi \in k[\delta]$, $\pi \neq 0$, such that $\pi.\omega = 0$.
- A finitely generated $k[\delta]$-module Ω can be written as a direct sum, equation (2).

$$\Omega = T \oplus \Phi \tag{2}$$

  T is a torsion submodule and $\Phi = \Omega/T$ is a free submodule.
- The rank of Ω, denoted by $rank(\Omega)$, is equal to the rank of the free submodule Φ which is equal to the cardinality of any basis of Φ.

Definition 3. A k-linear system Ω is a $k[\delta]$-module. A linear dynamics is k-linear system Ω with an input u such that the quotient module $\Omega/[u]$ is torsion, (Fliess *et al.*, 1995).

Finally, this formalism is more general than the classical Kalman state variable representation. For a time varying system given by the Kalman form (3), with $x \in \Re^n$, $u \in \Re^m$ and $y \in \Re^p$

$$\begin{cases} \dot{x} = A(t)x + B(t)u \\ y = C(t)x \end{cases} \tag{3}$$

it is easy to see it that (3) is equivalent to (4) in a module framework representation.

$$\underbrace{\begin{pmatrix} A(t) - I\delta & B(t) & 0 \\ C(t) & 0 & -I \end{pmatrix}}_{R(\delta,t)} \begin{pmatrix} x \\ u \\ y \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix} \tag{4}$$

The entries of the matrix $R(\delta, t)$ belong to a non commutative ring $k[\delta]$ and $\begin{pmatrix} x^t & u^t & y^t \end{pmatrix} \in \Omega$.

The structural properties of (4) are then translated to a module framework, see (Fliess, 1989). For example, controllability corresponds to the freeness of the module Ω.

2.2 Flatness

Roughly speaking, flatness consists on the possibility to parameterize every system variable (state variables, input variables and output variables) as function of fictive outputs and their derivatives. More precisely, consider the nonlinear model (5), where $x \in R^n, u \in R^m, z \in R^p$, and f is a smooth vector field of appropriate dimension and h is an analytic function.

$$\begin{cases} \dot{x} = f(x, u) \\ z = h(x, u) \end{cases} \tag{5}$$

The system (5) is said to be flat if and only if:

$$\exists y \in R^m; y = \lambda(x, u, \dot{u}, u^{(2)}, ..., u^{(r)})$$

such that:

$$\begin{cases} x = \varphi(y, \dot{y}, y^{(2)}, ..., y^{(k)}) \\ u = \phi(y, \dot{y}, y^{(2)}, ..., y^{(k+1)}) \\ z = \psi(y, \dot{y}, y^{(2)}, ..., y^{(k)}) \end{cases}$$

This differential parametrization contains all the input-state and state-output information of the given system which is useful for stabilization and trajectory tracking control design.

3. BOND GRAPH MODELLING OF LINEAR TIME VARYING SYSTEMS

A time varying bond graph model is composed of basic elements, associated with ports, I and C (both storage fields), R (dissipative field), MSe and MSf (sources). Elements $0, 1, TF$ (transformers) and GY (gyrators) compose the junction structure, which exchanges energy between the different parts of the dynamic system and is constrained to satisfy power conservation. The constitutive laws of these elements are presented hereafter in an algebraic framework.

3.1 *R element*

For a time varying linear R element, according to causality, two cases can be considered. If the flow f is given to the R element, then $e = R(t)f$.
If the effort e is given to the R element, then $\forall R(t) \neq 0, f = R^{-1}(t)e = (1/R(t))e$.
There is no preferential causality for the R element when $R(t) \neq 0, \forall t$.

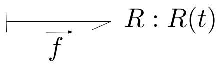


Fig. 1. Time varying dissipative linear element in resistance causality.

3.2 *C and I elements*

It is better to assign an integral causality to dynamical elements in order to perform integration instead of derivation. In this case one has the following situation:

Fig. 2. Time varying storage elements in integral causality.

With the assumption of linearity, their constitutive relations in integral causality assignment are of the form

$$e_C = \frac{1}{C(t)}\delta^{-1} f_c, \qquad f_I = \frac{1}{I(t)}\delta^{-1} e_I \tag{6}$$

and the associated states are $q_C = C(t)e_C$ and $p_I = I(t)f_I$.

