REJECTION OF UNKNOWN SINUSOIDAL DISTURBANCES IN NON-MINIMUM-PHASE NONLINEAR SYSTEMS

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Abstract: This paper deals with disturbance rejection with stability based on the estimated state and disturbances. The unknown disturbances are combination of sinusoidal disturbances with unknown frequencies, unknown phases and amplitudes. The only information of the unknown disturbances is the number of distinctive frequencies inside. The class of nonlinear systems considered in this paper consists of nonlinear systems in the output feedback form and the systems are nonminimum phase, ie, with unstable zero dynamics. Based on the adaptively estimated disturbances, a new control design is proposed for stabilization and disturbance rejection of the nonlinear system in output feedback form which has a nonminimum phase zero.

Keywords: Disturbances rejection, Non-minimum-phase systems, Nonlinear systems, Adaptive control, Uncertainty

1. INTRODUCTION

In engineering systems, there are deterministic disturbances, apart from random disturbances. Among the various types of deterministic disturbances, sinusoidal disturbances have attracted a large amount of research interests, from the estimation of the disturbance frequencies to the compensation or rejection of disturbances. It was until fairly recently that a global convergent estimation algorithm was proposed for estimation of a single frequency of the stand alone sinusoidal signal (Hsu *et al.*, 1999), and more recently an algorithm was proposed to estimate multiple frequencies from a sinusoidal signal using adaptive observers (Marino and Tomei, 2002). On the other hand, a series of results have been published for rejecting disturbances of unknown frequencies (Bodson *et al.*, 1994; Bodson and Douglas, 1997; Marino *et al.*, 2003). Two algorithms, a direct and an indirect one, are presented in (Bodson and Douglas, 1997) for disturbance compensation for stable linear time invariant systems. The indirect one estimates the disturbance frequency first and then to compensate it. Only the direct one ensures the complete compensation or asymptotic rejection of disturbances with unknown frequencies. The algorithm shown in (Marino *et al.*, 2003) ensures robust compensation of unknown disturbances for linear systems. For nonlinear systems, a result for strict feedback nonlinear system is shown in (Nikiforov, 1998) based on full state feedback. For nonlinear systems using output feedback, global rejection with stabilization is reported in (Ding, 2003)for minimum phase nonlinear systems in output feedback form.

This paper deals with asymptotic rejection of unknown sinusoidal disturbances for nonlinear systems in the output feedback form. The system is allowed to be nonminimum phase, and the stabilization of the system is considered together with the disturbance rejection. An indirect approach in design is adopted, with separate stages of estimation of disturbances and control design for disturbance rejection and stabilization. The control design makes use of the recent result (Ding, 2005) for exponentially convergent estimate for unknown sinusoidal disturbances in nonminimum phase nonlinear systems in the output feedback form. The estimated disturbance and frequencies asymptotically converge to their ideal values. The control design makes use of the estimated disturbances, and re-estimate the state variables. The re-estimation is needed to reduce the involvement of the variables in the filters for disturbance estimation in the differentiation and therefore simplify the control design. If the system is linear, then the control input can directly designed without the re-estimation as the control input is directly based on the estimates of state without the involvement of differentiation. In the control design, a restriction is imposed on the number of nonminimum phase zeros. The proposed control design allows only one nonminimum phase zero. This restriction is not due to the estimation or disturbance rejection methods proposed or used. It is due to the fact that there are very few control design methods even just for the stabilization of nonminimum phase nonlinear systems. As there are a quite number of filters are involved in the estimation and control design, an example is included in the paper to demonstrate the actual filters and observers used in the estimation and control. The simulation results for the demonstrated example are also included.

