DYNAMIC GAMES: ENGINEERING-BASED TOOLS FOR ANALYZING STRATEGIC ECONOMIC INTERACTIONS[1]

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Abstract: This paper gives a brief introduction to the theory of dynamic games as developed mostly by control theorists and engineers and presents some economic applications of this theory. The paper shows how the theory of dynamic games can be applied to problems of economic policy-making with heterogeneous policy-makers, whose behavior is characterized by strategic interactions. In particular, for the case of macroeconomic policy-making in a monetary union, it is illustrated how strategic interactions between governments of the union's member countries responsible for national fiscal policies and the common central bank responsible for the union's monetary policy can be studied in a fruitful way, using concepts and results from dynamic game theory as applied to a macroeconomic model.

Keywords: Dynamic games, optimization, monetary policy, fiscal policy, Nash equilibrium, cooperation, economics.

1. INTRODUCTION

Starting in the late 1960s, concepts, theories and methods of optimum control theory have been applied to economic problems. Especially in the 1970s and 1980s, there was an intensive dialogue between control engineers and applied mathematicians on the one hand and economists on the other, which has brought a lot of insights from control and systems theory into economics. More recently, however, it has increasingly been recognized that many problems of economic policy cannot be solved by uncritical adoption of optimum control concepts. In particular, economic policy problems are typically characterized by a multitude of decision-makers with non-identical interests. Moreover, disillusion with Keynesian activist policies have raised severe doubt about the possibilities of controlling an economy in a similar way as a physical object such as a rocket, for instance. Hence, economists have started to look for alternative sources of inspiration for their scientific work, such as biology, applied business and management, or psychology, for instance.

Nevertheless, there is still a need for a framework for quantitative economic policy problems. In this paper, we argue that dynamic game theory, which also originated from engineering and extends optimum control theory, can provide such a framework. We give a brief history of (Section 2) and introduction into dynamic game theory (Section 3) and sketch some of its applications in economics (Section 4). By means of a simple model (Section 5), we then show in more detail how the dynamic game approach can be used to derive insights into macroeconomic policy problems in the context of a monetary union (Section 6). Sensitivity analysis opens up the possibility of drawing more general conclusions from numerically specified simple models (Section 7). Section 8 concludes.

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2. THE DEVELOPMENT OF DYNAMIC GAME THEORY

The theory of dynamic games has basically two roots: dynamic optimization, in particular optimum control theory, and (static) game theory. The former originated in the engineering and applied mathematics scientific communities, while the latter arose from the collaboration of mathematicians and economists (most prominently, John von Neumann and Oskar Morgenstern). For many years, game theory was mainly a playing field for mathematicians due to its high degree of abstraction and complexity. Starting with the 1980s, however, its value for applied analyses in the social sciences and economics in particular was increasingly recognized, with three of its most distinguished researchers even honored by the Nobel Prize in 1993 (John C. Harsanyi, John F. Nash and Reinhard Selten). Eventually dynamic games have also found their way into textbooks of traditional (predominantly static) game theory; see, for instance, Fudenberg and Tirole, 1992).

In contrast to one-agent optimization problems (in the dynamic case: optimum control problems) and team decision problems (in the dynamic case: decentralized control problems), for a game the presence of at least two decision-makers with different objectives is constitutive. In the dynamic case, we can distinguish between differential games (with time being a continuous variable) and difference games (in discrete time). Figure 1 gives a schematic picture of the genesis of dynamic game theory.

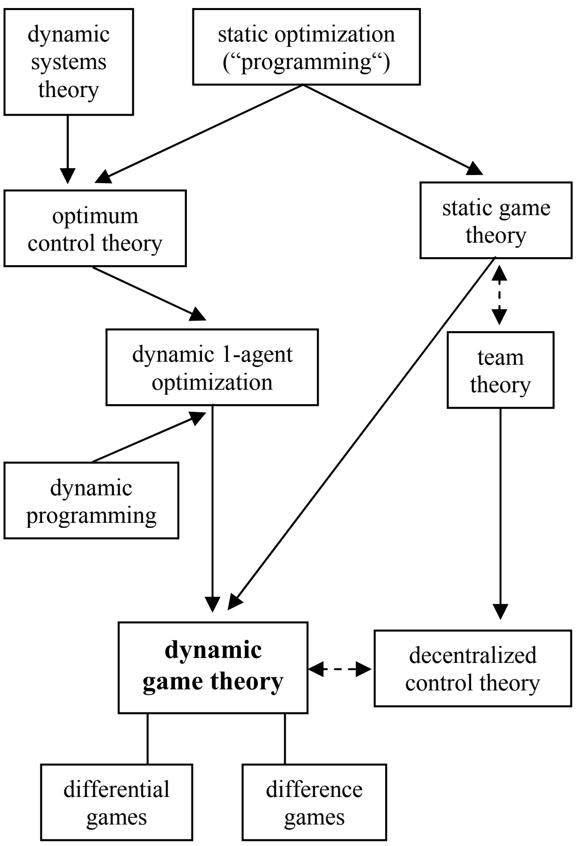

Fig. 1. Genesis of dynamic games

3. DYNAMIC GAME THEORY: A BRIEF OVERVIEW

Specifying a dynamic game requires the following model elements (see Başar, 1986; Başar and Olsder, 1999; Mehlmann, 1988; Dockner et al., 2000, for more details):

1. A set of **players** (decision-makers, agents): $\mathcal{N} = \{1,2,\ldots,n\}, i \in \mathcal{N} \subset \mathbb{N}$.

2. A **time interval** on which the game is defined. While time is considered a discrete variable for difference games $(\mathcal{T} = \{0,1,\ldots,T\})$, differential games are formulated in continuous time: $\mathcal{T} = [0,T], \mathcal{T} \subset \mathbb{R}$. The time horizon T can also be infinite or endogenous.

3. **Control variables** (decision, policy, instrument variables) for each player: $\mathbf{u}_i \subset \mathcal{U}_i, i \in \mathcal{N}$, where $\mathcal{U}_i$ is the control space of player i. We write $\mathbf{u} = (\mathbf{u}_1' \ldots \mathbf{u}_n')'$ as a vector. Due to the dynamics of the game, the control variables are time-dependent: $\mathbf{u}_i(t), t \in \mathcal{T}$. Let $\mathbf{u}_i(t) \in U_i \ \forall t \in \mathcal{T}$, where U_i is the action space of player i. In the dynamic game, the control trajectories (control functions) $\{\mathbf{u}_i(t), t \in \mathcal{T}\}$ are elements of the decision spaces $\mathcal{U}_i$.

4. In stochastic dynamic games there exists a **disturbance (noise) variable** $\mathbf{w} \in \mathcal{W}$. It is assumed that $\mathbf{w}(t) \in W \ \forall t \in \mathcal{T}$; the probability distribution of $\mathbf{w}$ is common knowledge of all players.