Multiplying both sides by δ gives

$$f_C = \delta C(t)e_C, \qquad e_I = \delta I(t)f_I \tag{7}$$

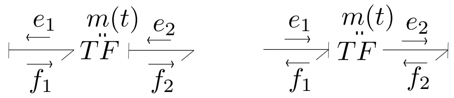
Using the multiplication rule given by equation 1 yields

$$\begin{cases} f_C = \underbrace{C(t)\delta}_{\text{C element}}\ e_C + \underbrace{\dot{C}(t)}_{\text{R element}}\ e_C \\[2ex] e_I = \underbrace{I(t)\delta}_{\text{I element}}\ f_I + \underbrace{\dot{I}(t)}_{\text{R element}}\ f_I \end{cases} \tag{8}$$

Equation (8) represents the constitutive relations in derivative causality assignment.

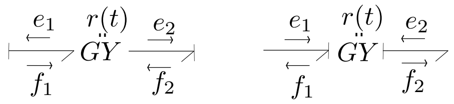
3.3 *TF and GY elements*

When assigning causality to an TF element, two situations occur, figure 3. Similarly, two configurations are available for the GY, figure 4.



$$\begin{cases} e_1 = m(t)e_2 \\ f_2 = m(t)f_1 \end{cases} \qquad \begin{cases} e_2 = 1/m(t)e_1 \\ f_1 = 1/m(t)f_2 \end{cases}$$

Fig. 3. Time varying transformer



$$\begin{cases} e_1 = r(t)f_2 \\ e_2 = r(t)f_1 \end{cases} \qquad \begin{cases} f_1 = 1/r(t)e_2 \\ f_2 = 1/r(t)e_2 \end{cases}$$

Fig. 4. Time varying gyrator

These enumerated elements are connected by 0 and 1 junctions and are constrained to obey to causality affectation procedure. Rearranging the relations obtained by applying the constitutive laws of the bond graph elements and the junctions laws leads generally to a state variable representation of the form

$$\delta x = A(t)x(t) + B(t)u(t) \tag{9}$$

where x is the state vector associated with the bond graph variables p and q and u is the input vector associated with the input sources. $A(t)$, $B(t)$ are matrices of appropriate dimensions with time varying entries coefficients.

3.4 *Example*

Consider a separately excited DC motor (SEDCM) depicted as in figure 5.

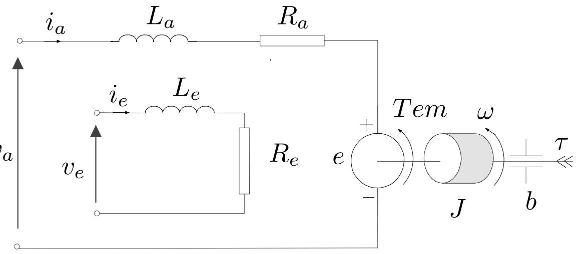


Fig. 5. Dynamic equivalent circuit of SEDCM

Assume that the electromechanical energy conversion is without losses; equation (10) holds.

$$\begin{cases} e = k_m \omega \\ Tem = k_m i \end{cases} \tag{10}$$

Considering a linear excitation windings characteristic, the excitation flux is $\Phi(t) = L_e i_e(t)$. If the excitation circuit is fed by a sinusoidal voltage of pulsation ϖ, then $i_e(t)$ can be assumed sinusoidal and the excitation flux is simplified to $\Phi(t) = \Phi_0(1 + \alpha sin\varpi t)$, (Rotella *et al.*, 2002). By using the standard bond graph formalism (Karnopp *et al.*, 1990) and the above hypothesis, the corresponding time varying bond graph is depicted as in figure 6.

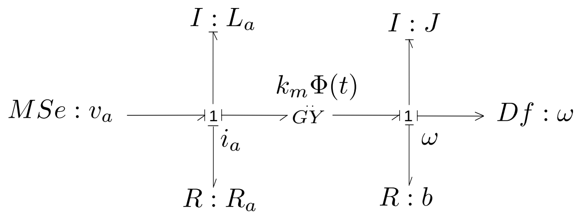


Fig. 6. Bond graph model of the SEDCM

4. FLATNESS OF LINEAR TIME VARYING BOND GRAPHS

Consider the change of coordinates given by

$$z(t) = P^{-1}(t)x(t) \quad \text{or} \quad x(t) = P(t)z(t) \quad (11)$$

Applying this change of coordinates to equation (9) leads to a new state variable representation of the form

$$\delta z(t) = \bar{A}(t)z(t) + \bar{B}(t)u(t) \quad (12)$$

where $\bar{A}(t) = P^{-1}(t)[A(t) - \delta]P(t)$ and $\bar{B}(t) = P^{-1}(t)B(t)$