2. PROBLEM FORMULATION

Consider a single-input-single-output nonlinear system which can be transformed into the output feedback form

$$\begin{aligned} \dot{x} &= A_c x + \phi(y) + b(u - \mu) \\ y &= Cx \end{aligned} \tag{1}$$

with

$$A_c = \begin{bmatrix} 0 & 1 & 0 & \dots & 0 \\ 0 & 0 & 1 & \dots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & 0 & \dots & 1 \\ 0 & 0 & 0 & \dots & 0 \end{bmatrix}, \quad C = \begin{bmatrix} 1 \\ 0 \\ \vdots \\ 0 \end{bmatrix}^T, b = \begin{bmatrix} 0 \\ \vdots \\ 0 \\ b_\rho \\ \vdots \\ b_n \end{bmatrix}$$

where $x \in R^n$ is the state vector, $u \in R$ is the control, ϕ, is a known nonlinear smooth vector field in R^n with $\phi(0) = 0$, $\mu \in R$ is a matched disturbance which is generated from an unknown exosystem

$$\begin{aligned} \dot{w} &= Sw \\ \mu &= l^T w \end{aligned} \tag{2}$$

with $w \in R^s$.

Remark 1. The coordinate-free geometric conditions for the existence of state transform for transforming a nonlinear system into (1) are specified in (Marino and Tomei, 1993). $b_\rho \neq 0$ indicates the nonlinear system before the transformation has a constant relative degree of ρ.

Assumption 1. The system has one nonminimum-phase zero, i.e., $\mathbf{B}(s) = \sum_{i=\rho}^{m} b_i s^{n-i} = (s + \beta_0)\sum_{i=1}^{m} \beta_i s^{m-i}$ with $m = n - \rho$, where $\beta_0 < 0$ and $\sum_{i=1}^{m} \beta_i s^{m-i}$ is Hurwitz.

Assumption 2. The eigenvalues of S are with zero real parts and are distinct.

Remark 2. Assumption 2 ensures that the disturbances are combination of sinusoidal signals including constant bias. The dimension of S decides the number of independent frequencies in the disturbances. It follows the assumption made on unknown exosystems in (Nikiforov, 1998; Serrani and Isidori, 2000; Ding, 2003). Unlike the neutral stable assumptions on exosystems in (Isidori and Byrnes, 1990; Isidori, 1995; Byrnes *et al.*, 1997), the dynamics are not assumed to be known.

Remark 3. As shown in (Ding, 2003), the unmatched disturbances in the nonlinear systems in the output feedback form can be transformed to the matched case of (1), if Assumption 2 is satisfied. In this paper, only the matched disturbance is considered for the convenience of presentation.

The stabilization problem solved in this paper is to use the exponentially convergent estimates of disturbances and the state to design a feedback control which ensures the overall stability the feedback control system and the output converges to zero.

3. PRELIMINARY RESULTS

As shown in (Ding, 2005), the disturbance and state can be estimated using the following filters. Define

$$\dot{p} = (A_c + kC)p + \phi(y) + bu - ky \tag{3}$$

$$\dot{\xi} = F\xi + G(p_1 - y) \tag{4}$$

$$\dot{\zeta} = F\zeta + G\hat{\psi}_1^T \xi \tag{5}$$

$$\dot{\hat{\psi}}_1 = \Gamma\xi(\xi - \zeta)^T PG \tag{6}$$

where $p \in R^n$, $k \in R^n$ is chosen so that $A_c + kC$ is Hurwitz, $\{F, G\}$ is a controllable pair, Γ is a positive definite matrix, and P is the positive definite matrix satisfying

$$PF + F^T P = -2I_s \tag{7}$$

Define $\hat{\psi}_i$, $i = 2, \ldots, \rho$,

$$\hat{\psi}_i^T = \hat{\psi}_{i-1}^T (F + G\hat{\psi}_1^T) + k_{i-1}\psi_1^T, \tag{8}$$

and

$$\begin{bmatrix} \hat{\psi}_{\rho+1}^T \\ \vdots \\ \hat{\psi}_n^T \end{bmatrix} = \hat{\psi}_z^T - \sum_{i=1}^{\rho} B^{\rho-i}\bar{b}\hat{\psi}_i^T \tag{9}$$

with B and $\bar{b}$ being given by

$$B = \begin{bmatrix} -b_{\rho+1}/b_\rho & 1 & \ldots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ -b_{n-1}/b_\rho & 0 & \ldots & 1 \\ -b_n/b_\rho & 0 & \ldots & 0 \end{bmatrix}, \quad \bar{b} = \begin{bmatrix} b_{\rho+1}/b_\rho \\ \vdots \\ b_n/b_\rho \end{bmatrix}$$