5. The **information pattern (structure)** of the game can be written as a vector, $\boldsymbol{\eta} = (\boldsymbol{\eta}_1' \ldots \boldsymbol{\eta}_n')'$, or defined as a set-valued function. Here $\boldsymbol{\eta}_i(t)$ is the information of player i in $t \in \mathcal{T}$ about $\mathbf{u}_1,\ldots,\mathbf{u}_n$ and $\mathbf{w}$ (information structure of player i); $\boldsymbol{\eta}_i \in \Omega_i$, where Ω_i is the information space of player i. The information structure of the game can be defined as a mapping $\boldsymbol{\eta} : (\mathsf{X}_{i \in \mathcal{N}} \mathcal{U}_i) \times \mathcal{W} \to (\mathsf{X}_{i \in \mathcal{N}} \Omega_i)$ that has to fulfill certain causality properties.

6. For each player $i \in \mathcal{N}$ **strategy variables**, $\gamma_i \in \Gamma_i$, are defined as mappings $\gamma_i : \boldsymbol{\eta} \to \mathbf{u}_i$, where Γ_i is the strategy space of player i.

7. **Objective functionals** (loss, payoff, cost functions) L_i are defined for each player $i \in \mathcal{N}$ as mappings $L_i : (\mathsf{X}_{i \in \mathcal{N}} \mathcal{U}_i) \times \mathcal{W} \to \mathbb{R}$. They can also be transformed to cost functionals on $\mathsf{X}_{i \in \mathcal{N}} \Gamma_i$ by

$$J_i(\gamma_1,\ldots,\gamma_n) = \mathrm{E}[L_i(\gamma_1(\boldsymbol{\eta}_1(\mathbf{u},\mathbf{w})),\ldots,\gamma_n(\boldsymbol{\eta}_n(\mathbf{u},\mathbf{w})),\mathbf{w})], \quad (1)$$

which leads to objective functionals in the strategic (normal) form of the dynamic game.

Apart from the solution concept, the elements given so far already contain a complete description of the dynamic game. However, as in systems theory generally, a **state variable** $\mathbf{x}$ is usually introduced in addition, which summarizes the information about the development of the system. We define it by

$$\mathbf{x}(t) = \mathbf{X}(t, \mathbf{x}(\tau), \mathbf{u}_1(\tau), \ldots, \mathbf{u}_n(\tau), \mathbf{w}(\tau), \tau < t),\ t > 0,\ \mathbf{x}(0) = \mathbf{x}_0, \quad (2)$$

where $\mathbf{X}$ is a mapping $\mathbf{X} : \mathcal{T} \times (\mathsf{X}_{i\in\mathcal{N}} \mathcal{U}_i) \times \mathcal{X} \times \mathcal{W} \to \mathcal{X}$ with causality properties and $\mathbf{x} \in \mathcal{X}$. $\mathbf{x}_0$ is the initial state, which is either given or (in a stochastic game) can be part of $\mathbf{w}$. Let $\mathbf{x}(t) \in X\ \forall t \in \tau$, where X is called the state space and is a subset of a finite-dimensional vector space. Sometimes also $\mathcal{X}$ is called state space; we call $\mathcal{X}$ the trajectory space of the game; its elements are the admissible state trajectories $\{\mathbf{x}(t), t \in \mathcal{T}\}$, which are defined for given trajectories of the control and disturbance variables and given initial state.

Additional stochastic variables, $\mathbf{y}_i \in \mathcal{Y}_i, i \in \mathcal{N}$, the **observation** (measurement, output) of player *i*, are related to the state variable $\mathbf{x}$. Here

$$\mathbf{y}_i(t) = \mathbf{Y}_i(t, \mathbf{x}(\tau), \mathbf{u}_1(\tau), \ldots, \mathbf{u}_n(\tau), \mathbf{w}(\tau), \tau \le t),\ i \in \mathcal{N},\ \tau \in \mathcal{T}. \quad (3)$$

Again, $\mathbf{Y}_i$ is a mapping fulfilling some causality assumptions, $\mathbf{Y}_i : \mathcal{T} \times (\mathsf{X}_{i\in\mathcal{N}} \mathcal{U}_i) \times \mathcal{X} \times \mathcal{W} \to \mathcal{Y}_i, i \in \mathcal{N}$, and $\mathbf{y}_i(t) \in Y_i\ \forall t \in \mathcal{T}$, where Y_i is the observation space of player *i*.

Essential for the interpretation is now the relation between the information structure of player *i* and his observations: Let $\mathbf{y} = (\mathbf{y}_1' \ldots \mathbf{y}_n')'$; then the information available to player *i* at time *t* to determine his (her) decision $\mathbf{u}_i(t)$ is given by

$$\eta_i(t) = \mathbf{C}_i(t, \mathbf{y}(\tau), \tau \le t), \quad (4)$$

where the mapping $\mathbf{C}_i : \mathcal{T} \times (\mathsf{X}_{i\in\mathcal{N}} \mathcal{Y}_i) \to \Omega_i, i \in \mathcal{N}$ is again causal.

As the state variable $\mathbf{x}$ can be expressed by means of $\mathbf{u}$ and $\mathbf{w}$, the objective functionals can also be defined using $\mathbf{x}$ as $\mathrm{L}_i : (\mathsf{X}_{i\in\mathcal{N}} \mathcal{U}_i) \times \mathcal{X} \times \mathcal{W} \to \mathbb{R}_i, i \in \mathcal{N}$, where L_i can be transformed to L_i. L_i is then part of the extensive-form description of the game.

The possibility of an endogenous determination of the time horizon of the dynamic game can be taken into account by the following definitions of a game's termination and its playability: Let $S \subset X$, $\mathbb{R}^+ = [0, \infty)$, $\Lambda \subset S \times \mathbb{R}^+$ with Λ called terminal set (target set) and $(\mathbf{x}_0, 0) \notin \Lambda$. Then we say that a dynamic game for a given *n*-tuple of strategies is **terminated** at time $T \in \mathbb{R}^+$ if and only if $T = \min\{t \in \mathbb{R}^+ : (\mathbf{x}(t), t) \in \Lambda\}$, and $T > 0$ is called terminal time of the dynamic game corresponding to the given *n*-tuple of strategies. Simple examples already show that not all dynamic games need to be terminated. Therefore we call an *n*-tuple of strategies **playable** in $(\mathbf{x}_0, 0)$ for a given *n*-person dynamic game with terminal set Λ iff it generates a state trajectory $\{\mathbf{x}(t), t \in \mathbb{R}^+\}$ for which there is a $t_1 < \infty$ such that $(\mathbf{x}(t_1), t_1) \in \Lambda$; the trajectory $\{\mathbf{x}(t)\}$ then is terminated in t_1.

An important special case, on which most analytical results are known, is the dynamic game with **perfect state information**. Such a game is characterized by the following properties for all $i \in \mathcal{N}$:

1. $\mathbf{X}(t, \alpha(\tau), \tau < t) = \int_0^t \alpha(\tau)\mathrm{d}\tau$.
2. $\mathbf{Y}_i(t, \mathbf{x}(\tau), \mathbf{u}_1(\tau), \ldots, \mathbf{u}_n(\tau), \tau \le t) = \mathbf{x}(t)$.
3. There is a function $g_i(\ldots)$ for which $L_i(\alpha) = \int_0^T g_i(\tau, \alpha(\tau))\mathrm{d}\tau$.

