

APPLICATIONS OF COUPLING ANALYSIS ON BIOREACTOR MODELS

Björn Halvarsson, Pär Samuelsson and Bengt Carlsson

*Division of Systems and Control,
Department of Information Technology,
Uppsala University,
PO Box 337, SE-751 05, Uppsala, Sweden
E-mail: Bjorn.Halvarsson@it.uu.se, Par.Samuelsson@it.uu.se,
Bengt.Carlsson@it.uu.se*

Abstract: In this paper the well-known Relative Gain Array (RGA) and the recently proposed Hankel Interaction Index Array (HIIA) are utilized for quantifying the degree of channel interaction in a multivariable bioreactor model, an activated sludge process configured for nitrogen removal. The HIIA can deal with plant structures where the RGA fails and can furthermore also be used to evaluate multivariable controller structures. It was found that the RGA method was unable to give reasonable input-output pairing suggestions in some cases while the HIIA method provided useful information in all of the considered cases. *Copyright ©2005 IFAC*

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1. INTRODUCTION

Many systems to be controlled are multivariable. This means that they have both multiple inputs as well as multiple outputs. Such systems are called multiple-input multiple-output (MIMO) systems. Compared to single-input single-output (SISO) systems, the control design procedure for MIMO systems is more elaborate. One reason for this is that different parts of a multivariable system may interact and cause couplings in the system. This means that a change in one input affects several outputs.

Often, an easy way to control a fairly decoupled MIMO system is to use a multi-loop strategy, i.e. to separate the control problem into several single-loop SISO systems and then use conventional SISO control on each of the loops, see Kinnaert (1995) and Wittenmark *et al.* (1995).

In real-life applications the considered MIMO system can be rather complex: in the chemical process indus-

try a complexity of several hundred control loops is not unusual, see Wittenmark *et al.* (1995). Often, it is not obvious how to choose a proper input-output pairing or the structure of a multivariable controller. The choice of pairing is crucial, since a bad choice may give unstable systems even though each loop separately is stable. This problem can arise due to interaction between the different loops. Generally, the stronger the interactions, the harder it is to obtain satisfactory control performance using a multi-loop strategy. Evidently, there is a need for a measure that can give advice when solving the pairing problem and that also quantifies the level of interaction occurring in the system.

One such measure is the well-known Relative Gain Array (RGA) developed by Bristol (1966). The RGA considers steady-state properties of the plant and gives a suggestion on how to solve the pairing problem in the case of a decentralized (diagonal) controller structure. It also indicates which pairings should be

avoided due to possible stability and performance problems.

A somewhat different approach was employed by Conley and Salgado (2000) when considering observability and controllability Gramians to measure channel interaction. In this way, the full dynamics of the considered system are incorporated in one single measure. Recently, Wittenmark and Salgado (2002) refined this work and proposed a new measure for channel interaction, the Hankel Interaction Index Array (HIIA). This measure seems to be able to overcome most of the disadvantages that the RGA possesses. The Gramian based approach is further discussed in Salgado and Conley (2004).

In this paper, the RGA and the HIIA will be employed in the selection of input–output signal pairings for a part of a MIMO bioreactor system: an activated sludge process configured for nitrogen removal. Modelling and control of the activated sludge process have been an intense research area in the last decade, see for example Olsson (1993), Lindberg and Carlsson (1996), Alex *et al.* (1999), Vanrolleghem *et al.* (1999), Samuelsson and Carlsson (2001), Yuan *et al.* (2002) and Jeppson and Pons (2004). The results from the RGA analysis will be compared with those of the HIIA and with results obtained from physical insights of the considered system. It is also discussed what additional conclusions that can be drawn from the HIIA analysis.

2. THE RELATIVE GAIN ARRAY (RGA)

The RGA for a quadratic plant is given by

$$\text{RGA}(\mathbf{G}) = \mathbf{G}(0) .* (\mathbf{G}(0)^{-1})^T \quad (1)$$

where $\mathbf{G}(0)$ is the steady-state transfer function matrix and “.*” denotes the Hadamard or Schur product (i.e. elementwise multiplication). Each element in the RGA can be regarded as the quotient between the open-loop gain and the closed-loop gain. The RGA element (i, j) is hence the quotient between the gain in the loop between input j and output i when all other loops are open and the gain in the same loop when all other loops are closed. For a full derivation of the RGA, see e.g. Bristol (1966), Kinnaert (1995) or Skogestad and Postlethwaite (1996).