The obtained form (12) becomes a controller form if one chooses the matrices $\bar{A}(t)$ and $\bar{B}(t)$ as

$$\bar{A} = \begin{pmatrix} 0 & 0 & 0 & .. & -a_0(t) \\ 1 & 0 & 0 & .. & -a_1(t) \\ 0 & 1 & 0 & .. & -a_2(t) \\ : & : & : & .. & : \\ 0 & 0 & 0 & .. & -a_{n-1}(t) \end{pmatrix}, \bar{B} = \begin{pmatrix} 1 \\ 0 \\ 0 \\ : \\ 0 \end{pmatrix}$$

It remains to calculate the corresponding change of coordinates matrix $P(t)$. Let it be

$$P(t) = \left[\, p_1(t)\ p_2(t)\ p_3(t)\ ..\ p_n(t) \,\right]$$

Therefore $P(t)\bar{B} = B \Longrightarrow p_1(t) = B$
and $P(t)\bar{A}(t) = [A(t) - \delta]P(t)$

$$\Longrightarrow \begin{cases} p_2(t) = [A(t) - \delta]B(t) \\ : \\ p_n(t) = [A(t) - \delta]^{n-1}B(t) \end{cases}$$

Consequently

$$P(t) = [B(t), (A(t)-\delta)B(t), ..., (A(t)-\delta)^{n-1}B(t)]$$

Remark 1. The matrix $P(t)$ has to be of full rank, which corresponds to controllability of (9) or the freeness of the module Ω represented by

$$[A(t) - \delta]x(t) + B(t)u(t) = 0 \quad (13)$$

The flat output is then clearly proportional to the last variable $z_n(t)$ of the obtained canonical form (12), with $\lambda(t) \in \mathbb{R}(t)$.

$$y(t) = \lambda(t)z_n(t) = (0, ..., \lambda(t))z(t) \quad (14)$$

In the original coordinates, the flat output is a linear combination of the state variables of the bond graph model described by

$$y(t) = C(t)x(t) = \sum_{i=1}^{n} c_i(t)x_i(t) \quad (15)$$

As a consequence of equations (11), (14) and (15), $C(t)P(t) = (0, ..., \lambda(t))$. Therefore, the row vector $C(t)$ satisfy the set of algebraic equations (16).

$$\begin{cases} C(t)[A(t) - \delta]^{l-1}B(t) = 0 \\ C(t)[A(t) - \delta]^{n-1}B(t) = \lambda(t) \end{cases} \quad (16)$$
$$\forall \quad l = 1, .., n-1$$

The entries of the column vectors $[A(t)-s]^{l-1}B(t)$ are obtained directly from the time varying bond graph model by means of formula (17).

$$\{[A(t) - \delta]^{l-1}B(t)\}_j = \sum_{H} G_l(u, x_j)(t) \quad (17)$$

Where $G_l(u, x_j)(t) = \prod_{i=n}^{1}(\tilde{G}_i(t) - \gamma\delta)$ denotes the gain of the k^{th} causal path of length l relating the input variable u to the state variable x_j.

- $\tilde{G}_i(t)$ is the constant part of the elementary causal path gain at the element i along the k^{th} causal path.
- $\gamma = 1$ if the elementary causal path contains a causal loop of length one (formed by a dynamical element and an R element) and $\gamma = 0$ otherwise.
- H denotes the set of all the causal paths of length k connecting the input variable u to the state variable x_j.

Remark 2.
- Due to the non commutative aspect, the gain of a causal path has to be calculated backward, i.e: from the output (sink) variable to the input (source) variable.
- The freeness of Ω means that every element $w \in \Omega$ is torsion, i.e:

$$\begin{bmatrix} A(t)^t + \delta \\ B(t)^t \end{bmatrix} w^t = 0$$

which is equivalent to the condition given in equation (16) for $w = C(t)x(t)$.

- An algorithm can be obtained for multi input bond graphs in a similar way.