This is a **deterministic** game; the probability distribution over $\mathcal{W}$ is degenerated to a (0,1)-distribution. In this case the state equation can be represented as a differential equation (for a differential game)

$$\dot{\mathbf{x}}(t) = \mathbf{f}(t, \mathbf{x}(t), \mathbf{u}_1(t), \ldots, \mathbf{u}_n(t)),\ \mathbf{x}(0) = \mathbf{x}_0 \quad (5)$$

or a difference equation (for a difference game)

$$\mathbf{x}(t+1) = \mathbf{f}(t, \mathbf{x}(t), \mathbf{u}_1(t), \ldots, \mathbf{u}_n(t)). \quad (6)$$

Here it has to be assumed that this equation has a unique solution for all admissible control trajectories $\{\mathbf{u}_i(t), t \in \mathcal{T}\}, i \in \mathcal{N}$. This is fulfilled under the usual information structures, for instance, if the functions $\mathbf{f}(\ldots)$ and $\gamma_i(.)$ are continuous and uniformly Lipschitz in their respective arguments. In a dynamic game with perfect state information, the states are observed directly by all players, $\mathbf{y}_i(t) = \mathbf{x}(t)\ \forall i \in \mathcal{N}$, and the objective functionals can be written as

$$\mathrm{L}_i = \int_0^T g_i(t, \mathbf{x}(t), \mathbf{u}_1(t), \ldots, \mathbf{u}_n(t))\mathrm{d}t \quad (7)$$

in the continuous-time case (with obvious modifications for the discrete-time case).

A more general representation of the state trajectory in the continuous-time case exists for certain **stochastic** differential games under additional assumptions about the function $\mathbf{f}$ and the stochastic disturbance process $\{\mathbf{w}(t), t \ge 0\}$. In particular, if $\{\mathbf{w}(t)\}$ is a Wiener process, the state equation can be written as stochastic differential equation in Itô form

$$\mathrm{d}\mathbf{x}(t)=\mathbf{f}(t,\mathbf{x}(t),\mathbf{u}_1(t),...,\mathbf{u}_n(t))\mathrm{d}t+\boldsymbol{\sigma}(t,\mathbf{x}(t))\mathrm{d}\mathbf{w}(t). \quad (8)$$

A unique solution of this equation exists for a given n-tuple of admissible control trajectories under conditions analogous to the deterministic case if, in addition, $\boldsymbol{\sigma}(...)$ is non-singular, continuous and uniformly Lipschitz.

Some examples of special **information patterns** for dynamic games are (the first five refer to dynamic games with perfect state information):

1. **Feedback** information pattern: $\eta_i(t)=\mathbf{C}_i(t,\mathbf{y}(\tau),\tau\le t)=\{\mathbf{x}(t)\}, i\in\mathcal{N}$.
2. **Closed-loop perfect-memory** information pattern: $\eta_i(t)=\{\mathbf{x}(\tau),\tau\le t\}, i\in\mathcal{N}$.
3. **Closed-loop no-memory** information pattern: $\eta_i(t)=\{\mathbf{x}(0),\mathbf{x}(\tau)\}, i\in\mathcal{N}$.
4. **Open-loop** information pattern: $\eta_i(t)=\{\mathbf{x}_0\}, i\in\mathcal{N}$.
5. ε**-delayed closed-loop** information pattern: $\eta_i(t)=\begin{cases}\{\mathbf{x}_0\} & \text{for } 0\le t\le\varepsilon,\\ \{\mathbf{x}(\tau),\ 0\le\tau\le t-\varepsilon\} & \text{for } \varepsilon<t,\end{cases}$, with $\varepsilon>0$ fixed; $i\in\mathcal{N}$.
6. **Stochastic** closed-loop perfect-memory information pattern with **total measurement sharing**: $\eta_i(t)=\{\mathbf{y}(\tau),\tau\le t\}, i\in\mathcal{N}$.
7. Stochastic feedback information pattern with full exchange of observations: $\eta_i(t)=\{\mathbf{y}(t)\}, i\in\mathcal{N}$.
8. Stochastic feedback information pattern **without measurement sharing**: $\eta_i(t)=\{\mathbf{y}_i(t)\}, i\in\mathcal{N}$.
9. **Stochastic** closed-loop perfect-memory information pattern with **delayed measurement sharing**: $\eta_i(t)=\{\mathbf{y}_i(\tau),\tau\le t;\ \mathbf{y}_j(\tau'),\tau'\le t-\varepsilon,\ j\in\mathcal{N},\ j\ne i\}, i\in\mathcal{N}$.

A dynamic game can be described in extensive form by giving $\mathcal{N}$, $\mathcal{T}$, $\mathcal{X}$ or X, $\mathcal{U}_i$ or U_i, $\mathcal{W}$, η, Γ_i, L_i (for $i\in\mathcal{N}$) and the state equations (5), (6), or (7). If the assumptions about the unique solvability of these differential (difference) equations are fulfilled, a unique solution of the respective functional differential (difference) equation can be determined for every fixed n-tuple of strategies $\gamma\in\mathrm{X}_{i\in\mathcal{N}}\Gamma_i$, in the deterministic continuous-time case, for example,

$$\dot{\mathbf{x}}(t)=\mathbf{f}(t,\mathbf{x}(t),\gamma_1(t,\mathbf{x}),...,\gamma_n(t,\mathbf{x})), \gamma_i\in\Gamma_i, i\in\mathcal{N}. \quad (9)$$

Then the respective control trajectories $\{\mathbf{u}_i(t)\}$ can be determined as $\mathbf{u}_i(.)=\gamma_i(.,\mathbf{x}), i\in\mathcal{N}$. This is then substituted, together with the (assumed) unique resulting state trajectory $\{\mathbf{x}(t)\}$ into the L_i-functions, assuming integrability of $\mathbf{g}_i$ in (7); in the stochastic case, the expected value for the random variables has to be taken. For fixed statistics of $\mathbf{w}$ and $\mathbf{x}_0$, from this we get mappings $J_i:\Gamma_i\times...\times\Gamma_n\to\mathbb{R}$, $i\in\mathcal{N}$ as in (1). The strategy spaces Γ_i and the cost functionals $J_i, i\in\mathcal{N}$, fully characterize the **normal form** description of the dynamic game, in which the information aspects of the game are suppressed. When defining **solution concepts** for dynamic games, one usually starts from the normal form, making it possible to use the same definitions as for static games for (both deterministic and stochastic) dynamic games, too.

In particular, an n-tuple of strategies $\gamma^*\in\mathrm{X}_{i\in\mathcal{N}}\Gamma_i$ is a (non-cooperative) **Nash equilibrium solution** iff

$$\gamma_i^*=\underset{\gamma_i\in\Gamma_i}{\arg\min}\, J_i(\gamma^{i*},\gamma_i)\ \forall i\in\mathcal{N}, \quad (10)$$

where $(\gamma^{i*},\gamma_i)\equiv(\gamma_1^*,...,\gamma_{i-1}^*,\gamma_i,\gamma_{i+1}^*,...,\gamma_n^*)$. For $n=2$, $\gamma^*\in\Gamma_1\times\Gamma_2$ is a **Stackelberg equilibrium solution** with player 1 as **leader** and player 2 as **follower** iff

$$\gamma_1^*=\underset{\gamma_1\in\Gamma_1}{\arg\min}\ \sup_{\gamma_2\in R_2(\gamma_2)} J_1(\gamma_1,\gamma_2) \quad (11)$$

