ADAPTIVE ESTIMATION OF UNKNOWN SINUSOIDAL DISTURBANCES IN NON-MINIMUM-PHASE NONLINEAR SYSTEMS

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Abstract: This paper deals with adaptive estimation of unknown disturbances in a class of nonminimum phase nonlinear systems. The unknown disturbances are combination of sinusoidal disturbances with unknown frequencies, unknown phases and amplitudes. The only information of the unknown disturbances is the number of distinctive frequencies inside. The class of nonlinear systems considered in this paper consists of nonlinear systems in the output feedback form and the systems may be nonminimum phase, ie, with unstable zero dynamics. An adaptive estimation algorithm is developed to give exponentially convergent estimates of the unknown disturbance and the system states. The asymptotic convergent estimates of unknown frequencies are also obtained. The proposed estimation algorithm works for both minimum phase and nonminimum phase nonlinear systems in output feedback form.

Keywords: Disturbance rejection, Estimation, Nonlinear systems, Adaptive systems, Uncertainty

1. INTRODUCTION

In engineering systems, there are deterministic disturbances, apart from random disturbances. Among the various types of deterministic disturbances, sinusoidal disturbances have attracted a large amount of research interests, from the estimation of the disturbance frequencies to the compensation or rejection of disturbances. Estimation and reconstruction of unknown disturbances have their importance for detection and monitoring, apart from the stabilization of a system and disturbance rejection. It was until fairly recently that a global convergent estimation algorithm was proposed for estimation of a single frequency of the stand alone sinusoidal signal (Hsu *et al.*, 1999), and more recently an algorithm was proposed to estimate multiple frequencies from a sinusoidal signal using adaptive observers (Marino and Tomei, 2002).

This paper deals with estimation of unknown sinusoidal disturbances for nonlinear systems in the output feedback form. The proposed algorithm allows the system to be nonminimum phase. A new set of filters are designed to extract the contribution of the disturbance to the states and to estimate disturbance and the frequencies. The estimation starts from the contribution to the output of the system, from which the disturbance characterization such as frequencies can be obtained. Based on this estimation, the contributions to other states can then be calculated and finally the unknown disturbance is reconstructed. The proposed estimation algorithm imposes no restriction on the number or the range of disturbance frequencies, and no restriction such as projection used in (Marino *et al.*, 2003) for the adaptive law for parameter estimation. The estimated disturbance and frequencies asymptotically converge to their true values. An illustrative example is included with simulation results shown in figures.

2. PROBLEM FORMULATION

Consider a singleinputsingleoutput nonlinear system which can be transformed into the output feedback form

$$\begin{aligned} \dot{x} &= A_c x + \phi(y) + b(u - \mu) \\ y &= Cx \end{aligned} \tag{1}$$

with

$$A_c = \begin{bmatrix} 0 & 1 & 0 & \dots & 0 \\ 0 & 0 & 1 & \dots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & 0 & \dots & 1 \\ 0 & 0 & 0 & \dots & 0 \end{bmatrix}, \quad C = \begin{bmatrix} 1 \\ 0 \\ \vdots \\ 0 \end{bmatrix}^T, b = \begin{bmatrix} 0 \\ \vdots \\ 0 \\ b_\rho \\ \vdots \\ b_n \end{bmatrix}$$

where $x \in R^n$ is the state vector, $u \in R$ is the control, ϕ, is a known nonlinear smooth vector field in R^n with $\phi(0) = 0$, $\mu \in R$ is a matched disturbance which is generated from an unknown exosystem

$$\begin{aligned} \dot{w} &= Sw \\ \mu &= l^T w \end{aligned} \tag{2}$$

with $w \in R^s$.

Assumption 1. The zeros of polynomial $\mathbf{B}(s) = \sum_{i=\rho}^{n} b_i s^{n-i}$ have non-zero real parts.

Remark 1. Assumption 1 only requires that $\mathbf{B}(s)$ has not zeros on the imaginary axis. It allows $\mathbf{B}(s)$ to have positive real parts, ie, the case of nonminimum phase systems.

Assumption 2. The eigenvalues of S are with zero real parts and are distinct.

Remark 2. Assumption 2 ensures that the disturbances are combination of sinusoidal signals including constant bias. It follows the assumption made on unknown exosystems in (Nikiforov, 1998; Ding, 2003).

