ROBUST H_∞ CONTROL OF A BILATERAL TELEOPERATION SYSTEM UNDER COMMUNICATION TIME-DELAY

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Abstract: An H_∞ approach to robust control of bilateral teleoperation systems under communication time-delay is considered. First stability conditions in both cases of constant and time-varying delays are given. Then, the problem of controller design that robustly stabilizes the system w.r.t environment uncertainties (and for any delay) is formulated as an H_∞ problem. When delay independent stability cannot be achieved, a way to determine the maximal allowed time-delay is provided. Simulation results point out the interest of the proposed methodology.

Keywords: Bilateral teleoperation systems, Time delay, H_∞ approach, Robust stability.

1. INTRODUCTION

This paper deals with bilateral teleoperation systems trough communication network, i.e. when the force sensed at the environment is reflected back to the master side to provide a good fidelity to the operator. However the incurred communication time delay may destabilize a bilaterally controlled teleoperator (Anderson and Spong, 1989). As briefly described below, many control schemes have been proposed to overcome the instability due to the communication time delay issue.

Passivity theory has been largely used to ensure the stability of time-delay teleoperation systems (Chopra *et al.*, 2003). In this approach, wave variable transformations are used to ensure passivity of the communication link, which allows to get passivity of the whole system. However, as pointed by Tanner and Niemeyer (2004) non- idealities can violate passivity. Moreover cautious digital implementations are necessary as passivity may be lost if no specific mechanism is done to handle missing packets (Berestesky *et al.*, 2004).

In (Niculescu *et al.*, 2002; Taoutaou *et al.*, 2003), frequency sweeping test is used to derive conditions on PI-type controller such that the global system is asymptotically stable. However, the study cannot be directly generalized for other types of controller. Authors have also proposed in (Fattouh and Sename, 2003*a*) a finite spectrum controller for bilateral teleoperation systems. However, the time-delay must be known and robustness is difficult to analyze.

In this work, an H_∞ approach is proposed. In this framework, Leung *et al.* (1995) have used μ-analysis and synthesis to design robust controllers for bilateral teleoperation systems. However, the time delay is treated as a disturbance on the system and not as a system parameter. As well, in (Fattouh and Sename, 2003*b*), the authors have designed an H_∞ impedance controller and provided a stability analysis w.r.t the time-delay, but considering it as an uncertainty which leads to a conservative result.

In this paper, the tracking behavior of a teleoperation system is controlled in the presence of environment and communication time-delay uncertainties. To our knowledge this problem has not been tackled before. First a necessary and sufficient condition to ensure the stability of the nominal system for any constant communication time-delay is derived and a sufficient criteria for stability w.r.t time-varying delay is then provided. The H_∞ framework allows us to design a controller that ensures robust stability w.r.t environment impedance uncertainties and for all constant delays. When such a solution cannot be obtained, a graphical Nyquist-type procedure (in the presence of environment uncertainties) is provided to determine the maximal delay uncertainty (added to a known constant one) that preserves the stability of the teleoperation system. The outline is as follows. A general representation of teleoperation systems is given in section 2, followed by a stability analysis of the nominal system in section 3. The robustness of the bilateral teleoperation system is studied in Section 4. Section 5 presents simulation results that support the theoretical work and conclusion is drawn in section 6.

2. SYSTEM REPRESENTATION

A bilateral teleoperation system can be represented as in Fig. 1 where: F_h is the force applied by human operator, F_e is the contact force with the environment and X_m, X_s are the position of master and slave manipulators respectively.

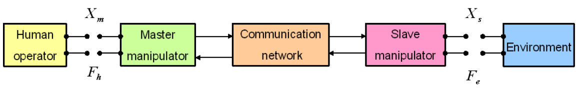

Fig. 1. Bilateral teleoperation system.

The operator commands a position forward to the environment through the master, the communication channel and the slave. Likewise, the force sensed at the environment is transmitted back to the human operator through these blocks. Notice that, since the teleoperator is controlled bilaterally, the arrows in Fig. 1 can be reversed. In this case the operator commands force forward to the environment and position is sent back to the master.

Generally, three controllers are designed for this system: two local controllers for master and slave manipulators in order to achieve desired master and slave compliances and second slave controller such that, in steady state, the slave position X_s is equal to the master position X_m and the global system is asymptotically stable. Here, both local controllers are assumed to be already designed and integrated in the master and slave transfer functions. Only the design and robustness of the second slave controller is thus tackled.