and

$$\gamma_2^*\in R_2(\gamma_1^*), \quad (12)$$

where $R_2(\gamma_1)\equiv\{\hat{\gamma}_2\in\Gamma_1 : J_2(\gamma_1,\hat{\gamma}_2)\le J_2(\gamma_1,\gamma_2)\ \forall\gamma_2\in\Gamma_2\}$ is the **reaction set** of the follower. When $R_2(.)$ is a singleton, a unique **reaction function** $\mathbf{R}_2:\Gamma_1\to\Gamma_2$ of the follower can be defined as

$$\mathbf{R}_2(\gamma_1)=\underset{\gamma_2\in\Gamma_2}{\arg\min}\, J_2(\gamma_1,\gamma_2)\ \forall\gamma_1\in\Gamma_1, \quad (13)$$

and conditions (11) and (12) can be replaced by

$$\gamma_1^*=\underset{\gamma_1\in\Gamma_1}{\arg\min}\, J_1(\gamma_1,\mathbf{R}_2(\gamma_1)) \quad (14)$$

and

$$\gamma_2^*=\mathbf{R}_2(\gamma_1^*). \quad (15)$$

In this case, the "leader" in the Stackelberg equilibrium obtains a result which is no worse than that of any Nash equilibrium solution of the game. This is no longer true if $R_2(.)$ is not a singleton, i.e. when the mapping $\mathbf{R}_2$ is not unique; then condition (10) protects the leader only against the worst choice from $R_2(\gamma_1)$. The concept of a Stackelberg equilibrium solution can be extended to games with $n>2$, with the possibility of introducing several hierarchical levels. The Stackelberg solution has an equilibrium property as the Stackelberg equilibrium solution of a (static or dynamic) game is equivalent to a (the so-called strong feedback) Nash equilibrium solution of a related dynamic game (Başar and Haurie, 1984).

When defining solution concepts of a dynamic game starting from the extensive instead of the normal form description of the game, **other equilibrium solutions** can be defined according to the information pattern assumed, such as open-loop or closed-loop Nash and Stackelberg equilibrium solutions, feedback equilibria, equilibria in memory strategies, etc. These equilibria then may have certain desirable properties such as subgame perfection or (strong or weak) time consistency or not. Particular assumptions about the possibilities of commitment can be made for the players at time $t = 0$, which then correspond to these different solution concepts (cf. Başar, 1989; Dockner and Neck, 1988, among others). Moreover, there are several non-cooperative (e.g., the consistent conjectural variations equilibrium) and cooperative solution concepts; for the latter, the assumption is that the players follow joint strategies backed by a binding agreement. Cooperative solutions usually are **efficient** (Pareto optimal): An n-tuple of strategies $\boldsymbol{\gamma}^* \in \mathsf{X}_{i\in\mathcal{N}}\Gamma_i$ is efficient iff

$$J_i(\boldsymbol{\gamma}^*) \leq J_i(\boldsymbol{\gamma}) \forall \boldsymbol{\gamma} \in \mathsf{X}_{i\in\mathcal{N}}\Gamma_i \; \forall i \in \mathcal{N} . \qquad (16)$$

Efficient solutions are usually obtained by solving an optimal control problem. Non-cooperative equilibrium solutions are generally inefficient; the question as to when and how efficient non-cooperative equilibria of dynamic games are possible is the object of much research. Another focus of intensive interest lies on problems of analytical and/or numerical computation of various equilibrium solutions of dynamic games.

4. SOME ECONOMIC APPLICATIONS OF DYNAMIC GAME THEORY

It is easy to see that dynamic games are very appropriate for many economic problems. The classical **microeconomic** oligopoly problem, in which two or more (but not many) firms compete against each others, is a typical game situation as these firms have obviously conflicting interests, although these interests are not necessarily completely antagonistic. For example, each firm aims at increasing its profit, possibly at the expense of its competitors, but all of them may be interested in increasing their entire market, i.e., in inducing consumers to buy more of their products. This means that their situation can be best described as a non-zero-sum game – a model that is more complicated than a zero-sum game, in which each player's gains are exactly the other player's losses. The same is true for a large variety of economic (and even most other social) problems.

Dynamic analyses have also found their ways into economics already many decades ago. For instance, for the oldest mathematical model in economics, Cournot's monopoly model, dynamic counterparts and variants were developed already in the 1920s and 1930s, using dynamic optimization methods (including optimum control theory). Since the oligopoly is the just the extension of the monopoly market from one supplier to two, it was to be expected that dynamic oligopoly models were developed as soon as there were mathematical tools available to solve such models. Hence it comes to no surprise that dynamic games find a wide area of applications in many areas of economics. For a recent overview, see (Dockner *et al.,* 2000), who give an extensive account of dynamic games in the fields of capital accumulation, oligopoly and industrial organization, marketing, and resources and environmental economics.

In the area of **macroeconomics** and the theory of **stabilization policy**, there are also several possibilities to introduce decision-makers with different interests (see, among others, Petit, 1990). In particular, different policy-making institutions, which are responsible for specific policy instruments and/or areas, may differ with respect to their preferences. On the national level, there may be conflicts between the government (which is usually responsible for fiscal policy) and the central bank, to which monetary policy is entrusted. For example, central banks are often highly adverse against inflation, while governments frequently put more emphasis on goals like full employment or high GDP growth. In an international context, governments of different countries may have different objectives, and problems of international policy coordination may arise. In this case, policy-makers of different countries may pursue primarily their own national interests and do not care about spillovers of their actions to other countries or even engage in "beggar-thy-neighbor"-policies. Stabilization theory even sometimes considers conflicts of interest between the government of a country and the (aggregate) private sector of that country or at least between the decisions of a country's policy-makers and the preferences of the majority of its citizens. Several other possibilities of divergent interests of policy-makers are conceivable, and dynamic game theory is a very appropriate tool to analyze (and sometimes help resolving) the resulting conflict situations.

An especially interesting problem of strategic interactions arises in the case of a monetary union, of which the European Economic and Monetary Union (EMU) is a prominent example. In a monetary union, national currencies (national central banks) are completely replaced by a common currency (common central bank). This implies that the exchange rate between the members of a monetary union is no longer available as an instrument of adjustment. In the following sections of this paper, we consider the design of stabilization policies for a small macroeconomic model of a monetary union consisting of two countries. We confine ourselves to the simple case of a monetary union consisting of two symmetric countries, i.e. countries of identical size with identical model parameters. It will be shown that the results of different solution concepts for a dynamic game between the common central bank and national fiscal policy-makers can provide insights into the structure of a policy conflict and its consequences under different assumptions about policy-makers' behavior in such a union. More details can be found in Neck and Behrens (2004b); see also Neck and Behrens (2004a).

Dynamic game models are usually much more complex than optimum control problems; hence only in rare cases analytical solutions for these models are available. Therefore, even for small macroeconomic models, numerical solutions or approximations to them are the best one can hope for. Here we use the OPTGAME algorithm (Behrens and Neck, 2003a) to analyze the macroeconomic policy problem for the two-country monetary union. The OPTGAME algorithm is designed for approximating solutions of dynamic games with a finite planning horizon. It solves discrete-time LQ (linear-quadratic) games, and approximates the solutions of nonlinear-quadratic difference games by iteration. At present, the algorithm calculates the open-loop and the feedback Nash equilibrium solution and the cooperative Pareto-optimal solutions for an arbitrary number of players; extensions to other solution concepts are being implemented.