where

$$\mathrm{vec}(\hat{\psi}_z) = \frac{|\hat{\Sigma}|}{\sigma + |\hat{\Sigma}|^2}\mathrm{adj}(\hat{\Sigma})\mathrm{vec}(\hat{\psi}_1 k_z^T) \tag{10}$$

with

$$\hat{\Sigma} = (F + G\hat{\psi}_1^T)^T \otimes I_{(n-\rho)} - I_s \otimes B \tag{11}$$

$$\dot{\sigma} = -\lambda_\sigma \sigma, \qquad \sigma(0) = \sigma_0 \tag{12}$$

for some positive reals λ_σ and σ_0. The notations $|\cdot|$ and $\mathrm{adj}(\cdot)$ are used to denote the determinant and the adjoint matrix of a matrix respectively. The following theorem summarize the results of the disturbance and state estimation.

Theorem 3.1 Based on the filters (4), (5), (6) and estimates shown in (8) and (10), the estimates of the state and the disturbance of (1) are given by

$$\hat{x} = p + \hat{\psi}^T \xi \tag{13}$$

$$\hat{\mu} = \hat{\psi}_u^T \xi \tag{14}$$

where

$$\hat{\psi}^T = [\hat{\psi}_1, \ \ldots, \ \hat{\psi}_n]^T \tag{15}$$

$$\hat{\psi}_u^T = \frac{1}{b_\rho}[\hat{\psi}_{\rho+1}^T - \hat{\psi}_\rho^T (F + G\hat{\psi}_1^T) - k_\rho \psi_1^T] \tag{16}$$

and the estimate of exosystem matrix $F + G\psi_1^T$ is given by

$$\hat{F}_o = F + G\hat{\psi}_1^T \tag{17}$$

There exist positive real constants λ_x, d_x, λ_μ, d_μ, λ_F, and d_F such that

$$\|x(t) - \hat{x}(t)\| \leq d_x e^{-\lambda_x t} \tag{18}$$

$$\|\mu(t) - \hat{\mu}(t)\| \leq d_\mu e^{-\lambda_\mu t} \tag{19}$$

$$\|F_o - \hat{F}_o(t)\| \leq d_F e^{-\lambda_F t} \tag{20}$$

Proof. See (Ding, 2005).

4. STABILIZATION WITH DISTURBANCE REJECTION OF NONLINEAR SYSTEMS

When $\phi(y)$ is a genuine nonlinear term, the control design presented in the above section will not be able to ensure the stability of the closed loop control system. The observer backstepping (Krstic *et al.*, 1995) is a powerful tool to deal with the control design of nonlinear systems in the output feedback form, but it does not apply to the system (1) considered in this paper, as it is nonminimum phase, not even for the case that there is no disturbances.

To deal with the nonminimum phase, a state transform is introduced with the resultant system shown in the following lemma.

Lemma 4.1: Consider a state transform defined by

$$r = T^{-1}x \tag{21}$$

with

$$T = \beta_0 I + A_c$$

Then in r-coordinate, the system model (1) is described by

$$\begin{aligned} \dot{r} &= A_c r + \phi_r(y) + \begin{bmatrix} 0_{\rho\times 1} \\ \beta \end{bmatrix} (u - \mu), \\ y &= C_r r \end{aligned} \tag{22}$$

where $C_r = [\beta_0, 1, 0, \ldots, 0]$, $\phi_r(y) = T^{-1}\phi(y)$ and $\beta = [\beta_1, \ldots, \beta_m]^T$.

Proof: From $\sum_{i=\rho}^m b_i s^{n-i} = (s + \beta_0)\sum_{i=1}^m \beta_i s^{m-i}$, it is straightforward to verify that

$$\begin{bmatrix} 0_{(\rho-1)\times 1} \\ b \end{bmatrix} = T \begin{bmatrix} 0_{\rho\times 1} \\ \beta \end{bmatrix}. \tag{23}$$