In the case of a 2×2 system, the following RGA matrix is obtained:

$$\text{RGA}(\mathbf{G}) = \begin{bmatrix} \lambda & 1 - \lambda \\ 1 - \lambda & \lambda \end{bmatrix}. \quad (2)$$

Depending on the value of λ , five different cases occur (see Kinnaert (1995)); the main conclusion is to select a pairing so that the relative gain is positive and as close to one as possible. Negative pairings should definitely be avoided.

As previously seen, the RGA provides a very simple way of characterizing interactions present in a MIMO

linear system. The RGA gives a suggestion on how to pair the input and output signals if a *decentralized* controller is to be used. It may also give warnings in terms of large RGA elements when stability and robustness problems may occur.

However, the RGA suffers from at least two main disadvantages (as will be illustrated later in this paper):

- (1) The RGA only considers one separate frequency;
- (2) The RGA fails to give reliable information in the case of triangular plants.

Concerning the first of these drawbacks, it would of course be better to have an interaction measure that considers information given by all the interesting frequencies.

When dealing with triangular plants the RGA fails to give reliable information: In this case the RGA does not indicate the presence of couplings through off-diagonal elements: a triangular plant either gives an RGA that equals the identity matrix or the anti-identity matrix. In Kinnaert (1995) it is mentioned that some authors do not regard this as being a drawback since the RGA still gives the *best possible* decentralized controller structure. This is certainly true, but if the objective is to find the best possible controller among all controller structures – MIMO controllers included – then this feature of the RGA is a clear drawback.

3. THE HANKEL INTERACTION INDEX ARRAY (HIIA)

In the previous section it was seen that the RGA suffers from some important disadvantages. In the light of this, Conley and Salgado (2000) proposed a new interaction measure based on Gramians, able to handle both of the above-mentioned pitfalls. Recently, a modified version of the interaction measure was suggested by Wittenmark and Salgado (2002) where the Hankel norm is used.

Consider a linear system, with inputs given by the $n \times 1$ vector $\mathbf{u}(t)$ and outputs given by the $p \times 1$ vector $\mathbf{y}(t)$. The system can be described as a state-space realization

$$\begin{aligned} \dot{\mathbf{x}}(t) &= \mathbf{A}\mathbf{x}(t) + \mathbf{B}\mathbf{u}(t) \\ \mathbf{y}(t) &= \mathbf{C}\mathbf{x}(t) + \mathbf{D}\mathbf{u}(t) \end{aligned} \quad (3)$$

where \mathbf{A} , \mathbf{B} , \mathbf{C} and \mathbf{D} are matrices of dimension $n \times n$, $n \times m$, $p \times n$ and $p \times m$, respectively. $\mathbf{x}(t)$ is the state vector. The controllability Gramian, $\mathbf{\Gamma}_c$, and the observability Gramian, $\mathbf{\Gamma}_o$, for the system given in (3) are defined as

$$\mathbf{\Gamma}_c = \int_0^\infty e^{\mathbf{A}\tau} \mathbf{B} \mathbf{B}^T e^{\mathbf{A}^T \tau} d\tau \quad (4)$$

$$\mathbf{\Gamma}_o = \int_0^\infty e^{\mathbf{A}^T \tau} \mathbf{C}^T \mathbf{C} e^{\mathbf{A}\tau} d\tau. \quad (5)$$

These are measures of how hard it is to control and to observe the states of the given system. As shown by Conley and Salgado (2000), it is possible to split the system given by $(\mathbf{A}, \mathbf{B}, \mathbf{C}, \mathbf{D})$ into fundamental subsystems $(\mathbf{A}, \mathbf{B}_j, \mathbf{C}_i, \mathbf{D}_{ij})$ where \mathbf{B}_j is the j :th column in \mathbf{B} , \mathbf{C}_i is the i :th row in \mathbf{C} and \mathbf{D}_{ij} is the (i, j) :th element of \mathbf{D} . Then for each of these, the controllability and the observability Gramian can be calculated. The controllability and observability Gramians for the full system will then be the sum of the Gramians for all the subsystems.

Unfortunately, both the controllability and the observability Gramian will depend on the chosen state-space realization. However, the eigenvalues of the product of these will not.