5. THE MODEL

In the following description of the macroeconomic model, capital letters indicate nominal values, while lower case letters correspond to real values. The superscripts ^{d} and ^{s} denote demand and supply, respectively. The model consists basically of short-run deviations from an exogenous long-run growth path due to Keynesian features of goods and financial markets. The two countries under consideration are linked both through national goods markets (exports and imports of goods and services) and through the integrated money (and, implicitly, other financial) markets. Three active policy-makers are considered: the governments of the two countries and the common central bank of the monetary union.

The goods market for each country is modeled by a short-run income-expenditure equilibrium relation (IS curve). Real output in country i ($i = 1,2$) at time t ($t = 1,...,T$) is given as the sum of the long-run equilibrium level of real output, $\bar{y}_{it}$, and the short-term deviation there from, $\tilde{y}_{it}$, i.e.,

$$y_{it} = \bar{y}_{it} + \tilde{y}_{it} \quad (16)$$

where

$$\bar{y}_{it} = (1+\theta)\bar{y}_{i(t-1)}, \; \bar{y}_{i0} \text{ given,} \quad (17)$$

$$\tilde{y}_{it} = \frac{\delta_i(P_{jt} - P_{it})}{P_{it}} - \gamma_i(r_{it} - \theta) + \rho_i\tilde{y}_{jt} - \eta_i\tilde{f}_{it} + z_{it}, \quad (18)$$

for $i \neq j$ ($i, j = 1,2$). P_{it} ($i = 1,2$) denotes country i's general price level, r_{it} ($i = 1,2$) country i's real interest rate, and $\tilde{f}_{it}$ ($i = 1,2$) country i's (short-term deviation from a zero) real fiscal surplus (if negative, its fiscal deficit). $\tilde{f}_{it}$ ($i = 1,2$) in (18) is country i's fiscal policy instrument, i.e. its control variable. The natural real growth rate, $\theta \in [0,1]$, is assumed to be equal to the natural real rate of interest. The parameters δ_i, γ_i, ρ_i, η_i, $i = 1,2$, in (18) are assumed to be positive. The variables z_{1t} and z_{2t} are non-controlled exogenous variables and represent exogenous demand-side shocks on the goods market.

For $t = 1,...,T$, the current real rate of interest for country i ($i = 1,2$) is given by

$$r_{it} = R_{Et} - X_{it}, \quad (19)$$

where R_{Et} denotes the common nominal rate of interest determined by the common central bank of the monetary union, and X_{it} ($i = 1,2$) represents country i's rate of inflation. The long-run equilibrium and the natural (nominal and real) interest rate, $\bar{R}_{Et} = \bar{r}_{it} = \theta$, are "inflation-free", i.e. $\bar{X}_{it} = 0$ for $i = 1,2$.

The general price levels and inflation rates for $i = 1,2$ and $t = 1,...,T$ are determined according to an expectations-augmented Phillips curve, i.e. the rate of inflation depends positively on expected inflation and on goods market excess demand:

$$P_{it} = (1 + X_{it})P_{i(t-1)}, \quad P_{i0} \text{ given,} \quad (20)$$

$$X_{it} = X_{it}^e + \xi_i\tilde{y}_{it}, \quad (21)$$

where ξ_1 and ξ_2 are positive parameters. X_{it}^e ($i = 1,2$) denotes the rate of inflation of country i ($i = 1,2$) expected to prevail during time period t, which is formed at (the end of) time period $t - 1$, $t = 1,...,T$. Inflationary expectations are formed according to the hypothesis of adaptive expectations:

$$X_{it}^e = \varepsilon_i X_{i(t-1)} + (1 - \varepsilon_i)X_{i(t-1)}^e, \quad (22)$$

where $\varepsilon_i \in [0,1]$ for $i = 1,2$ are positive parameters determining the speed of adjustment of expected to actual inflation.

We also define average variables for output and inflation in the monetary union as

$$y_{Et} = \omega y_{1t} + (1-\omega)y_{2t}, \quad \omega \in [0,1], \quad (23)$$

$$X_{Et} = \omega X_{1t} + (1-\omega)X_{2t}, \quad \omega \in [0,1]. \quad (24)$$

Real money demand in country i ($i = 1,2$) is the sum of long-run and short-run real money demand:

$$m_{it}^d = \bar{m}_{it}^d + \tilde{m}_{it}^d. \quad (25)$$

Short-run real money demand is determined by a Keynesian money demand function:

$$\widetilde{m}_{it}^{d} = \kappa_i \widetilde{y}_{it} - \lambda_i \left(R_{Et} - \theta\right). \tag{26}$$

Here κ_i, λ_i $(i = 1,2)$ are positive parameters and R_{Et} denotes the common nominal interest rate. In accordance with the long-run equilibrium relations, $\overline{y}_{it} = y_{it}$, $\widetilde{y}_{it} = 0$, $\overline{X}_{it} = 0$ and $\overline{r}_{it} = \theta$ $(i = 1,2)$, long-run equilibrium money demand is given by

$$\overline{m}_{it}^{d} = \kappa_i \overline{y}_{it}. \tag{27}$$

This leaves us with the following relationship for the long-run demand for money in country i $(i = 1,2)$:

$$\overline{M}_{it}^{d} = P_{it}\overline{m}_{it}^{d} = P_{it}\kappa_i\left(1+\theta\right)\overline{y}_{i(t-1)}. \tag{28}$$

In a monetary union, the sum of the countries' money demands has to be equal to the monetary union's money supply. Here we assume the money market always to clear in the short-run, too, and hence

$$M_{Et}^{s} = M_{1t}^{d} + M_{2t}^{d}. \tag{29}$$

This leads to the LM curve for the monetary union:

$$M_{Et}^{s} = \kappa_1 y_{1t} P_{1t} + \kappa_2 y_{2t} P_{2t} - \left(\lambda_1 P_{1t} + \lambda_2 P_{2t}\right)\left(R_{Et} - \theta\right). \tag{30}$$

The government budget constraint is given as an equation for government debt of country i $(i = 1,2)$,

$$D_{it} = \left(1 + R_{E(t-1)}\right)D_{i(t-1)} - F_{it} - \beta_i \widetilde{B}_{Et},\ D_{i0} \text{ given}, \tag{31}$$

where the nominal fiscal surplus of country i $(i = 1,2)$ is determined by the identity

$$F_{it} = P_{it} f_{it} = P_{it} \widetilde{f}_{it}. \tag{32}$$

$\widetilde{B}_{Et}$ denotes the short-term deviation of high-powered money, B_{Et}, from its long-run equilibrium level, $\overline{B}_{Et}$. The long-run (equilibrium) stock of high-powered money is assumed to grow at the constant natural rate θ. Hence,

$$B_{Et} = \overline{B}_{Et} + \widetilde{B}_{Et} = \left(1+\theta\right)\overline{B}_{E(t-1)} + \widetilde{B}_{Et}. \tag{33}$$