3. STABILITY ANALYSIS

In view of previous section, let P_m and P_s be the stable transfer functions of master and slave manipulators including the local controllers, Z_e be the environment impedance, $h_1 \geq 0$ and $h_2 \geq 0$ be time delays of communication forward and backward channels respectively, and C be the second slave controller. With these notations, Fig. 1 can be redrawn as shown in Fig. 2.

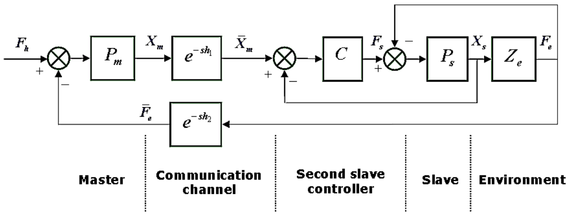

Fig. 2. Considered control structure

Definition 1. Consider the bilateral teleoperation system of Fig. 2. This system is said to be *asymptotically stable* if:

(1) The transfer function from $\bar{X}_m$ to X_s is asymptotically stable with unitary gain.
(2) The transfer function from F_h to X_m is asymptotically stable either for any time-delay or for a bounded time-varying delay.

3.1 Analysis of Condition 1 in Definition 1

The control scheme for the system $\bar{X}_m/X_s$ is shown in Fig. 3 where $W_1(s)$ is a weighting function reflecting the desired tracking performance.

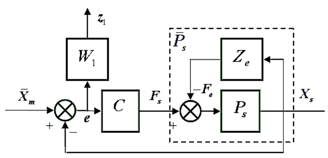

Fig. 3. Master → Slave positions system.

Noting $\bar{P}_s = \frac{P_s}{1+P_s Z_e}$, the sensitivity and complementary sensitivity functions are given by

$$S_s := \frac{1}{1+C\bar{P}_s}; \quad T_s := \frac{X_s}{X_m} = \frac{C\bar{P}_s}{1+C\bar{P}_s} \tag{1}$$

Using an ad hoc choice of the weighting function W_1, the condition 1 can be expressed as the following H_∞ problem: find a controller C that ensures internal stability and

$$\|W_1 S_s\|_\infty < 1 \tag{2}$$

3.2 Analysis of Condition 2 in Definition 1

From block diagrams 2 and 3, the transfer function from F_h to X_m can be described in the block diagram of Fig. 4 where $h = h_1 + h_2$. Hence,

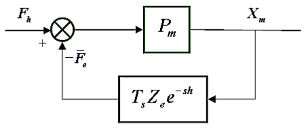

Fig. 4. Operator force → Master position system.

$$T_m := \frac{X_m}{F_h} = \frac{P_m}{1 + P_m T_s Z_e e^{-sh}} \tag{3}$$

Case of constant time-delay. Our objective now is to provide a condition that ensures asymptotically stability for any time delay h. Let us first recall the following result (Niculescu, 2001).

Lemma 2. Let $P(s)$ and $Q(s)$ be two polynomials in complex variable s satisfying $Q(s)$ is stable, and $\deg_s[P(s)] < \deg_s[Q(s)]$.
Then, the polynomial $Q(s) + P(s)e^{-s\tau}$ is stable for all $\tau \geq 0$ if and only if

$$|Q(j\omega)| > |P(j\omega)|, \forall \omega \in \Re \tag{4}$$

In view of above lemma, the following proposition is obtained.

Proposition 3. Consider the closed-loop system of Fig. 4. Let

$$P_m(s) = \frac{N_m(s)}{D_m(s)},\ \ T_s(s) = \frac{N_s(s)}{D_s(s)} \tag{5}$$

Assume that P_m and T_s are stable and such that $deg_s(N_m N_s Z_e) < deg_s(D_m D_s)$, then the transfer function T_m given in (3) is asymptotically stable for any time delay h if and only if

$$\|W_2 T_s\|_\infty < 1,\ where\ W_2 = P_m Z_e \tag{6}$$

Proof: The transfer function T_m given in (3) can be rewritten as follows

$$T_m = \frac{N_m D_s}{D_m D_s + N_m N_s Z_e e^{-sh}} \tag{7}$$

Due to the assumptions, the two conditions of Lemma 2 are satisfied. Therefore, using Lemma 2, system (3) is asymptotically stable for all time delay h if and only if

$$|D_m D_s(j\omega)| > |N_m N_s Z_e(j\omega)|, \forall \omega \in \Re \tag{8}$$

which is equivalent to condition (6). $\triangle$

Case of time-varying time-delay. Here, the result of Kao and Lincoln (2004) is used to give a stability test when time-varying bounded delay is considered. The following proposition states this result.