The Hankel norm for a system with transfer function $\mathbf{G}(s)$ is defined as

$$\|\mathbf{G}(s)\|_H = \sqrt{\lambda_{max}(\mathbf{\Gamma}_c \mathbf{\Gamma}_o)} = \sigma_1^H \quad (6)$$

where σ_1^H is the maximum Hankel singular value. Hence, this measure is invariant with respect to the state-space realization and it is therefore well suited as a combined measure for controllability and observability. In Wittenmark and Salgado (2002) it is shown that the Hankel norm of $\mathbf{G}(s)$ given in (6) can also be interpreted as a gain between past inputs and future outputs. Then, if the Hankel norm is calculated for each fundamental subsystem and arranged in a matrix $\tilde{\Sigma}_H$ given by

$$[\tilde{\Sigma}_H]_{ij} = \|\mathbf{G}_{ij}(s)\|_H \quad (7)$$

this matrix can be used as an interaction measure. In Wittenmark and Salgado (2002) a normalized version, the Hankel Interaction Index Array (HIIA), is proposed:

$$[\Sigma_H]_{ij} = \frac{\|\mathbf{G}_{ij}(s)\|_H}{\sum_{kl} \|\mathbf{G}_{kl}(s)\|_H}. \quad (8)$$

With this normalization, the sum of all elements in Σ_H is one. If the intention is to use a decentralized controller then the HIIA can be used and interpreted in the same way as the RGA. Even though not directly stated by Wittenmark and Salgado (2002), expected performance for different controller structures can certainly be compared by summing the elements in Σ_H : Clearly, due to the normalization, the aim is to find the simplest controller structure that gives a sum as near one as possible. In the slightly different interaction measure, the participation matrix (PM), proposed by Conley and Salgado (2000) this is used. See Salgado and Conley (2004) for a further discussion of the PM.

When $\mathbf{G}_{ij} = 0$ the Gramian product, $\mathbf{\Gamma}_c^{(j)} \mathbf{\Gamma}_o^{(i)}$, will be zero and so will the corresponding element in the matrix Σ_H . This implies that the structure of Σ_H will be the same as the structure of \mathbf{G} and thus, non-diagonal elements will not be hidden as in the case of the RGA. Hence, the HIIA can also be used to evaluate other controller structures than just the diagonal, decentralized, ones.

4. THE BIOREACTOR MODEL

In the complex process of wastewater treatment, many different cause-effect relationships exist, and therefore, there are many possible choices of input and output signals, see Olsson and Jeppsson (1994). Consequently, this can motivate the study of wastewater treatment plant models with respect to the selection of input and output signals.

From a theoretical point of view, the bioreactor models are non-linear multivariable systems that may contain a significant degree of coupling. Hence, this also gives an interesting opportunity to test the performance of the methods for input-output pairing selection discussed in the previous sections.

The objective in this paper is to find suitable control structures. If the couplings between the different control handles in the system are sufficiently low, then a controller selection involving several decoupled SISO controllers may be suitable. If this is not the case, a MIMO controller structure will provide a better solution. The MIMO solution will, however, generally be much more complex. Both the RGA and the HIIA method will be used in the sequel.

The considered model is a simplified version of the IAWQ Activated Sludge Model No. 1 (ASM1) that models an activated sludge process configured for nitrogen removal. ASM1 is thoroughly described by Henze *et al.* (1987). In this study the bioreactor consists of two tanks of equal volume (one anoxic and one aerobic of 1000 m³ each) and a settler, see Figure 1. The influent flow rate, Q , is 18446 m³/day. The model is valid in the medium time-scale (i.e. hours to days). For a discussion of the model parameters, see Halvarsson (2003).

Two different processes, nitrification and denitrification, are simultaneously being performed. To get an indication of how well these processes are being performed the effluent ammonium concentration ($S_{NH,2}(t)$) and the nitrate concentration ($S_{NO,2}(t)$), respectively, can be considered. Hence, these concentrations are selected as output signals. The considered input signals are the concentration of dissolved oxygen (DO set point, $S_{O,2}(t)$) in the aerobic compartment and the internal recirculation flow rate ($Q_i(t)$). According to Ingildsen (2002) the denitrification is mainly influenced by $Q_i(t)$ (among the selected input signals) while the nitrification is mainly influenced by $S_{O,2}(t)$. Hence, if the couplings between

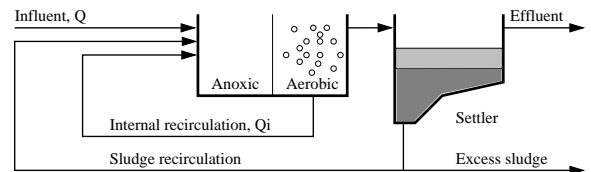


Fig. 1. A basic activated sludge process (ASP) configured for nitrogen removal.