Remark 3. As shown in (Ding, 2003), the unmatched disturbances in the nonlinear systems in the output feedback form can be transformed to the matched case of (1), if Assumption 2 is satisfied.

The estimation problem considered in this paper is to estimate the disturbance μ, the state x and the unknown disturbance frequencies characterized by the eigenvalues of S.

3. PRELIMINARY DESIGN AND ANALYSIS

If the disturbance does not exist in (1), the system (1) is in the linear-observer-error format (Isidori, 1995). In that case, a state observer can be designed as

$$\dot{p} = (A_c + kC)p + \phi(y) + bu - ky \tag{3}$$

where $p \in R^n$, $k \in R^n$ is chosen so that $A_c + kC$ is Hurwitz. The difficulty in the state estimation is due to the unknown disturbance μ. Based on Assumption 2 and the design of k, $A_c + kC$, and S have exclusive eigenvalues, and therefore there exists a solution $Q \in R^{n \times s}$ of the following matrix equation for given S

$$QS = (A_c + kC)Q + bl^T \tag{4}$$

Define

$$q(w) = Qw \tag{5}$$

then (4) guarantees

$$\dot{q} = (A_c + kC)q + b\mu \tag{6}$$

Since S is unknown, the solution Q from (4) does not exist and the filter (6) cannot be implemented due to the unknown disturbance μ. But the two equations (4) and (6) are important in the reformulation of the estimation problem through the property stated in the following lemma (Ding, 2003).

Lemma 3.1 The state variable x can be expressed as

$$x = p - q + \epsilon \tag{7}$$

where p is generated from (3) with q and ϵ satisfying (6) and (8) satisfying

$$\dot{\epsilon} = (A_c + kC)\epsilon \tag{8}$$

The state estimation is solved if an estimate of q is provided. Referring to (6), the problem which is going to be solved is to estimate both the state and the unknown input to a nonminimum phase linear dynamic system. The solution depends on the characteristics of the matched disturbance μ.

For the convenience of filter design for adaptive estimation, the exosystem (2) is to be reformulated.

Choose a controllable pair $\{F, G\}$ with $F \in R^{s\times s}$ Hurwitz and $G \in R^s$. For a matrix S satisfying Assumption 2, there exists a solution $M \in R^{s\times s}$ of the following equation

$$MS - FM = GQ_{(1)} \tag{9}$$

where $Q_{(i)}$ denotes the ith row of Q. Introduce a state transform of the exosystem

$$\eta = Mw \tag{10}$$

it follows that

$$\begin{aligned} \dot{\eta} &= (FM + GQ_{(1)})w \\ &= (F + G\psi_1^T)\eta \\ &:= F_o\eta \\ q_1 &= \psi_1^T\eta \end{aligned} \tag{11}$$

where $\psi_1^T = Q_{(1)}M^{-1}$. In the new coordinate η, q can then be expressed as

$$q = QM^{-1}\eta := [\psi_1, \dots, \psi_s]^T\eta \tag{12}$$

and

$$\mu = l^T M^{-1}\eta := \psi_u^T\eta \tag{13}$$

Relating q and μ expressed in (12) and (13) to the dynamics shown in (6)gives

$$\psi_i^T F_o = \psi_{i+1}^T + k_i\psi_1^T, \text{ for } i = 1, \dots, \rho - 1 \tag{14}$$

and

$$\begin{aligned} &\psi_i^T F_o = \psi_{i+1}^T + k_i\psi_1^T + b_i\psi_u^T, \text{ for } i = \rho, \dots, n-1 \\ &\psi_n^T F_o = k_n\psi_1^T + b_n\psi_u^T \end{aligned} \tag{15}$$

Define

$$\psi_z^T := [\psi_{\rho+1}, \dots, \psi_n]^T - \sum_{i=1}^{\rho} B^{\rho-i}\bar{b}\psi_i^T \tag{16}$$

where B and $\bar{b}$ are given by

$$B = \begin{bmatrix} -b_{\rho+1}/b_\rho & 1 & \dots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ -b_{n-1}/b_\rho & 0 & \dots & 1 \\ -b_n/b_\rho & 0 & \dots & 0 \end{bmatrix}, \quad \bar{b} = \begin{bmatrix} b_{\rho+1}/b_\rho \\ \vdots \\ b_n/b_\rho \end{bmatrix}$$