It is also easy to check that

$$[\beta_0, 1, 0, \ldots, 0] = C_r T. \tag{24}$$

To complete the proof, one only needs to show

$$A_c = T^{-1} A_c T \tag{25}$$

which is equivalent to $TA_c = A_c T$. From the structure of T, it follows that

$$TA_c = (\beta_0 I + A_c)A_c = A_c(\beta_0 I + A_c) = A_c T. \tag{26}$$

The control design is then carried with the system (22). Although the exponentially converging estimates of states and disturbances are available for control design as shown in (14), they are not in the convenient form for control design using observer backstepping. With the estimated disturbances, it is now able to design an observer

$$\dot{\hat{r}} = A_o\hat{r} + \phi_r(y) + [0_{1\times\rho}, \beta^T]^T(u - \hat{\mu}) - k_r y \quad (27)$$

where $A_o = A_c + k_r C_r$ with $k_r \in R^n$ being chosen such that A_o is Hurwitz.

For the observer error $e_r = r - \hat{r}$, it can be obtained that

$$\dot{e}_r = A_o e_r + [0_{1\times\rho}, \beta^T]^T(\hat{\mu} - \mu) \quad (28)$$

Backstepping will be used to design control input based on the transformed system (22). Since the first state variable of (22), r_1 is not available, the backstepping design will start from its estimate $\hat{r}_1$. In the following, define

$$z_1 = \hat{r}_1 \quad (29)$$

$$z_i = \hat{r}_i - \alpha_{i-1}, \; i = 2, \ldots, \rho + 1 \quad (30)$$

$$z_{\rho+2} = 0 \quad (31)$$

where α_i are referred to as stabilizing functions. The control design starts from the dynamics of z_1:

$$\dot{z}_1 = \dot{r}_1 - \dot{\epsilon}_1 \quad (32)$$

$$= r_2 + \phi_{r,1}(y) - A_{o,1}e_r \quad (33)$$

$$= z_2 + \alpha_1 + \phi_{r,1}(y) + e_{r,2} - A_{o,1}e \quad (34)$$

where $A_{o,1}$ denotes the first row of A_o. Based on the above, α_1 is designed as

$$\alpha_1 = -c_1 z_1 - d_1 z_1 - \phi_{r,1}(y) \quad (35)$$

which results in

$$\dot{z}_1 = z_2 - c_1 z_1 - d_1 z_1 e_{r,2} - A_{o,1}e \quad (36)$$

Before moving to step 2, consider the dynamics of y

$$\dot{y} = \beta_0(\hat{r}_2 + \phi_{r,1}(y)) + \hat{r}_{r,3} + \phi_{r,2}(y) + e_y \quad (37)$$

where $e_y = \beta_0 e_{r2} + e_{r3}$. Then considering $\alpha_1 = \alpha_1(y, \hat{r}_1)$, it is obtained that

$$\begin{aligned}\dot{z}_2 &= \dot{r}_2 + \dot{e}_{r,2} - \frac{\partial\alpha_1}{\partial\hat{r}_1}\dot{\hat{r}}_1 - \frac{\partial\alpha_1}{\partial y}\dot{y} \\ &= (1 - \frac{\partial\alpha_1}{\partial y})(z_3 + \alpha_2) + \phi_{r,2}(y) - \frac{\partial\alpha_1}{\partial\hat{r}_1}\dot{\hat{r}}_1 \\ &\quad -\frac{\partial\alpha_1}{\partial y}(\beta_0(\hat{r}_2 + \phi_{r,1}(y)) + \phi_{r,2}(y) + \epsilon_y) \\ &\quad +e_{r,3} - A_{o,2}e_r - \frac{\partial\alpha_1}{\partial y}e_y \end{aligned} \quad (38)$$

Therefore, α_2 is designed as

$$\begin{aligned}&(1 - \frac{\partial\alpha_1}{\partial y})\alpha_2 \\ &= z_1 - c_2 z_2 - d_2[1 + (\frac{\partial\alpha_1}{\partial y})^2]z_2 - \phi_{r,2}(y) \\ &\quad +\frac{\partial\alpha_1}{\partial y}(\beta_0(\hat{r}_2 + \phi_1(y)) + \phi_2(y)) + \frac{\partial\alpha_1}{\partial\hat{r}_1}\dot{\hat{r}}_1 \end{aligned} \quad (39)$$