Proposition 4. (Kao and Lincoln, 2004) Consider the closed-loop system in Fig 4 where h is time-varying. The system is stable for any time-varying delay $0 \leq h(t) \leq \delta_{max}$ if

$$\left|\frac{N_m N_s Z_e}{D_m D_s + N_m N_s Z_e}\right| < \frac{1}{\delta_{max}\omega}, \forall \omega \in [0, \infty] \tag{9}$$

4. ROBUST DESIGN AND ANALYSIS

In this section, robustness w.r.t environment and communication time-delay uncertainties is analyzed. We assume here that the environment impedance Z_e belongs to some admissible set Ξ, and that the time-delay is constant and defined as $h = h_0 + \tau$ where h_0 is known and constant , and τ represents the unknown delay uncertainty.
Our aim is to find conditions s.t T_s and T_m are robustly stable for all $Z_e \in \Xi$ in both following cases: first for all positive delay h, and otherwise for a maximal delay uncertainty τ_{max} (added to a constant one h_0).
First, according to Fig. 3, T_s is only subject to environment uncertainties (not delay one). The environment impedance is assumed to belong to a set Ξ of multiplicative input uncertainties, defined as:

$$\Xi = \{Z_e = Z_e^0(1 + W_e\Delta)\} \tag{10}$$

where Δ is the uncertainty matrix s.t $\|\Delta\|_\infty < 1$, and W_e the uncertainty weight. Now, define the following set of transfer functions:

$$\tilde{P}_s = \{\bar{P_s} : Z_e \in \Xi\} \tag{11}$$

The set Ξ is said to be admissible, if $\tilde{P}_s^0$ (for nominal impedance Z_e^0) and $\tilde{P}_s$ have the same unstable poles.

Proposition 5. Consider the system of Fig. 3 with the family of transfer functions (11) and Ξ is admissible. Assume that the system is internally stable for nominal impedance Z_e (which is ensured by (2)), then the system is internally stable for all $Z_e \in \Xi$ if

$$\|W_{IU} T_s\|_\infty < 1 \tag{12}$$

where W_{IU} is a weighting transfer function satisfying

$$|W_{IU}(j\omega)| \geq \max_{Z_e \in \Xi}\left|\frac{\tilde{P}_s(j\omega)}{\tilde{P}_s^0(j\omega)} - 1\right|,\ \ \forall \omega \in \Re \tag{13}$$

and $\tilde{P}_s^0 = \tilde{P}_s$ for nominal impedance Z_e.

proof: Using (13), the family of transfer functions (11) can be written as follows:

$$\tilde{P}_s = \tilde{P}_s^0(1 + W_{IU}\Delta) \tag{14}$$

where $\tilde{P}_s^0 = \tilde{P}_s$ for nominal impedance Z_e and Δ is a variable stable transfer function satisfying $\|\Delta\|_\infty < 1$. From (14) and robust control theory (Zhou *et al.*, 1996), the robust stability condition for T_s w.r.t multiplicative input uncertainties is given by (12). $\triangle$

4.1 Robust design w.r.t environment uncertainties

In view of (2), (6) and (12), the following theorem is the main result to ensure nominal performance and robust stability.

Theorem 6. Consider the system of Fig. 2 with the family of transfer functions (11) and Ξ is admissible. Define the uncertainty weight as:

$$|W_4(j\omega)| = \max\left\{|W_{IU}(j\omega)|, \max_{Z_e\in\Xi}|W_2(j\omega)|\right\}, \quad \forall\omega\in\Re \quad (15)$$

The teleoperation system is robustly asymptotically stable for any time-delay according to Definition 1 if there exists a controller C that ensures internal stability of T_s and

$$\left\| \begin{matrix} W_1 S_s \\ W_4 T_s \end{matrix} \right\|_\infty < 1 \quad (16)$$

which can be solved as a mixed sensitivity problem.
proof: If (16) is satisfied, then $\|W_1 S_s\|_\infty < 1$ which means that nominal performance is achieved. Furthermore we have

$$\|W_{IU} T_s\|_\infty < 1 \text{ and } \|W_2 T_s\|_\infty < 1, \ \forall Z_e \in \Xi$$

Therefore, using Propositions 3 and 5, we can conclude that T_s and T_m are asymptotically stable for all $Z_e \in \Xi$ and for all constant delay h.