It can be shown from (15) that

$$\psi_z^T F_o = B\psi_z^T + k_z\psi_1^T \tag{17}$$

where

$$k_z = [k_{\rho+1}, \dots, k_n]^T - \sum_{i=1}^{\rho} B^{\rho-i}\bar{b}k_i + B^\rho\bar{b}$$

Using the notation $\otimes$ for the Kronecker product of matrices and vec$(\cdot)$ for the vector obtained by rolling out the column vectors of a matrix, and using the identity $\text{vec}(ABC^T) = (A \otimes C)\text{vec}(B)$ (Graybill, 1983), from (17), it is obtained that

$$[F_o^T \otimes I_{(n-\rho)} - I_s \otimes B]\text{vec}(\psi_z) = \text{vec}(\psi_1 k_z^T) \tag{18}$$

and

$$\text{vec}(\psi_z) = \Sigma^{-1}\text{vec}(\psi_1 k_z^T) \tag{19}$$

where

$$\Sigma = F_o^T \otimes I_{(n-\rho)} - I_s \otimes B \tag{20}$$

4. FILTER DESIGN AND DISTURBANCE ESTIMATION

Based on the analysis of the influence of disturbance in the state variables through Q, a set of filters and estimation algorithm are proposed for state variables and the input disturbance. From the analysis in the previous section, it is clear that q and μ can be estimated or evaluated if η and ψ_1 are available. For the estimation of η and ψ_1, the following filters and adaptive law are designed as

$$\dot{\xi} = F\xi + G(p_1 - y) \tag{21}$$

$$\dot{\zeta} = F\zeta + G\hat{\psi}_1^T\xi \tag{22}$$

$$\dot{\hat{\psi}}_1 = \Gamma\xi(\xi - \zeta)^T PG \tag{23}$$

where Γ is a positive definite matrix, and P is the positive definite matrix satisfying

$$PF + F^T P = -2I_s \tag{24}$$

The following lemma describes the properties of the estimates.

Lemma 4.1 The estimates ξ and $\hat{\psi}_1$ converge to η and ψ_1 respectively. Furthermore the errors of the estimates are bounded by exponentially decaying functions, ie, there exist some positive real constants d_ξ, d_ψ, λ_ξ and λ_ψ such that

$$\|\eta(t) - \xi(t)\| < d_\xi e^{-\lambda_\xi t} \tag{25}$$

$$\|\psi_1 - \hat{\psi}_1(t)\| < d_\psi e^{-\lambda_\psi t} \tag{26}$$

Proof. Let us define $e_\xi = \zeta - \xi$. It can be obtained from (22) and (21) that

$$\dot{e}_\xi = Fe_\xi + C\epsilon \tag{27}$$

From (8), it can be seen that ϵ is exponentially decaying. In fact, putting e_ξ and ϵ together gives,

$$\begin{bmatrix} \dot{e}_\xi \\ \dot{\epsilon} \end{bmatrix} = \begin{bmatrix} F & GC \\ 0 & (A_c + kC) \end{bmatrix} \begin{bmatrix} e_\xi \\ \epsilon \end{bmatrix} \tag{28}$$

Since F and $(A_c + kC)$ are Hurwitz, the system (28) is exponentially stable, and therefor there exist positive reals d_1 and λ_1 such that (25) is satisfied.

To establish the convergence of $\hat{\psi}_1$, define

$$e = \xi - \eta \tag{29}$$

From (21) and (22), it follows that

$$\dot{e} = Fe + G\psi_1^T e_\xi - GC\epsilon + G\xi^T \tilde{\psi}_1 \tag{30}$$

where $\tilde{\psi}_1 = \psi - \hat{\psi}_1$. Define

$$\bar{e} = \begin{bmatrix} e \\ e_\xi \\ \epsilon \end{bmatrix} \tag{31}$$

Based on (8), (28) and (23), the adaptive systems can be arranged in the following format,

$$\begin{aligned} \dot{\bar{e}} &= \bar{A}\bar{e} + \Omega(t)^T \tilde{\psi}_1 \\ \dot{\tilde{\psi}}_1 &= -\Gamma\Omega(t)\bar{P}\bar{e} \end{aligned} \tag{32}$$