$Q_i(t)$ and $S_{O,2}(t)$ are low, then the denitrification and the nitrification process may be considered separately when choosing controller structure and thus, SISO controllers may be selected.

Three different operating points were selected¹. These correspond to the input signals:

- $\mathbf{u}_1 = [10000 \text{ m}^3/\text{day} \quad 2 \text{ mg/l}]^T$,
- $\mathbf{u}_2 = [36892 \text{ m}^3/\text{day} \quad 2 \text{ mg/l}]^T$,
- $\mathbf{u}_3 = [50000 \text{ m}^3/\text{day} \quad 2 \text{ mg/l}]^T$.

Since both the RGA and the HIIA are defined for linear models, the simplified ASM1 model was linearized around each operating point using the MATLAB function *linmod*. In a small neighbourhood of each operating point the linearized model will mimic the characteristics of the nonlinear system. Thus, the analysis in the following sections is strictly valid only in the above mentioned neighbourhoods. However, as can be seen in the lower part of Figure 2, the operational maps can be divided into two different regions where the process shows different stationary characteristics. It is therefore probable that each operating point describes the corresponding area fairly well.

The obtained linear models can be represented in standard state-space form as:

$$\begin{aligned} \Delta \dot{\mathbf{x}}(t) &= \mathbf{A} \Delta \mathbf{x}(t) + \mathbf{B} \Delta \mathbf{u}(t) \\ \Delta \mathbf{y}(t) &= \mathbf{C} \Delta \mathbf{x}(t) \end{aligned} \quad (9)$$

where $\mathbf{x}(t)$ is the state vector given by

$$\mathbf{x}(t) = [S_{\text{NH},1}(t) \quad S_{\text{NH},2}(t) \quad S_{\text{NO},1}(t) \quad S_{\text{NO},2}(t) \quad S_{\text{S},1}(t) \quad S_{\text{S},2}(t)]^T \quad (10a)$$

where the elements are the concentrations of ammonium ($S_{\text{NH},n}$), nitrate ($S_{\text{NO},n}$) and readily biodegradable substrate ($S_{\text{S},n}$) in compartment n in the bioreactor. The operator Δ refers to the deviation from the operating point. For a more thorough description see Halvarsson (2003). The input signal vector $\mathbf{u}(t)$ is given by:

$$\mathbf{u}(t) = \begin{bmatrix} Q_i(t) \\ S_{O,2}(t) \end{bmatrix} \quad (10b)$$

and the output signal vector is:

$$\mathbf{y}(t) = \begin{bmatrix} S_{\text{NH},2}(t) \\ S_{\text{NO},2}(t) \end{bmatrix} \quad (10c)$$

and

$$\mathbf{C} = \begin{bmatrix} 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \end{bmatrix}. \quad (10d)$$

The steady-state operational maps for the model, are shown in Figure 2. The output signals, $S_{\text{NH},2}(t)$ and $S_{\text{NO},2}(t)$ are plotted against the two input signals $S_{O,2}(t)$ and $Q_i(t)$.

The operational maps in Figure 2 clearly indicate that different controller structures should be used in the

different operating points, at least in the lower operating point, \mathbf{u}_1 , compared to the upper operating points, \mathbf{u}_2 and \mathbf{u}_3 . Note, however, that these operational maps can only be used to give an indication of the interactions in the system.

5. RGA ANALYSIS

The steady-state RGA matrices for the linearized model in the three operating points are:

$$\text{RGA}(\mathbf{G}_{\mathbf{u}_3}(0)) = \begin{bmatrix} 0.0055 & 0.9945 \\ 0.9945 & 0.0055 \end{bmatrix} \quad (11a)$$

$$\text{RGA}(\mathbf{G}_{\mathbf{u}_2}(0)) = \begin{bmatrix} 0.0051 & 0.9949 \\ 0.9949 & 0.0051 \end{bmatrix} \quad (11b)$$

$$\text{RGA}(\mathbf{G}_{\mathbf{u}_1}(0)) = \begin{bmatrix} 0.0041 & 0.9959 \\ 0.9959 & 0.0041 \end{bmatrix}. \quad (11c)$$

Apparently, the RGA suggests the anti-diagonal pairing $S_{\text{NH},2}(t)$ - $S_{O,2}(t)$ and $S_{\text{NO},2}(t)$ - $Q_i(t)$ for all of the three operating points. This contradicts the results from the operational maps in Figure 2.