5. DIFFERENTIAL PARAMETRIZATION

Definition 4. A ring bond graph over a ring $k[\delta^{-1}]$ and a $k[\delta^{-1}]$-module Ω means a bond graph described by a set of equations of the form (18).

$$e_i = \sum_{j=1}^{n} a_{i,j} e_j + \sum_{k=1}^{m} b_{i,k} u_k, \quad i = 1, ..., n \quad (18)$$

$a_{i,j}$, $b_{i,k}$ are in $k[\delta^{-1}]$ and e_i, e_j and u_k are in Ω which denote here the effort or flow variables associated with the dynamical elements in integral causality and the effort or flow variables associated with the input sources. Note that the ring $k[\delta^{-1}]$ is not commutative if k is not a field a constants.

Indeed, equation (9) can be written as

$$\delta x(t) = [A(t)\delta^{-1}]\delta x(t) + B(t)u(t) \quad (19)$$

with $k = \mathbb{R}(t)$. According to definition 4, Mason loop rule can not be used because it is based on Crammer rule which holds only in the commutative case. An extension to this rule to the non commutative case is presented hereafter. It is based on Riegle's gain formula which is originally introduced in the framework of non-commutative sets of algebraic equations (Pliam and Lee, 1995). This rule is also applied on variational bond graphs in (Achir *et al.*, 2004).

Definition 5. For a ring bond graph over a non commutative ring or field, the gain expression is given by (20).

$$T = \sum_{k \in H} G_{(k)} \quad (20)$$

where $G_{(k)}$ is called the causal path product of the k^{th} causal path from an input (source) element to an output (sink) element, and given by (21), where the product is taken in order over the n elements in the k^{th} causal path. A_i^k is the i^{th} bond gain (operator) along the k^{th} causal path, and $S_i^{(k)}$ is the self-gain of the element immediately following the bond $A_i^{(k)}$ with the remaining bonds along the k^{th} path removed.

$$G_k = \prod_{i=n}^{1} (1 - S_i^k)^{-1} A_i^k \quad (21)$$

Definition 6. The self gain of a ring bond graph dynamic element is equal to the sum of all the gains of the causal loops that cross the considered element.

Remark 3. The self-gain and bond gain is function of t and δ or δ^{-1} depending if it is calculated on the direct or inverse time varying bond graph model.

The input output relation obtained using equation (20) is given by the gain expression (22).

$$y = T(t, \delta^{-1})u \quad (22)$$

If the systems is flat, equation (22) is always invertible and enables us to get the input output differential parametrization given by equation (23).

$$u = T^{-1}(t, \delta)y \quad (23)$$

6. APPLICATION TO SEDCM

Consider again the time varying bond graph model of section 2. The state vector is $x = (p_{L_a}, p_J) \equiv (x_1, x_2)$.

6.1 Flat output of the SEDCM

A particular expression of equation (15) is given by (24).

$$y = c_1(t)x_1 + c_2(t)x_2 \quad (24)$$

It further verifies the conditions of equation (16), whose coefficients are determined by analyzing the causal paths of different lengths.

$$\begin{cases} c_1(t)G_1(v_a, x_1) + c_2(t)G_1(v_a, x_2) = 0 \\ c_1(t)G_2(v_a, x_1) + c_2(t)G_2(v_a, x_2) = \lambda(t) \end{cases} \quad (25)$$

- Causal paths of length 1
  $G_1(v_a, x_2) = 0$ because it does not exist any causal path of length 1 connecting the bond graph element $(I : J)$ to the input source. This implies that $c_1(t) = 0$
  $G_1(v_a, x_1) = 1$
- Causal paths of length 2
  Since $c_1(t) = 0$ it remains to calculate $G_2(v_a, x_2)$ which is equal to $k_m\Phi(t)\frac{1}{L_a}$. It is sufficient to choose $\lambda(t) = k_m\Phi(t)\frac{1}{L_a}$ to get $y = x_2$.