In step 3, from the variables in $\alpha_2 = \alpha_2(y, \hat{r}_1, \hat{r}_2)$, it can be obtained that

$$\begin{aligned}\alpha_3 &= (1 - \frac{\partial\alpha_1}{\partial y})z_2 - c_3 z_3 - d_3[1 + (\frac{\partial\alpha_2}{\partial y})^2]z_3 \\ &\quad -\phi_{r,3}(y) + \frac{\partial\alpha_2}{\partial y}(\beta_0(\hat{r}_2 + \phi_{r,1}(y)) \\ &\quad +\hat{r}_{r,3} + \phi_{r,2}(y)) + \sum_{j=1}^{2}\frac{\partial\alpha_2}{\partial\hat{r}_j}\dot{\hat{r}}_j \end{aligned} \quad (40)$$

and similarly for $i = 4, \ldots, \rho + 1$,

$$\begin{aligned}\alpha_i &= z_{i+1} - c_i z_i - d_i[1 + (\frac{\partial\alpha_{i-1}}{\partial y})^2]z_i - \phi_{r,i}(y) \\ &\quad +\frac{\partial\alpha_{i-1}}{\partial y}(\beta_0(\hat{r}_2 + \phi_{r,1}(y)) + \hat{r}_3 + \phi_{r,2}(y)) \\ &\quad +\sum_{j=1}^{i-1}\frac{\partial\alpha_{i-1}}{\partial\hat{r}_j}\dot{\hat{r}}_j \end{aligned} \quad (41)$$

Finally, the control input is given by

$$u = \hat{\mu} + \frac{\alpha_{\rho+1} - \hat{r}_{\rho+2}}{\beta_1} \quad (42)$$

Remark 5. The control design needs $\rho + 1$ steps for a system of relative degree ρ. The additional step, step 1, is used to stabilize the unstable zero-dynamics.

The evaluation of α_2 requires $(1 - \frac{\partial\alpha_1}{\partial y}) \neq 0$. From (35), it follows that

$$(1 - \frac{\partial\alpha_1}{\partial y}) = (1 + \frac{\partial\phi_{r,1}(y)}{\partial y}) \quad (43)$$

Therefore, the following assumption is needed to make the proposed control design feasible.

Assumption 3. $|1 + \frac{\partial\phi_1(y)}{\partial y}| \neq 0$, $\forall y \in R$.

The following theorem summarizes the stability result for the proposed control design (42).

Theorem 4.2 The proposed control (42) stabilizes the nonlinear system (1) and completely rejects the unknown disturbances if the system (1) satisfies Assumptions 1, 2 and 3.

Proof: Consider a Lyapunov candidate

$$V_z = \frac{1}{2}\sum_{i=1}^{\rho+1} z_i^2 \quad (44)$$

It can be shown that, for the designed stabilizing functions α_i and the control input,

$$\begin{aligned}\dot{V}_z = & -\sum_{i=1}^{\rho+1} c_i z_i^2 - \sum_{i=1}^{\rho+1}[1+(\frac{\partial \alpha_{i-1}}{\partial y})^2] z_i^2 \\ & +\sum_{i=1}^{\rho+1} z_i(e_{r,i+1} - A_{o,i}e_r + \frac{\partial \alpha_{i-1}}{\partial y} e_y) \\ \le & -2c_z V_z + e_z \end{aligned} \quad (45)$$

where $\frac{\partial \alpha_0}{\partial y}$ is set as 0 for the convenience of notation, and

$$c_z = \min_{i=1,\dots,\rho+1}\{c_i\} \quad (46)$$

$$e_z = \sum_{i=1}^{\rho+1} \frac{1}{4d_i}(\|e_{r,i+1} - A_{o,i}e_r\|^2 + \|e_y\|^2) \quad (47)$$

Notice from (28) that e_r is the output of a stable linear system with the input which is bounded by a decaying exponential function. It can be shown that e_r is also bounded by a decaying exponential function and therefore e_y is also bounded by a decaying exponential function. Furthermore, it can be concluded from (45) with invoking Lemma 4B in (Krstic *et al.*, 1995) that V_z is bounded by a decaying exponential function. Hence, z_i, for $i = 1, \dots, \rho+1$, are bounded and converge to zero asymptotically.