$\widetilde{B}_{Et}$ is the control variable of the monetary union's common central bank. This change in high-powered money, $\widetilde{B}_{Et}$, is distributed as seigniorage to the two countries according to given positive parameters $\beta_1 \in [0,1]$ and $\beta_2 := 1 - \beta_1$. Assuming a constant money multiplier, ψ, the broad money supply of the monetary union is given by

$$M_{Et}^{s} = \psi B_{Et}. \tag{34}$$

Both national fiscal authorities are assumed to care about stabilizing inflation, output, debt, and fiscal deficits of their own countries, i.e., they aim at zero inflation, natural output growth, zero government debt and a balanced budget at each time t. The common central bank is interested in stabilizing inflation and output in the monetary union and in a low variability of its supply of high-powered money. Hence, the individual objective (loss) functions of the national governments $(i = 1,2)$ and of the common central bank are given by

$$J_i = \tfrac{1}{2}\sum_{t=1}^{T}\left(\tfrac{1}{1+\theta}\right)^t \Big(\alpha_{iy}\left(y_{it} - \overline{y}_{it}\right)^2 + \alpha_{iX}\left(X_{it} - \overline{X}_{it}\right)^2 + \alpha_{iD}\left(D_{it} - \overline{D}_{it}\right)^2\Big) + \tfrac{1}{2}\sum_{t=1}^{T}\left(\tfrac{1}{1+\theta}\right)^t \alpha_{if}\widetilde{f}_{it}^{2}, \tag{35}$$

$$J_E = \tfrac{1}{2}\sum_{t=1}^{T}\left(\tfrac{1}{1+\theta}\right)^t \Big(\alpha_{Ey}\left(y_{Et} - \overline{y}_{Et}\right)^2 + \alpha_{EX}\left(X_{Et} - \overline{X}_{Et}\right)^2 + \alpha_{EB}\widetilde{B}_{Et}^{2}\Big), \tag{36}$$

where all weights α are positive numbers $\in [0,1]$. The joint objective function for the calculation of the cooperative Pareto-optimal solution is given by

$$J = \mu_1 J_1 + \mu_2 J_2 + \mu_E J_E, \quad (\mu_1, \mu_2, \mu_E \geq 0,\ \mu_1 + \mu_2 + \mu_E = 1). \tag{37}$$

The parameters of the model are specified in the simplest possible way for a symmetric monetary union (Table 1). For the parameter ε_i, Table 1 gives the value for our benchmark case; it will be varied later on. The target values assumed for the objective variables of the players are given in Table 2; they are basically the long-run equilibrium values of the respective variables. The initial values of the state variables of the dynamic game model are shown in Table 3.

Table 1: Parameter values for a symmetric monetary union, $i = 1,2$

T	θ	$\delta_i, \gamma_i, \rho_i, \varepsilon_i, \omega, \beta_i$	ξ_i	λ_i	ψ	η_i, κ_i, α's	μ_i, μ_E
20	0.03	0.5	0.25	0.15	2.0	1.0	0.33

Table 2: Target values for a symmetric monetary union, $i = 1,2$ and $t = 1,\ldots,T$,

$\overline{y}_{it}$	$\overline{y}_{Et}$	$\overline{X}_{it}$	$\overline{X}_{Et}$	$\overline{D}_{it}$	$\overline{\widetilde{f}}_{it}$	$\overline{\widetilde{B}}_{Et}$
$(1+\theta)^t$	$(1+\theta)^t$	0	0	0	0	0

Table 3: Initial values ($t = 0$) for a symmetric monetary union, $i = 1,2$

$\bar{y}_i$	$\tilde{y}_i$	p_i	X_i	D_i	R_E	$\bar{B}_E$	$\tilde{f}_i$	$\tilde{B}_E$
1	0	1	0	0	θ	1	0	0

Equations (16) – (37) constitute a nonlinear dynamic game with a finite planning horizon, where the objective functions are quadratic in the paths of deviations of state and control variables from their respective desired values.

6. SOME RESULTS OF FISCAL AND MONETARY POLICY GAMES

Next, we discuss the results of various solution concepts of this dynamic game. We assume a temporary positive symmetric demand shock influencing the economies of the two countries in the same way. In particular, we assume that autonomous real output (GDP) in both economies rises by 1.5 % of GDP above the long-run equilibrium path for the first four periods and less (declining) for the next three periods: $z_{i0} = 0$, $z_{i1} = z_{i2} = z_{i3} = z_{i4} = 0.015$, $z_{i5} = 0.01$, $z_{i6} = 0.005$, $z_{i7} = 0.0025$, and $z_{it} = 0$ for $t \geq 8$, $i = 1,2$.

Without policy intervention, the demand side shock leads to higher output and to higher inflation during the first periods (compared to the long-run equilibrium path), but lower output and inflation afterwards (see the path denoted as "uncontrolled" in Figures 3 and 4). The uncontrolled dynamic system adjusts in dampened oscillations, approaching the long-run path only slowly. The small size of the deviations of national output and inflation from their equilibrium paths shows that, even without policy intervention, there is sufficient negative feedback in the system to reduce the impact of the shock on output to not more than one fifth of the original shock in the case of a temporary symmetrical shock. The main mechanism working into this direction is the strong reaction of the rate of interest, which rises from 3 % to values around 6 % in the first four periods, but falls quickly afterwards to its long-run value of 3 %. Due to the symmetry of the economies and of the shock, the reactions of all variables are identical in both economies.

When policy-makers are assumed to react on this shock according to their preferences as expressed by their objective functions, outcomes depend on the assumptions made about the behavior of all the other policy-makers. Here we consider two non-cooperative equilibrium solutions of the resulting dynamic game, namely the open-loop Nash and the feedback Nash equilibrium solution, and one cooperative solution, the Pareto-optimal collusive solution (assuming identical weights, $\mu_i = 1/3$, $i = 1,2,E$ for the three players).

For the three solution concepts considered, Figures 2 and Fig. 3 show the trajectories of the control variables – real fiscal surplus for either country and additional high-powered money for the central bank, respectively. Figures 4 and 5 show the trajectories of the state (and target) variables' deviations from long-run equilibrium output and inflation, respectively. The common nominal rate of interest exhibits a behavior very similar to the uncontrolled case. For obvious reasons, all country-specific variables of our symmetrically modeled monetary union show exactly the same time paths for both countries.

As can be seen from Figures 2 – 5, both fiscal and monetary policies react on the positive demand shock in a restrictive and hence counter-cyclical way: both countries create a fiscal surplus during the first six periods and alternate between periods of fiscal deficit and surplus afterwards, and the central bank decreases its supply of high-powered money during the first five years and increases it afterwards. This results in a reduction of additional output (and hence of excess demand loss) and inflation for the first four years as compared to the uncontrolled solution; in fact, in the cooperative solution inflation is nearly reduced to one half of its uncontrolled values. Oscillations of these variables are dampened more strongly than in the uncontrolled solution. This statement holds for later periods, too.