where $\tilde{\psi}_1 = \psi_1 - \hat{\psi}_1$,

$$\bar{A} = \begin{bmatrix} F & G\psi_1^T & -GC \\ 0 & F & GC \\ 0 & 0 & (A_c + kC) \end{bmatrix}$$

$$\Omega(t) = \begin{bmatrix} \xi G^T & 0 & 0 \end{bmatrix}$$

$$\bar{P} = \begin{bmatrix} P & & \\ & \gamma_1 P & \\ & & \gamma_2 P_\epsilon \end{bmatrix}$$

with γ_1 and γ_2 being positive reals and P_ϵ being the positive definite matrix satisfying

$$P_\epsilon(A_c + kC) + (A_c + kC)^T P_\epsilon = -2I \tag{33}$$

Let

$$\bar{P}\bar{A} + \bar{A}\bar{P} = -\bar{Q} \tag{34}$$

A direct evaluation gives

$$\bar{Q} = \begin{bmatrix} 2I_s & -PG\psi_1^T & PGC \\ -\psi_1 G^T P & 2\gamma_1 I_s & -\gamma_1 PGC \\ C^T G^T P & -\gamma_1 C^T G^T P & 2\gamma_2 I_n \end{bmatrix} \tag{35}$$

From the structure of $\bar{Q}$, $\bar{Q}$ can be made positive definite by choosing a sufficient large γ_1 and then a sufficient large γ_2.

Define

$$V = \bar{e}^T \bar{P}\bar{e} + \tilde{\psi}_1^T \Gamma^{-1} \tilde{\psi}_1 \tag{36}$$

Then from (32), it follows that

$$\dot{V} = -\bar{e}^T \bar{Q}\bar{e} \tag{37}$$

Therefore $\bar{e}$ and $\tilde{\psi}_1$ are bounded and the invariant set theorem (Khalil, 2002) ensures that $\lim_{t\to\infty} \bar{e}(t) = 0$.

The consistent excitation condition of $\Omega(t)$ is needed to establish the convergence of $\hat{\psi}_1$. From the definition of η in the previous section, it can be seen that η is persistently excited, ie, there exist two positive reals T and γ_3 such that

$$\int_t^{t+T} \eta(\tau)\eta(\tau)^T d\tau \geq \gamma_3 I_s > 0, \forall t \geq 0 \tag{38}$$

With

$$\begin{aligned} \int_t^{t+T} \Omega(\tau)\Omega(\tau)^T d\tau &= \|G\|^2 \int_t^{t+T} \xi(\tau)\xi(\tau)^T d\tau \\ &= \|G\|^2 \int_t^{t+T} (\eta(\tau) - e_\xi(\tau))(\eta(\tau) - e_\xi(\tau))^T d\tau \end{aligned} \tag{39}$$

and the fact that η is bounded and e_ξ converges to 0 exponentially, it can be concluded that there exist a $t_o > 0$ and a γ_4 with $0 < \gamma_4 < \gamma_3\|G\|^2$ such that

$$\int_t^{t+T} \Omega(\tau)\Omega(\tau)^T d\tau \geq \gamma_4 I_s > 0, \forall t \geq t_0 > 0 \tag{40}$$

Since $\bar{e}(t_0)$ and $\tilde{\psi}_1(t_0)$ are bounded, applying Lemma B.2.3 (Marino and Tomei, 1995) leads that $(\bar{e}, \tilde{\psi}_1) = 0$ is a globally exponentially stable equilibrium point for (32), which implies (26).

With the estimates $\hat{\psi}_1$ and ξ for ψ_1 and η respectively, the following algorithms are proposed for estimation of ψ_i, $i = 2, \ldots, n$, and finally for q and μ. For $i = 2, \ldots, \rho$,

$$\hat{\psi}_i^T = \hat{\psi}_{i-1}^T (F + G\hat{\psi}_1^T) + k_{i-1}\psi_1^T, \tag{41}$$

and

$$\begin{bmatrix} \hat{\psi}_{\rho+1}^T \\ \vdots \\ \hat{\psi}_n^T \end{bmatrix} = \hat{\psi}_z^T - \sum_{i=1}^{\rho} B^{\rho-i}\bar{b}\hat{\psi}_i^T \tag{42}$$

where

$$\text{vec}(\hat{\psi}_z) = \frac{|\hat{\Sigma}|}{\sigma + |\hat{\Sigma}|^2} \text{adj}(\hat{\Sigma})\text{vec}(\hat{\psi}_1 k_z^T) \tag{43}$$

with

$$\hat{\Sigma} = (F + G\hat{\psi}_1^T)^T \otimes I_{(n-\rho)} - I_s \otimes B \quad (44)$$
$$\dot{\sigma} = -\lambda_\sigma \sigma, \quad \sigma(0) = \sigma_0 \quad (45)$$

for some positive reals λ_σ and σ_0. Notations $|\cdot|$ and adj$(\cdot)$ are used to denote the determinant and the adjoint matrix of a matrix respectively. The following theorem summarize the results of the disturbance and state estimation.