4.2 Robustness analysis w.r.t environment and delay uncertainties

In this section we consider the case where the delay independent stability condition (6) cannot be achieved. We assume that a controller C has been designed for T_s to achieve nominal performance (2) (and if possible robust stability (12)).
The environment uncertainties are considered of the form (10) and the delay is s.t $h = h_0 + \tau$, where τ represents the uncertain part of the delay.
Now, as the delay uncertainties only affect T_m, a robust stability analysis is performed to determine the maximal delay uncertainty τ_{max} that preserves stability of the teleoperation scheme in Fig 2-4, in the presence of environment uncertainties. A graphical method, based on the Nyquist plot, and due to Tsypkin and Fu (1993), is here considered.
Following the method in (Tsypkin and Fu, 1993), we will note:

$$W_\tau = P_m T_s Z_e e^{-sh} \text{ and } W_0 = P_m T_s Z_e^0 e^{-sh_0} \quad (17)$$

Then:

$$W_\tau = W_0(1 + W_e\Delta)e^{-s\tau} \quad (18)$$

The procedure to determine the maximal allowed delay uncertainty is given below.

Proposition 7. (Tsypkin and Fu, 1993) Let us considered the teleoperation scheme of Fig 4. Assume that environment uncertainties of the form Ξ as well as delay uncertainty are considered ($h = h_0 + \tau$). Then, the maximal allowed delay τ_{max} that preserves robust stability (in the presence of environment uncertainties) can be determined by the following procedure:

Step 1: Draw the Nyquist plot of the nominal system W_0.
Step 2: Define the uncertainty circles as, $\forall\omega \in \Re$:

$$\mathscr{Z}(\omega) = \mathscr{C}[W_0(j\omega), |W_0(j\omega)||W_e(j\omega)|], \quad (19)$$

and plot the "blurred" Nyquist plot.
Step 3: Define Ω the set of ω s.t. $\mathscr{Z}(w)$ intersects $\mathscr{C}[0,1]$ and compute the minimum angle $\theta(\omega)$ from the intersections to the negative real axis. Then:

$$\tau_{max} = \min_{\omega\in\Omega} \frac{\theta(\omega)}{\omega} \quad (20)$$

Remark 8. Note here that, finding C that also solves the robust stability condition (12) is not necessary as the previous Proposition ensures its robust stability w.r.t environment and delay uncertainties.

5. ILLUSTRATIVE EXAMPLE

Consider the following dynamics of the master and the slave manipulators

$$\begin{cases} M_m\dot{v}_m = F_h + u_m \\ M_s\dot{v}_s = -F_e + u_s \end{cases} \quad (21)$$

where v_m and v_s are the angular velocities for the master and the slave respectively, u_m and u_s are the respective motor torques, M_m and M_s are the respective inertias, F_h is the operator torque and F_e is the environment torque.
In order to stabilize the above system, Anderson and Spong (1989) have proposed the following PI control law

$$\begin{cases} u_m = -B_m v_m - B_{s_1}(v_m - v_s) - K_s\int(v_m - v_s)dt \\ u_s = -B_{s_2} v_s - \alpha_f F_e + B_{s_1}(v_m - v_s) + K_s\int(v_m - v_s)dt \end{cases} \quad (22)$$

where $M_m = 0.4kg$, $M_s = 1kg$ $B_m = 3N/m$, $B_{s_2} = 0.2N/m$, $Z_e = 1$, $\alpha_f = 0.5$ and K_s and B_{s_1} are the parameter of the PI controller which must be chosen such that the closed-loop system is stable.
In the presence of communication time delay $h \geq 0$, Niculescu *et al.* (2002) have ensured that for $K_s = 5$ and $B_{s_1} = 2.8$, the closed-loop system is stable for all $h > 0$. However, when the admittance of the environment changes to $Z_e = 2$, the system becomes unstable. In this case, choosing $K_s = 12$ and $B_{s_1} = 2.8$, the closed-loop system is proved to be stable for all $h < 0.3027$ sec.