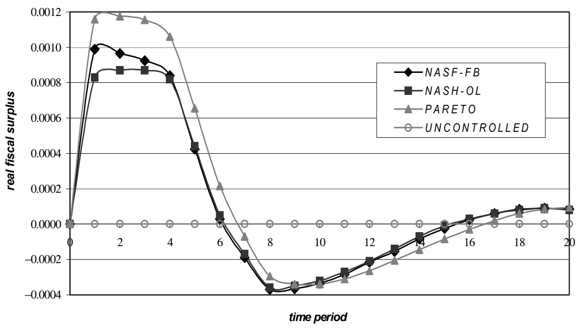

Fig. 2. Country i's fiscal surplus for i = 1, 2 and ε = 0.5

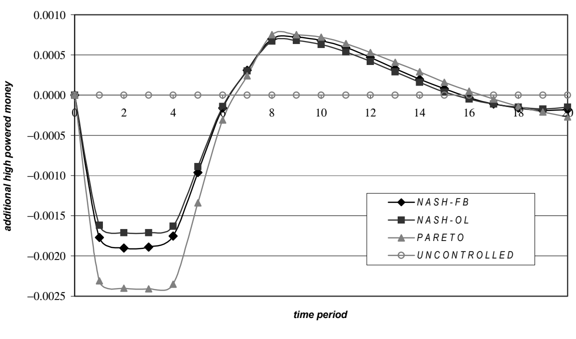

Fig. 3. Additional high-powered money for ε = 0.5

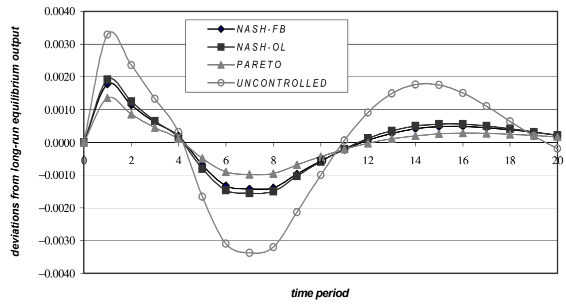

Fig. 4. Country i's output-deviation from its long-run equilibrium level for $i = 1, 2$ and $\varepsilon = 0.5$

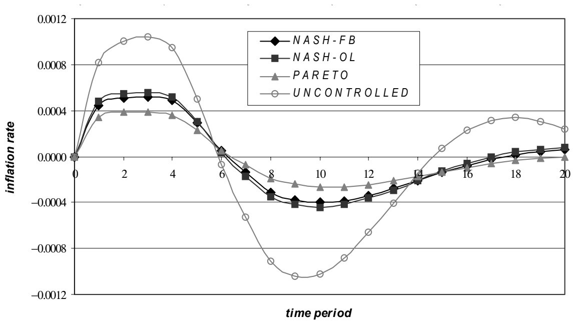

Fig. 5. Country i's inflation rate for $i = 1, 2$ and $\varepsilon = 0.5$

The magnitude of the (absolute) values of the instruments involved is rather small: the fiscal surplus/deficit created is to the order of one tenth (or less) of one percentage point of GDP, for example. Also changes of the monetary base in periods 1 to 4 amount to only about 0.25 percent of its stock. These small policy reactions are due to the strong self-stabilizing forces of the model used, acting especially through the interest rate channel.

When we compare the non-cooperative (Nash) equilibrium solutions and the cooperative (Pareto-optimal) solution, another interesting observation can be made: All show the same qualitative behavior, and the two non-cooperative Nash equilibrium solutions are rather close together in terms of all control and state variables. The Pareto-optimal collusive solution, although not too distant from the other two, entails more active policy-making (higher fiscal surplus and money reduction in the first periods). This policy-mix does not change the path of the rate of interest (higher fiscal surplus decreases, lower money supply increases the (short-run) interest rate, ceteris paribus), but does so for the paths of output and inflation: both are closer to their long-run values and hence contribute more to reaching the common goal in the cooperative than in the non-cooperative solutions.

7. SENSITIVITY ANALYSIS

Given these insights, one can conduct a sensitivity analysis of the results, i.e., investigate whether the results change substantially if some elements of the model and/or of the objective function change. For instance, it is of interest how the dynamics of the model and the results of the policy game depend on the way inflationary expectations are formed. To do so, we retain the assumption of adaptive expectations, i.e., equation (22), but vary the parameter ε_i between 0 and 1 by considering the different solutions for $\varepsilon_i = 0$ (static expectations), $\varepsilon_i = 0.25$, $\varepsilon_i = 0.5$ (the previous benchmark case), $\varepsilon_i = 0.75$, and $\varepsilon_i = 1$ (myopic expectations). Again, we assume the same value of ε_i for both countries $i = 1, 2$ in each of these cases.

Detailed results on the reactions of the model's variables can be found in Neck and Behrens (2004b). Here only the main results are summarized. Consider first the uncontrolled solution, i. e. the development of the variables without intervention of the monetary union's policy-makers. For both the actual and the expected rate of inflation, the following pattern of dependence upon ε_i can be observed: as ε_i increases, oscillations of these variables become wider (larger amplitude) and faster (shorter period). Expected inflation lags behind actual inflation, except for the case of $\varepsilon_i = 0$, where inflation is just determined by excess demand. Oscillations of output, price level and inflation are more pronounced (less dampened) when inflationary expectations react on actual inflation and contribute to determining the trajectory of actual inflation. In this case, these variables converge more slowly towards the long-run equilibrium path. Output oscillations exhibit higher frequency, too, the larger is ε_i. With stronger reactions of expectations to actual inflation, the overall effects on the price level are stronger. Inflation lags behind output in the present model, instead of the parallel movement of these two variables prevalent in the model without inflationary expectations.

If we include policy-makers' strategic interactions and consider the solutions of the dynamic game with different values of the inflationary expectations parameter, we find that the optimal reactions of fiscal policy and monetary policy in different solution concepts yield similar trajectories for the control and the endogenous variables, with even smaller differences between paths from models with different values of ε_i. The counter-cyclical behavior of all policy instruments (controls) holds for all values of ε_i. They are applied more vigorously for increasing values of ε_i, due to larger deviations of the target variables from their desired values. There is a trade-off between control and target variables: higher deviations of the latter in the uncontrolled solution call for more active policies, which in turn imply larger instrument costs. This trade-off is less favorable if inflationary expectations react more strongly to actual inflation.

The time paths of the target variables output and inflation in the different dynamic game solutions resulting from these policies are also qualitatively similar for different ε_i-models. Excess demand, actual and expected inflation are still oscillating, hence these variables are not fully counteracted by policies, due to the trade-off between control and target variables. The qualitative pattern of dependence of oscillations' amplitude and frequency is the same as in the uncontrolled case. But policy actions smooth the time paths of these variables. This is particularly true for output deviations, which are reduced rather quickly after exhibiting a peak in the first period.

For the rates of inflation, the amplitudes of the first oscillations are reduced to roughly one half of those in the uncontrolled case, and even more towards the end of the planning horizon. Expected inflation again lags behind actual inflation when ε_i is positive. A higher value of ε_i makes stabilization of inflationary expectations, inflation and output more costly than in the benchmark case. Equilibrium and optimal time paths of the target variables vary more strongly for high values of ε_i than for low values (or even $\varepsilon_i = 0$). The differences between the non-cooperative and the cooperative solutions are minor; in particular, the qualitative behavior is similar in all cases considered.

Apart from the investigation of the effects of the adaptation parameter in the equation determining inflationary expectations, we did a lot of further sensitivity analysis in order to check how robust the conclusions of the model are with respect to parameter values. For example, as shown in Neck and Behrens (2004a), we investigated the effect of the influence of expected on actual inflation. Moreover, since the length and size of the assumed demand shock were arbitrary, several scenarios with different time patterns of shocks were tried. The results are mostly as expected: for a one-period shock, countercyclical policies are short-lived, with a quick adjustment of the system to the equilibrium path. A negative shock of the same absolute size as the positive shock of Section 6 results in time paths for all variables that deviate nearly exactly as much from the long-run path as in the benchmark solution of Section 6, but in the opposite direction.