Theorem 4.2 Based on the filters (21), (22), (23) and estimates shown in (41) and (43), the estimates of the state and the disturbance of (1) are given by

$$\hat{x} = p + \hat{\psi}^T \xi \quad (46)$$
$$\hat{\mu} = \hat{\psi}_u^T \xi \quad (47)$$

where

$$\hat{\psi}_u^T = \frac{1}{b_\rho}[\hat{\psi}_{\rho+1}^T - \hat{\psi}_\rho^T(F + G\hat{\psi}_1^T) - k_\rho \hat{\psi}_1^T] \quad (48)$$

and the estimate of exosystem matrix $F + G\psi_1^T$ is given by

$$\hat{F}_o = F + G\hat{\psi}_1^T \quad (49)$$

There exist positive real constants λ_x, d_x, λ_μ, d_μ, λ_F, and d_F such that

$$\|x(t) - \hat{x}(t)\| \le d_x e^{-\lambda_x t} \quad (50)$$
$$\|\mu(t) - \hat{\mu}(t)\| \le d_\mu e^{-\lambda_\mu t} \quad (51)$$
$$\|F_o - \hat{F}_o(t)\| \le d_F e^{-\lambda_F t} \quad (52)$$

Proof. Define, for the convenience of expression, that an estimate is an exponentially convergent estimate if the estimation error is bounded by a decaying exponential function.It is to be established that the estimates for the state and for the disturbance are exponentially convergent estimate. Let $\tilde{F}_o = F_o - \hat{F}_o$. From (11) and (49), it can be obtained that

$$\|\tilde{F}_o\| = \|G\tilde{\psi}_1^T\| \le \sqrt{\sum_{i=1}^{s}\sum_{j=1}^{s} G_i^2 \tilde{\psi}_j^2}$$
$$= \|G\|\|\tilde{\psi}_1\| \le \|G\| d_2 e^{-\lambda_2 t} \quad (53)$$

Hence, (52) is established. Let $\tilde{\psi}_i = \psi_i - \hat{\psi}_i$. From (14) and (41), it follows that, for $i = 2, \ldots, \rho$,

$$\|\tilde{\psi}_i\| = \| - \tilde{\psi}_{i-1}^T F_o - \hat{\psi}_{i-1}^T \tilde{F}_o + k_{i-1}\tilde{\psi}_1^T\|$$
$$\le \|\tilde{\psi}_{i-1}\|\|F_o\| + \|\hat{\psi}_{i-1}\|\|\tilde{F}_o\| + |k_{i-1}|\|\tilde{\psi}_1\| \quad (54)$$

Since $\hat{\psi}_1$ and $\hat{F}_o$ are exponentially convergent estimates, and $\hat{\psi}_1$ is bounded, it can be concluded from (54) that $\hat{\psi}_2$ is an exponentially convergent estimate. To use (54) recursively, it can be obtained that $\hat{\psi}_i$, for $i = 2, \ldots, \rho$ are exponentially convergent.

It can be shown that $|\hat{\Sigma}|$ and $\text{adj}(\hat{\Sigma})\text{vec}(\hat{\psi}_1 k_z^T)$ are exponentially convergent estimates of $|\Sigma|$ and $\text{adj}(\Sigma)\text{vec}(\psi_1 k_z^T)$ respectively, as they are functions of the elements of $\hat{\psi}_1$ obtained by multiplications and additions. From (19) and (43), it can be obtained that