6. HIIA ANALYSIS

The HIIA is a scaling dependent tool. This motivates a scaling of the systems before the HIIA is considered. A reasonable scaling procedure is to scale the systems so that the maximum deviation from the average point of the considered variables lies in the interval $[-1, 1]$ (for a detailed description of this scaling procedure, see Halvarsson (2003)). If all of the three operating

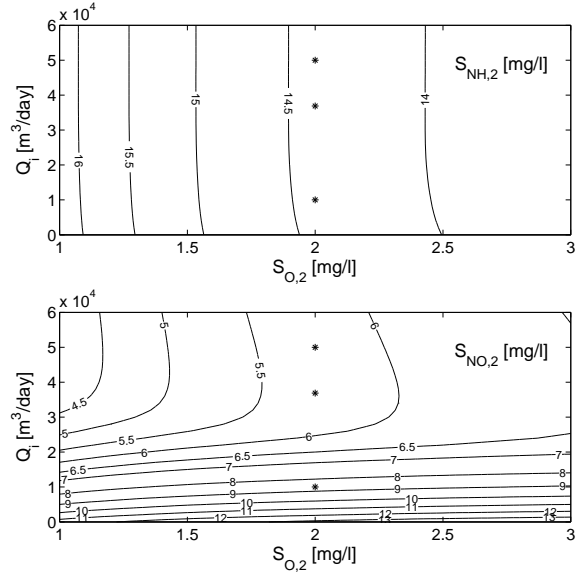


Fig. 2. Steady-state operational maps for the considered bioreactor model. The upper plot shows the level curves for the first output signal, the outgoing ammonium concentration, $S_{\text{NH},2}$, and the lower one shows the outgoing nitrate concentration, $S_{\text{NO},2}$. The operation points are indicated in the plots.

¹ These operating points do not necessarily correspond to feasible choices concerning an optimal operation of the plant. Instead, these are chosen in order to illustrate different interaction points.

points are scaled in the same way the following steady state transfer function matrices are obtained:

$$\mathbf{G}_{\mathbf{u}_3}^{scaled}(0) = \begin{bmatrix} 0.0004 & -0.7048 \\ 0.0748 & 0.6674 \end{bmatrix} \quad (12a)$$

$$\mathbf{G}_{\mathbf{u}_2}^{scaled}(0) = \begin{bmatrix} -0.0001 & -0.7050 \\ -0.0135 & 0.6422 \end{bmatrix} \quad (12b)$$

$$\mathbf{G}_{\mathbf{u}_1}^{scaled}(0) = \begin{bmatrix} -0.0313 & -0.7069 \\ -4.9176 & 0.4526 \end{bmatrix}. \quad (12c)$$

Furthermore, since the HIIA is a dynamic measure that considers all possible frequencies while the considered model is only valid in a limited frequency band it is also reasonable to perform a band-pass filtering before calculating the HIIA. This was carried out using a simple first-order low-pass filter, $F(s)$, given by:

$$F(s) = \frac{0.001}{s + 0.001} \quad (13)$$

where s is the Laplace-variable. This filter has a 3 dB cut-off frequency at approximately 10^{-3} rad/s which is reasonable since the considered bioreactor model is valid for frequencies ranging from approximately 10^{-5} rad/s up to 10^{-3} rad/s. Note also that this filter does not introduce any additional scaling in the steady state. The filtering can be expressed as:

$$\mathbf{G}^{filtered} = \mathbf{G}F. \quad (14)$$

If the systems are scaled in the suggested way and filtered using the low-pass filter F given in (13) before the HIIA is calculated, then the following HIIA matrices, Σ_H , are obtained for the three operating points:

$$\Sigma_H^{\mathbf{u}_3} = \begin{bmatrix} 0.0003 & 0.4869 \\ 0.0517 & 0.4611 \end{bmatrix} \quad (15a)$$

$$\Sigma_H^{\mathbf{u}_2} = \begin{bmatrix} 0.0001 & 0.5181 \\ 0.0099 & 0.4719 \end{bmatrix} \quad (15b)$$

$$\Sigma_H^{\mathbf{u}_1} = \begin{bmatrix} 0.0052 & 0.1157 \\ 0.8050 & 0.0741 \end{bmatrix}. \quad (15c)$$