6.2 Differential parametrization of the SEDCM

First, consider the sub-bond graph of figure (7-a). By writing the junction equation, one obtains the relation (26).

$$f_1 = \frac{1}{L_a}\delta^{-1}[1 + \frac{R_a}{L_a}\delta^{-1}]^{-1}e_1 \quad (26)$$

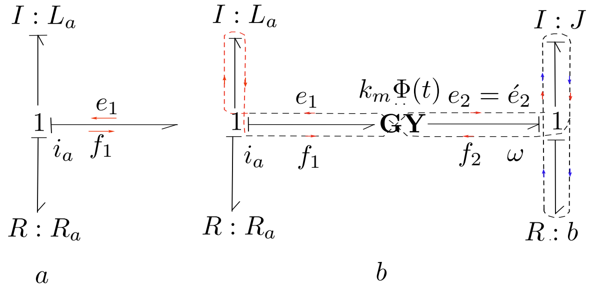


Fig. 7. Sub-bond graph associated with the variational bond graph model of the SEDCM

Now, consider the element $I : J$ of figure (7-b). Its self gain can be calculated by summing the gain of the two causal loops $I : J \rightleftharpoons R : b$ and $I : J \rightleftharpoons GY : k_m\Phi(t) \rightleftharpoons I : La$.
The calculation of the first loop gain is straightforward since all the operators are constant and is equal to $S_1 = -\frac{b}{J}\delta^{-1}$.
For the second loop gain, start with any split variable (source and sink variable), let it be e_2 and e'_2 and calculate the transfer relation.

$$\begin{aligned} e_2 &= k_m\Phi(t)f_1 = k_m\Phi(t).\frac{1}{L_a}\delta^{-1}[1+\frac{R_a}{L_a}\delta^{-1}]^{-1}e_1 \\ &= k_m\Phi(t).\frac{1}{L_a}\delta^{-1}[1+\frac{R_a}{L_a}\delta^{-1}]^{-1}k_m\Phi(t)f_2 \\ &= k_m\Phi(t).\frac{1}{L_a}\delta^{-1}[1+\frac{R_a}{L_a}\delta^{-1}]^{-1}k_m\Phi(t)(-\frac{1}{J}\delta^{-1})e'_2 \end{aligned}$$

Therefore, the self gain of the element $I : J$ is equal to

$$S_2 = S_1+\frac{k_m}{L_a}\Phi(t)\delta^{-1}[1+\frac{R_a}{L_a}\delta^{-1}]^{-1}(-\frac{k_m}{J}\Phi(t)\delta^{-1})$$

Now, it is possible to carry out the calculations for the whole bond graph of figure 6.

$$\omega = \frac{1}{J}\delta^{-1}(1-S_2)^{-1}k_m\Phi(t).\frac{1}{L_a}\delta^{-1}(1+\frac{R_a}{L_a}\delta^{-1})^{-1}v_a$$

Inverting and considering $y = J\omega$ one gets

$$v_a = (1-\frac{R_a}{L_a}\delta^{-1})(\frac{k_m}{L_a}\Phi(t)\delta^{-1})^{-1}(1-S_2)\delta y$$

Replacing S_2 and simplifying gives

$$v_a = \left[(\delta+\frac{R_a}{L_a})(\frac{k_m}{L_a}\Phi(t))^{-1}(\delta+\frac{b}{J})+\frac{k_m}{J}\Phi(t)\right]y$$

Using the multiplication rule, given by equation (1) and simplifying gives

$$\begin{aligned} v_a = \frac{L_a}{k_m\Phi(t)}\Bigg[&\ddot{y}+(\frac{R_a}{L_a}-\frac{\dot{\Phi}(t)}{\Phi(t)}+\frac{b}{J})\dot{y} \\ &+(\frac{R_a b}{L_a J}-\frac{\dot{\Phi}(t)b}{\Phi(t)J}+\frac{k_m^2}{L_a J}\Phi(t))y\Bigg] \end{aligned}$$

The state output parametrization is calculated similarly by considering the variable x_2 as an input variable. From the bond graph of figure 6, it follows

$$\omega = \frac{1}{J}\delta^{-1}(1-S_1)^{-1}k_m\Phi(t).\frac{1}{L_a}x_1$$

Inverting and replacing S_1

$$x_1 = \left(\frac{k_m}{L_a}\Phi(t)\right)^{-1}(\delta+\frac{b}{J})y$$

Therefore

$$x_1 = \frac{L_a\Phi(t)}{k_m}(\dot{y}+\frac{b}{J}y)$$

Finally, all the system variables x_1 and u are expressed in terms of the flat output $y = x_2$ and its derivatives.

CONCLUDING REMARKS

The presented results open new issues for analysis of structural properties of time varying linear systems taking into account the non commutativity aspect. The use of bond graphs (graphical concepts) simplifies the flat output identification and parametrization and reduce calculus because it takes advantage of the scarcity of state equations matrices of the bond graph. Further, an extension to multi-input will be considered.

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