$$\begin{aligned}
&\text{vec}(\psi_z) - \text{vec}(\hat{\psi}_z)\\
&= \frac{1}{|\Sigma|}\text{adj}(\Sigma)\text{vec}(\psi_1 k_z^T) - \frac{|\hat{\Sigma}|}{\sigma + |\hat{\Sigma}|^2}\text{adj}(\hat{\Sigma})\text{vec}(\hat{\psi}_1 k_z^T)\\
&= \frac{\sigma \text{adj}(\Sigma)\text{vec}(\psi_1 k_z^T)}{|\Sigma|(\sigma + |\hat{\Sigma}|^2)} + \frac{(|\hat{\Sigma}| - |\Sigma|)\text{adj}(\hat{\Sigma})\text{vec}(\hat{\psi}_1 k_z^T)}{|\Sigma|(\sigma + |\hat{\Sigma}|^2)}\\
&\quad + \frac{|\hat{\Sigma}|[(\text{adj}(\Sigma)\text{vec}(\psi_1 k_z^T)\text{adj}(\hat{\Sigma})\text{vec}(\hat{\psi}_1 k_z^T)]}{|\Sigma|(\sigma + |\hat{\Sigma}|^2)} \quad (55)
\end{aligned}$$

It can be shown that each of the three terms in (55) is bounded by a decaying exponential function, as σ is a decaying exponential function. Therefore it can be concluded from (42) that $\hat{\psi}_i$, $i = \rho + 1, \ldots, n$ are exponentially convergent estimates, and hence

$$\hat{\Psi} := [\hat{\psi}_1, \ldots, \hat{\psi}_n] \quad (56)$$

is exponentially convergent. Finally from

$$\begin{aligned}
\|x - \hat{x}\| &= \|\epsilon - \Psi^T\eta + \hat{\Psi}^T\xi\|\\
&= \|\epsilon - \Psi^T(\eta - \xi) + (\hat{\Psi}^T - \Psi^T)\xi\|\\
&\le \|\epsilon\| + \|\Psi\|\|\eta - \xi\| + \|\hat{\Psi} - \Psi\|\|\xi\| \quad (57)\\
\|\mu - \hat{\mu}\| &= \|\psi_u^T\eta - \hat{\psi}_u^T\xi\|\\
&\le \|\psi_u\|\|\eta - \xi\| + \|\psi_u - \hat{\psi}_u\|\|\xi\| \quad (58)
\end{aligned}$$

it can be concluded that $\hat{x}$ and $\hat{\mu}$ are exponentially convergent estimates of x and μ respectively.

5. AN EXAMPLE

Consider a nonlinear system in output feedback form

$$\begin{aligned}
\dot{x}_1 &= x_2 - y^3 + (u - \mu)\\
\dot{x}_2 &= -(u - \mu)\\
y &= x_1 \quad (59)
\end{aligned}$$

where μ is a sinusoidal disturbance generated by

$$\begin{aligned}
\dot{w} &= \begin{bmatrix} 0 & \omega \\ -\omega & 0 \end{bmatrix} \quad w(0) = w_0\\
\mu &= l^T w \quad (60)
\end{aligned}$$

with ω, l and w_0 unknown. It is easy to see that the system (59) are in the format of (1) with

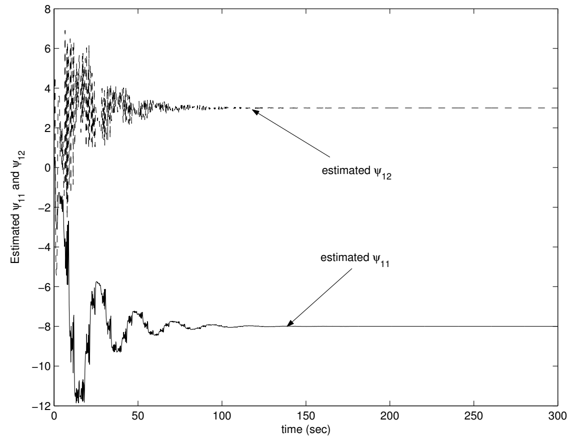

Fig. 1. Estimate of ψ_1

$\phi(y) = [-y^3\ 0]^T$ and $b = [1\ -1]^T$. The system is nonminimum phase with the nonminimum phase zero at $s = 1$.