Based on the above discussion, the master and slave transfer functions with local controllers are given by

$$P_m = \frac{1}{0.4s^2 + 3s + 5}, \quad P_s = \frac{1}{s^2 + 0.2s + 2.8} \quad (23)$$

The impedance of the environment is modelled as $Z_e = B_e s + K_e$, where $0 \leq B_e \leq 2$ and $0 \leq K_e \leq 4$ (the nominal value is $Z_e = s + 2$), which includes the free motion case (without contact).
The nominal communication time-delay is chosen as $h_0 = 1.5sec$.

5.1 *Robust design*

The H_∞ problem to be solved is (16). First the tracking weight W_1 is chosen as : $W_1(s) = \frac{s/M_s+wb}{1+wb\varepsilon}$ with $\varepsilon = 10^{-4}$, $M_s = 1$ and $wb = 0.3$.
Then the weight W_4, representing the robust stability constraint (see (15)), is chosen as represented in Fig 5.

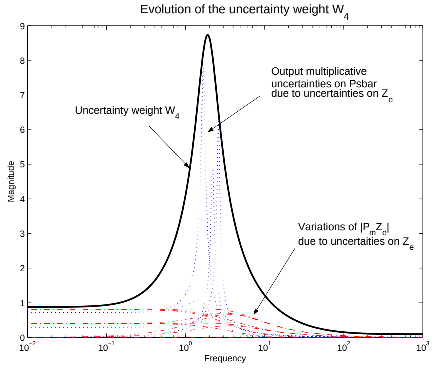

Fig. 5. Uncertainty weight W_4

Solving the H_∞ problem (16) leads to a controller solution C (of order 5) where the solved mixed sensitivity problem is represented in Fig 6.

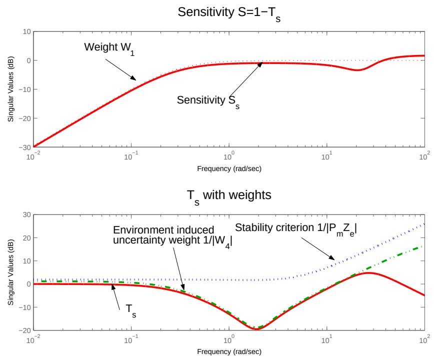

Fig. 6. Sensitivity functions S_s and T_s with weights

5.2 *Stability analysis w.r.t time-varying delay*

Applying the result in Proposition 4, it is shown that the teleoperation scheme in Fig 4 remains stable for all time-varying delay s.t. $0 \leq h(t) \leq 0.23sec.$, as shown in Fig 7.

5.3 *Robustness analysis w.r.t time-delay uncertainties*

Here we consider the case where the delay independent stability condition (6) is not satisfied, e.g. when the environment impedance chosen as $Z_e = B_e s + K_e$ with $0 \leq B_e \leq 6$ and $0 \leq K_e \leq 10$. A controller C has been designed for T_s to achieve nominal performance (2). However the solution of the H_∞ problem leads to $\|W_2 T_s\|_\infty = 1.99$ (for $Z_e = Z_e^{max}$).
Now, using the procedure described in Proposition 7, it will be shown that the system is robustly stable for

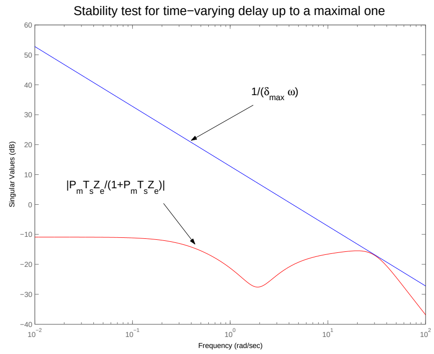

Fig. 7. Stability test w.r.t time-varying delays

all delays s.t. $h = h_0 + \tau_{max}$, where τ_{max} is the delay uncertainty. In Fig 8 the Nyquist plot of the nominal model W_0 is plotted as well as the uncertainty circles (w.r.t environment uncertainties) with the "stability" circle $\mathscr{C}[0,1]$.