If a decentralized controller structure is to be used, the HIIA analysis suggests the same input-output pairings as the RGA, i.e the anti-diagonal pairing in all of the considered operating points. However, since $[\Sigma_H]_{22}$ is large for \mathbf{u}_2 and \mathbf{u}_3 this indicates that $S_{O,2}$ affects both outputs, $S_{NH,2}$ and $S_{NO,2}$. This in turn means that the suggested decentralized controller structure could be insufficient to provide good control performance. Instead, improved control performance can be expected if a (multivariable) triangular controller structure that also includes the impact $S_{O,2}$ has on $S_{NO,2}$ is used.

In the lowest operating point, \mathbf{u}_1 , the HIIA also suggests a triangular controller structure, even though not as strongly as for \mathbf{u}_2 and \mathbf{u}_3 . In fact, in this operating point a decentralized controller may be good enough since the sum of the anti-diagonal HIIA elements is 0.9207 which is close to one.

Concerning the scaling procedure, it was found that reasonable small changes in the scaling matrices (for instance, $\pm 40\%$ in the element that scales Q_i) do not alter the HIIA recommendations.

7. DISCUSSION

In the RGA analysis of the bioreactor model it was seen that the RGA method did not provide reasonable input-output pairings in all of the considered operating points. The reason for this can be found if the steady-state gain matrices for the considered systems are studied. Triangular systems will always give the same RGA, namely the identity matrix (under the assumption that the rows in the transfer function matrix are permuted to get nonzero elements along the diagonal before calculating the RGA). The transfer function matrices of the (scaled) model are almost right under triangular, see (12a)–(12c). Therefore, the structure of the RGA will be similar for all of them: almost the anti-identity matrix. The RGA matrices are given in equations (11a)–(11c), and evidently they are all very close to the anti-identity matrix.

Obviously, the HIIA provides an interaction analysis that goes deeper than the RGA is able to. When considering the information given by the HIIA there is no longer any contradiction with the steady-state results in the operational maps in Figure 2. This can also be seen as a confirmation that the applied scaling procedure is reasonable. Note once again, that these steady-state operational maps can merely be used to give an indication of the interactions in the system, and what a reasonable controller structure may look like.

Compared to the RGA, the HIIA possesses several advantages. Evidently, the HIIA is able to deal with special transfer function matrix structures such as the analysed nearly triangular ones. The HIIA does not require decentralized (diagonal) controller structures as the RGA does. Instead, the HIIA considers each subsystem in the model independently. Therefore, the HIIA can be used to suggest MIMO controller structures as seen in Section 6. The RGA method is unable to do this.

It was also observed that the HIIA method is scaling-dependent. This means that some effort must be spent on finding proper scaling matrices. However, this is not necessarily a drawback, since this gives an opportunity for the user to weight the considered signals according to his own choice. The RGA method is scaling-independent and does not offer this possibility.

Based on the RGA results in this particular case, it should not be concluded that the couplings are low between the DO set point ($S_{O,2}(t)$) and the internal recirculation flow rate ($Q_i(t)$) independent of operation point. Instead, the operational maps indicate that there are some couplings between the nitrification and the denitrification process. A MIMO controller structure can therefore be expected to give better control performance compared to a solution involving decentralized control. The HIIA analysis supports this view, and also suggests possible controller structure selections.

8. CONCLUSIONS

The RGA method provides a simple way to decide how a set of input signals should be utilized to control a given set of output signals. Often this method performs well, but in the analysis of the considered bioreactor model, it was clearly seen that the RGA method does not work properly in all cases. The reason for this was found to be the almost triangular structure of the transfer function matrices. From this it can be concluded that the RGA should be used with care. It is advisory to include an examination of the structure of the considered transfer function matrices in the RGA analysis.

Furthermore, the newly suggested HIIA method was employed to quantify the level of interactions occurring between the inputs and outputs in the considered bioreactor systems. It was shown that for the HIIA method to give reasonable information, the considered systems had to be both scaled in a physically relevant way and low-pass filtered. The filtering was performed to select the frequency band of interest. When treating the systems according to this procedure, the HIIA method suggested the same decentralized controller structure as the RGA, but it also gave suggestions on other controller structures that may perform better. The RGA is unable to give this extra information.

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