From (35), it follows

$$\begin{aligned} z_2 &= \hat{r}_2 - \alpha_1 \\ &= y + \phi_{r,1}(y) + c_1 z_1 + d_1 z_1 - \beta_0 z_1 \\ &\quad -\beta_0 e_{r,1} - e_{r,2} \end{aligned} \quad (48)$$

Above relation together with Assumption 3 ensure that $\lim_{t\to\infty} y(t) = 0$, which further implies $\lim_{t\to\infty} \hat{r}_2 = 0$. Then with $\lim_{t\to\infty} z_3(t) = 0$ and $\lim_{t\to\infty} \alpha_2(t) = 0$, it can concluded that $\lim_{t\to\infty} \hat{r}_3 = 0$. This reasoning can be applied iteratively to conclude that $\lim_{t\to\infty} \hat{r}_i = 0$, $i = 4, \dots, \rho+1$. The convergence to zero of the remaining variables in the observer (27) can be established from the fact that $\sum_{i=1}^{m} \beta_i s^{m-i}$ is Hurwitz. Finally from $r = \hat{r} + e_r$, it follows that $\lim_{t\to\infty} r(t) = 0$, and then $\lim_{t\to\infty} x(t) = 0$.

5. AN EXAMPLE

Consider a nonlinear system in output feedback form

$$\begin{aligned} \dot{x}_1 &= x_2 - y^3 + (u-\mu) \\ \dot{x}_2 &= -(u-\mu) \\ y &= x_1 \end{aligned} \quad (49)$$

where μ is a sinusoidal disturbance generated by

$$\begin{aligned} \dot{w} &= \begin{bmatrix} 0 & \omega \\ -\omega & 0 \end{bmatrix} \quad w(0) = w_0 \\ \mu &= l^T w \end{aligned} \quad (50)$$

with ω, l and w_0 unknown. It is easy to see that the system (49) are in the format of (1) with $\phi(y) = [-y^3 \ 0]^T$ and $b = [1 \ -1]^T$. The system is nonminimum phase with the nonminimum phase zero at $s = 1$. The filters for disturbance estimation are designed as

$$\dot{p} = \begin{bmatrix} k_1 & 1 \\ k_2 & 0 \end{bmatrix} p + \begin{bmatrix} -y^3 \\ 0 \end{bmatrix} - \begin{bmatrix} k_1 \\ k_2 \end{bmatrix} y \quad (51)$$

$$\dot{\xi} = \begin{bmatrix} -f_1 & 1 \\ -f_2 & 0 \end{bmatrix} \xi + \begin{bmatrix} 0 \\ g \end{bmatrix} (p_1 - y) \quad (52)$$

$$\dot{\zeta} = \begin{bmatrix} -f_1 & 1 \\ -f_2 & 0 \end{bmatrix} \zeta + \begin{bmatrix} 0 \\ g \end{bmatrix} \hat{\psi}_1^T \xi \quad (53)$$

$$\dot{\hat{\psi}}_1 = \Gamma \xi (\xi - \zeta)^T P \begin{bmatrix} 0 \\ g \end{bmatrix} \quad (54)$$

With $\hat{\psi}_1$, $\hat{\psi}_z$ is calculated by

$$\hat{\psi}_z = (k_1 + k_2 - 1)\frac{|\hat{\Sigma}|}{|\hat{\Sigma}|^2 + \sigma} \mathrm{adj}(\hat{\Sigma})\hat{\psi}_1 \quad (55)$$

where

$$\hat{\Sigma} = F^T + [0 \ g]^T \hat{\psi}_1 - I \quad (56)$$

Finally, $\hat{\psi}_u$ is given by

$$\hat{\psi}_u = -\hat{\psi}_2 - k_1 + F^T \hat{\psi}_1 + \hat{\psi}_1^T G \hat{\psi}_1 \quad (57)$$

with $\hat{\psi}_2 = \hat{\psi}_2 - \hat{\psi}_1$

For the control design, the state transform is given by

$$r = \begin{bmatrix} -1 & -1 \\ 0 & -1 \end{bmatrix} x \quad (58)$$

and the transformed system is described by

$$\begin{aligned} \dot{r} &= \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix} r + \begin{bmatrix} y^3 \\ 0 \end{bmatrix} + \begin{bmatrix} 0 \\ 1 \end{bmatrix} (u - \mu) \\ y &= [\,-1 \ 1\,] r \end{aligned} \quad (59)$$