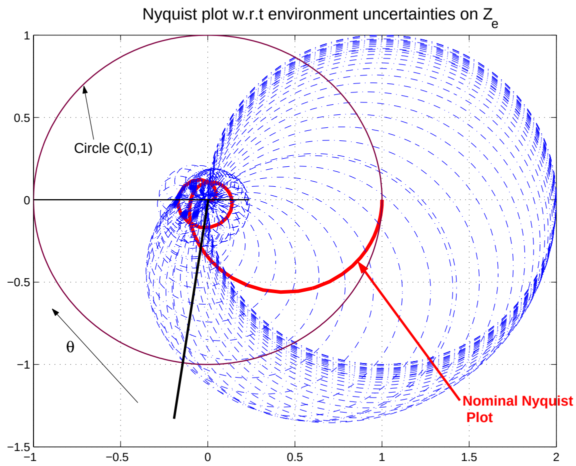

Fig. 8. "Blurred" Nyquist plot of W_τ

Using the method described in Proposition 7, the maximal delay uncertainty that ensures robust stability (w.r.t environment uncertainties) is given by:

$$\tau_{max} = 7.34sec,$$

which proves that the teleoperation scheme will remain stable for all delays up to $h = 8.84sec$.
Notice that, in this case, the previous stability criterion w.r.t time-varying delay gives $d_{max} = 0.19sec$, which is much more conservative.
Let us notice that, applying the previous Proposition 7 with a nominal delay equal to $7sec$ leads to a maximal delay uncertainty that preserves stability equal to $\tau_{max} = 1.60sec$, which proves that this methods is quite independent of the prespecified nominal delay value.

5.4 *Simulation tests*

Here, simulations in time-domain are provided. The environment is s.t $Z_e = B_e s + K_e$ where $0 \leq B_e \leq 2$ and $0 \leq K_e \leq 4$, and the communication time-delay is here time-varying and s.t. $0.9 \leq h \leq 2.1$. A step disturbance of magnitude 0.2 is applied at time $t = 60sec$. that

represents an increase of the external force (i.e of the contact at the environment).
Fig. 9, 10 and 11 show that the slave position pursuits the master position for variable communication time delay and nul, nominal and maximal environment impedance, which proves the robustness of the proposed scheme. Note that a good disturbance attenuation property is obtained.

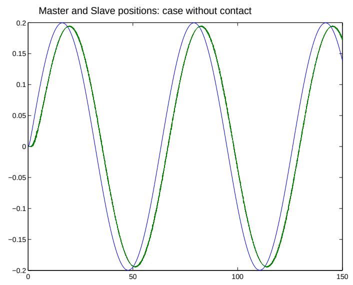

Fig. 9. Master - Slave positions ($F_e = 0$)

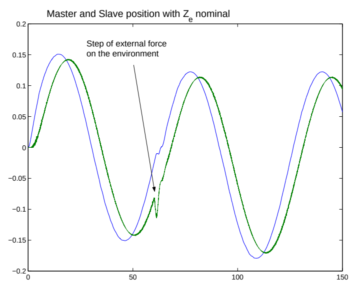

Fig. 10. Master - Slave positions: Z_e nominal

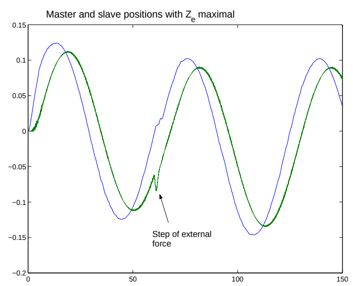

Fig. 11. Master - Slave positions: Z_e maximal

These simulation test emphasize the interest of the proposed approach, where uncertainties in the environment impedance and in the communication time-delay are taken into account to design a robust teleoperation scheme. As a trade-off the teleoperation system behaves quite slowly.

6. CONCLUSIONS

In this paper stability criteria w.r.t constant and time-varying delay have been provided. The robust control design of a bilateral teleoperation system for all communication time delay and in the presence of environment uncertainties has been solved as an H_∞ mixed-sensitivity problem. When delay independent stability is not achieved, the maximal uncertainty (upon a constant communication delay) that keeps stability has been obtained using a Nyquist graphical method. The next step is to deal with control design that ensures robust performance w.r.t both environment and communication delay uncertainties.

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