The filters for disturbance estimation can be designed accordingly. The simulation study has been carried out for the disturbance estimation. The simulation results shown below are for the settings $k_1 = -3$, $k_2 = -2$, $f_1 = 3$, $f_2 = 2$, $g = 1$, $\Gamma = 1000I$. The settings for the disturbance are $\omega = 1$, $w_0 = [0, 1]^T$, i.e., the disturbance is set as $\mu(t) = \sin t$. The estimate for ψ_1 is shown in Figure 1, where $\hat{\psi}_1$ converges to $[-8, 3]^T$, the correct value for ψ_1. In fact, it is easy to check that the eigenvalues of $(F + G[-8, 3]^T)$ are $\pm 1j$.The estimate of the disturbance is shown in Figure 2, with a clear convergence to the disturbance. An enlargement of a section of Figure 2 is shown in Figure 3.

8. CONCLUSIONS

A disturbance estimation algorithm has been proposed for nonlinear systems in the output feedback form. With the estimation of the unknown disturbance, the system state can also be estimated. The big difference between the proposed method and the methods in the literature for nonlinear systems is that the algorithm proposed in this paper works for the nonminimum phase nonlinear systems, and the others do not. The nonminimum phase makes the estimation much more diffcult. Despite the diffculty of dealing with nonminimum phase, the proposed algorithm achieves exponentially convergent estimates of the disturbance and its characteristic matrix, from which the disturbance frequencies can be calculated. The results presented in this paper will be very useful in control design for rejecting unknown disturbance.

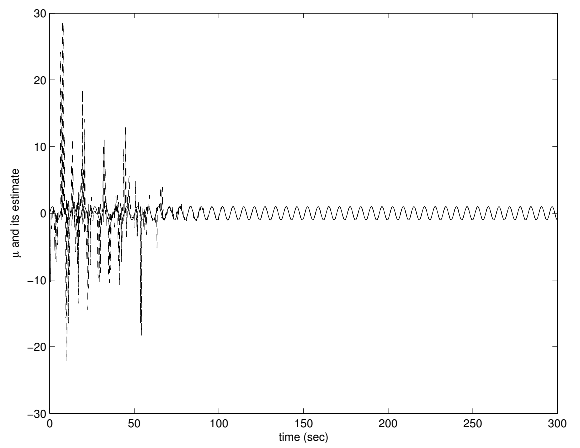

Fig. 2. Estimate of the disturbance

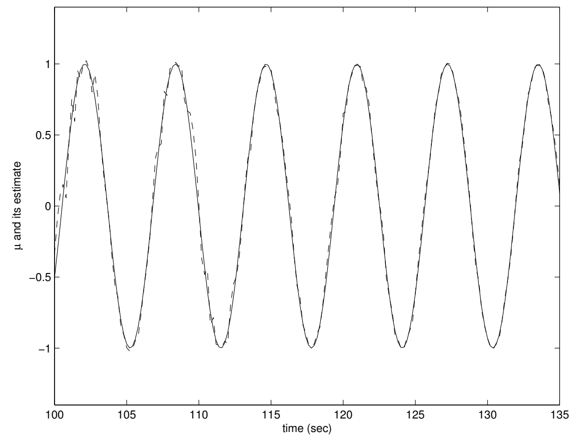

Fig. 3. An enlargement of a part of Figure 2

REFERENCES

Ding, Z. (2003). Global stabilization and disturbance suppression of a class of nonlinear systems with uncertain internal model. *Automatica* **39**(3), 471–479.

Graybill, F. G. (1983). *Matrices with Applications in Statistics*. 2nd ed.. Wadsworth. California, USA.

Hsu, L., R. Ortega and G. Damm (1999). A globally convergent frequency estimator. *IEEE Trans. Automat. Contr.* **44**, 698–417.

Isidori, A. (1995). *Nonlinear Control Systems*. 3rd ed.. Springer-Verlag. Berlin.

Khalil, H. K. (2002). *Nonlinear Systems*. 3rd ed.. Prentice Hall. New Jersey.

Marino, R. and P. Tomei (1995). *Nonlinear Control Design: Geometric, Adaptive, and Robust*. Prentice-Hall. London.

Marino, R. and P. Tomei (2002). Global estimation of n unknown frequencies. *IEEE Trans. Automat. Contr.* **47**(8), 1324–1328.

Marino, R., G. L. Santosuosso and P. Tomei (2003). Robust adaptive compensation of biased sinusoidal disturbances with unknown frequencies. *Automatica* **39**, 1755–1761.

Nikiforov, V. O. (1998). Adaptive non-linear tracking with complete compensation of unknown disturbances. *European Journal of Control* **4**, 132–139.