The observer for the control design is given by

$$\begin{aligned} \dot{\hat{r}} = & \begin{bmatrix} -k_{r1} & k_{r1}+1 \\ -k_{r2} & k_{r2} \end{bmatrix} \hat{r} + \begin{bmatrix} y^3 \\ 0 \end{bmatrix} - \begin{bmatrix} k_{r1} \\ k_{r2} \end{bmatrix} y \\ & + \begin{bmatrix} 0 \\ 1 \end{bmatrix} (u - \hat{\psi}_u^T \xi) \end{aligned} \quad (60)$$

The final control design is given by

$$\begin{aligned} \alpha_1 &= -c_1 \hat{r}_1 - d_1 \hat{r}_1 - y^3 \quad (61) \\ u &= \hat{\psi}_u^T \xi + (1 + \frac{\partial \alpha_1}{\partial y})^{-1}\{-\hat{r}_1 - c_2 z_2 \end{aligned}$$

$$
\begin{aligned}
&-d_2[(\frac{\partial\alpha_1}{\partial y})^2+1]z_2+\frac{\partial\alpha_1}{\partial\hat{r}_1}\dot{\hat{r}}_1 \\
&+\frac{\partial\alpha_1}{\partial y}(-\hat{r}_2-y^3)\}
\end{aligned}
\tag{62}
$$

The simulation study has been carried out for the estimation and control design shown in this example. The simulation results shown below are for the settings $k_1 = -3$, $k_2 = -2$, $f_1 = 3$, $f_2 = 2$, $g = 1$, $\Gamma = 1000I$, $k_{r1} = 5$, $k_{r2} = 2$, $c_1 = d_1 = c_2 = d_2 = 1$. The settings for the disturbance are $\omega = 1$, $w_0 = [0,1]^T$, i.e., the disturbance is set as $\mu(t) = \sin t$. The control input and the system output are shown in Figure 1, in which the output converges to zero with the input to asymptotically cancel the disturbance.

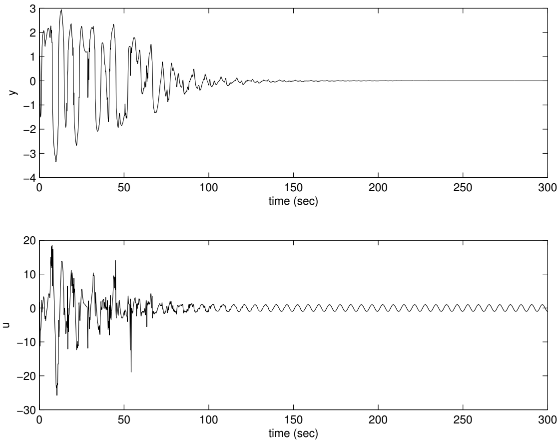

Fig. 1. Control input and system output

6. CONCLUSIONS

An indirect approach has been presented to rejecting unknown sinusoidal disturbances and stabilization of nonlinear systems in the output feedback form. The big difference between the proposed methods and the methods in the literature for nonlinear systems is that our algorithms work for the nonminimum phase nonlinear systems, and the others do not. The nonminimum phase makes the estimation and control design much more difficult. One can compare the results shown in this paper and the one shown in (Ding, 2003) where the disturbance rejection is achieved for the same system, but with minimum phase. The nonminimum phase causes the involvement of vector and matrix manipulation for the estimation of disturbances, and the re-estimation of the states for the control design. The proposed algorithms work for both the minimum phase and nonminimum phase systems.

Despite the difficulty of nonminimum phase, the proposed algorithm ensures the asymptotic rejection or complete compensation of unknown sinusoidal disturbances. Although the control design restricts to the case of one nonminimum phase zero, the restriction is very difficult to relax without compromising the global stability, because of the structural properties required for nonlinear control design.

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