Sensitivity analyses for several parameters yield interesting results, especially as more variation between the member countries of the monetary union can be introduced in this way. For example, we varied the slope of the Phillips curve (the short-run aggregate supply curve) from $\xi_{1,2} = 0.25$ to $\xi_1 = 1$ and $\xi_2 = 0.1$. This serves also to model an asymmetric transmission of a symmetric shock. Country 1 exhibits a faster reaction of prices and inflation on the shock than before, country 2 a slower one. The reaction of the output gap (deviation from long-run level) and inflation in country 1 is both faster and shorter-lived than in country 2, hence higher price flexibility (a steeper short-run supply curve) acts like an automatic stabilizer. This scenario calls for stronger reactions of policy-maker 2 (the less effective one) and slightly weaker reactions of (the more effective) policy-maker 1 than in the benchmark scenario, whereas the reaction of the central bank is nearly unchanged. Country 1 stabilizes quicker and with smaller amplitude than country 2 (and both countries in the benchmark scenario).

Another scenario examines the sensitivity with respect to the parameters λ_i , which are changed from 0.15 to $\lambda_1 = 0.15$ (as before) and $\lambda_2 = 0$. This assumes an interest-inelastic demand for money in country 2, which makes country 2's fiscal policy less effective. In the uncontrolled solutions, the objective variables output and inflation deviate considerably less from the equilibrium paths than in the benchmark scenario, showing that this rather "classical" scenario exhibits strong self-stabilizing forces. This, together with the lower effectiveness of fiscal policy, implies a smaller reaction of the budget surplus than in the benchmark case; also monetary policy is less active here. Nevertheless, due to the automatic stabilizers, the system becomes more stable here than in the benchmark solution.

Further scenarios investigated do not differ from the benchmark solution with respect to the uncontrolled solution, but only through their policy reactions. For example, we assumed the fiscal multipliers in the two countries to be different: $\eta_1 = 1.5$ and $\eta_2 = 0.5$. This scenario results in a less active fiscal policy-maker in country 1 and a more active one for country 2, both compared to the benchmark. The central bank is nearly not affected but follows virtually the same policy strategy than in the benchmark. A final scenario assumes a different objective function than in the benchmark scenario: the central bank is assumed to be only concerned about price stability in the union and not about output. Interestingly, the controls and the controlled objective variables are nearly the same as in the benchmark scenario.

To summarize, the most striking result of this sensitivity analysis are the small differences between the different scenarios. More variation can be generated only when supply shocks and/or asymmetric shocks will enter the picture. Due to lack of space, however, these extensions are beyond the scope of this paper.

8. CONCLUDING REMARKS

In this paper, an introductory overview of dynamic game theory and of some application to economic problems was given. It was shown that dynamic games provide adequate models for situations with several decision-makers having distinct preferences, which are very common in economics and other social sciences. Moreover, dynamic games are natural extensions of optimum control problems, which are all too familiar to the engineering community. Hence, further developments and applications of dynamic game theory and applications can provide another chance for mutually fruitful dialogues between economists and control engineers.

More specifically, we have applied dynamic game theory and the OPTGAME algorithm to a simple macroeconomic model of fiscal and monetary policies in a two-country monetary union and obtained some insights into the design of economic policies in the case of a symmetric excess demand shock. In particular, optimal policies of both the governments and the common central bank are counter-cyclical but not very active, at least for the model under consideration. The outcomes of the different solution concepts of dynamic game theory are rather close to each other. In all cases, there are trade-offs between the vigor of policy actions and the smoothing effect on target variables. If private agents' inflationary expectations react more strongly to actual inflation, this complicates the stabilization task of macroeconomic policies in the monetary union.

The model considered here refers to a very simple monetary union of two symmetric countries only. In order to derive results that are valid for a particular monetary union, one can extend the model (for example, to a larger number of member countries of the monetary union) and calibrate parameters for that particular union. For example, Haber et al. (2002) use a large calibrated model of the global economy (the MSG2 model) to obtain insights into the policy conflicts between the governments (fiscal policy) and the central bank (the European Central Bank; monetary policy) in the European Economic and Monetary Union. A related approach is pursued by van Aarle et al. (2002). Hence, dynamic game applications may gain realism and even lead to results that are off direct interest to actual policy-makers looking for advice in designing policy actions. This is again a fruitful field for interdisciplinary cooperation between mathematicians, engineers and economists.

9. REFERENCES

Başar, T. (1986). A Tutorial on Dynamic and Differential Games. In: *Dynamic Games and Applications in Economics* (T. Başar (Ed.)), 1–25. Springer-Verlag, Berlin

Başar, T. (1989). Time Consistency and Robustness of Equilibria in Non-Cooperative Dynamic Games. In: *Dynamic Policy Games in Economics* (F. van der Ploeg and A. de Zeeuw (Eds.)), 9–54. North-Holland, Amsterdam.

Başar, T. and A. Haurie (1984). Feedback Equilibria in Differential Games with Structural and Modal Uncertainties. In: *Advances in Large Scale Systems 1* (J.B. Cruz (Ed.)), 163–201. Greenwich CT.

Başar, T. and G.J. Olsder (1999). *Dynamic Noncooperative Game Theory.* 2nd ed., SIAM, Philadelphia.

Behrens, D. A. and R. Neck (2003). *Approximating Equilibrium Solutions for Multi-Player Difference Games Using the OPTGAME Algorithm.* Working Paper, University of Klagenfurt, Klagenfurt.

Dockner, E.J., S. Jorgensen, N.V. Long and G. Sorger (2000). *Differential Games in Economics and Management Science.* Cambridge University Press, Cambridge, UK.

Dockner, E.J. and R. Neck (1988). *Time-Consistency, Subgame-Perfectness, Solution Concepts and Information Patterns in Dynamic Models of Stabilization Policies*. Discussion Paper, University of Saskatchewan, Saskatoon.

Fudenberg, D. and J. Tirole (1992). *Game Theory.* MIT Press, Cambridge, MA.

Haber, G., R. Neck and W. J. McKibbin (2002). Global Implications of Monetary and Fiscal Policy Rules in the EMU. *Open Economies Review* **13** (4), 363 – 379.

Mehlmann, A. (1988). *Applied Differential Games*. Plenum Press, New York.

Neck, R. and D. A. Behrens (2004a). Macroeconomic Policies in a Monetary Union: A Dynamic Game. *Central European Journal of Operations Research* **12** (2), 171–186.

Neck, R. and D. A. Behrens (2004b). *A Macroeconomic Policy Game for a Monetary Union with Adaptive Expectations.* Working Paper, Ludwig Boltzmann Institute for Economic Policy Analyses, Vienna.

Petit, M. L. (1990). *Control Theory and Dynamic Games in Economic Policy Analysis.* Cambridge University Press, Cambridge.

van Aarle, B., G. Di Bartolomeo, J. Engwerda and J. Plasmans (2002). Monetary and Fiscal Policy Design in the EMU: An Overview. *Open Economies Review* **13** (4), 321 – 340.