IMPROVED OPTIMAL CONTROL OF DRY CLUTCH ENGAGEMENT

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Abstract: Optimal control of the engagement of a dry clutch is examined paying particular attention to the driver's comfort. Due to the strong constraints and large errors of the torque control for internal combustion engines, only the normal force on the clutch disks is considered as controlled input. The resulting analytically derived controller is tested on a highly realistic nonlinear model proving very good performance. *Copyright*[©] 2005 *IFAC*

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1. INTRODUCTION

The engagement control of automotive dry clutch is getting attention from automotive industry as a mean of enhancing the comfort of manual transmission (MT) passenger cars. The increasing torque of modern engines coupled with the stiffness reduction of the driveline make very hard for the driver to meet standard comfort requirements with a manually operated clutch, particularly in the standing start scenario where the energy dissipated in the clutch is maximal. The introduction of an automated manual transmission (AMT) or, eventually, a clutch-by-wire system is seen as a mean of easing the driver task and, thus, enhancing his satisfaction about the car.

The control of powertrain systems is an ample and well-established research field covering thermic, chemical, mechanical and electric systems. The driveline alone has been the subject of different studies concerning, for example, optimal shift strategies, hybrid vehicle management, engine torque estimation, dry clutch controlled sliding and engagement control.

Literature on dry clutch engagement control for AMT transmissions is quite ample and many different approaches have been proposed: quantitative feedback theory (Sliker and Loh, 1996), fuzzy control (Tanaka and Wada, 1995) (Shuiwen *et al.*, 1995), model predictive control (Bemporad *et al.*, 2001) and decoupling control (Garofalo *et al.*, 2001). Also optimal control of a dry clutch has been studied in some detail (Glielmo and Vasca, 2000) and (Garofalo *et al.*, 2002).

To our best knowledge all the solutions proposed in control literature use either the engine and clutch torque, or the engine torque alone. The torque of an internal combustion engine under dynamic charge is difficult to master and heavily

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constrained particularly at the low rotation speeds found during a standing start making such an approach impractical. Moreover, even admitting perfect control of the engine torque, the proposed solutions leave residual oscillations of the powertrain after the engagement.

The main contributions of this paper are: obtaining a closed form for a finite-time optimal control based an improved version of the optimality criterion introduced by Glielmo (Glielmo and Vasca, 2000) avoiding all residual oscillations using the clutch as only controlled input and introducing a novel nonlinear LuGre model of the clutch and the rest of the driveline driveline. Numerical results obtained this highly realistic model with actual car parameters highlight the good performances of the controller.

2. DYNAMIC MODEL

2.1 Simulation model

The scheme of a powertrain is shown in Fig. 1.

Engine torque is transmitted through the driveline to the wheels, friction forces between the wheels and the ground accelerate the vehicle mass. Since an internal combustion engine has a minimal rotational speed, called idle speed, the clutch has to assure a smooth transition from zero to minimal speed. Apart this basic task the clutch also allows easier gear shifting and temporary decoupling of the engine and the powertrain.

The dynamic equations describing the system in Fig. 1 are given by:

$$\begin{cases} J_e \dot{\omega}_e = \Gamma_e - \Gamma_c \\ J_g \dot{\omega}_g = \Gamma_c - \Gamma_d / \alpha \\ J_{tl} \dot{\omega}_{tl} = \Gamma_d / 2 - k_{tl} \theta_{tl} - \beta_{tl} (\omega_{tl} - \omega_{wl}) \\ J_{tr} \dot{\omega}_{tr} = \Gamma_d / 2 - k_{tr} \theta_{tr} - \beta_{tr} (\omega_{tr} - \omega_{wr}) \\ J_{wl} \dot{\omega}_{wl} = k_{tl} \theta_{tl} + \beta_{tl} (\omega_{tl} - \omega_{wl}) - R_w F_{xl} \quad (1) \\ J_{wr} \dot{\omega}_{wr} = k_{tr} \theta_{tr} + \beta_{tr} (\omega_{tr} - \omega_{wr}) - R_w F_{xr} \\ \dot{\theta}_{tl} = \omega_{tl} - \omega_{wl} \\ \dot{\theta}_{tr} = \omega_{tr} - \omega_{wr} \\ M \dot{v} = F_{xl} + F_{xr} \end{cases}$$

where ω_e is the engine rotational speed; ω_g the gearbox speed; ω_{tr} and ω_{tl} the right and left transmission speeds; ω_{wr} and ω_{wl} right and left wheel speeds; v the vehicle speed; θ_{tr} and θ_{tr} the right and left transmission torsion; Γ_e the engine torque; Γ_c the clutch torque and Γ_d the differential torque; F_{xr} and F_{xl} the right and left longitudinal wheel friction forces; k_{tr} and k_{tl} the right and left transmission stiffness coefficients; β_{tr} and β_{tl} the right and left transmission damping coefficients; R_w the wheel radius and α the gearbox reduction.

The clutch torque, Γ_c , is defined by the normal force controlled LuGre model. This solutions as-

sures a simple continuous, albeit nonlinear, alternative to the linear piecewise switching model usually proposed in literature.

$$\dot{z}_{c} = \omega_{e} - \omega_{g} - \sigma_{0c} \frac{|\omega_{e} - \omega_{g}|}{g_{c}(\omega_{d} - \omega_{g})} z_{c}$$
$$\Gamma_{c} = F_{n} \left[\sigma_{0c} z_{c} + \sigma_{1c} e^{\left(\frac{\omega_{e} - \omega_{g}}{\omega_{dc}}\right)^{2}} \dot{z}_{c} + \sigma_{2c}(\omega_{e} - \omega_{g}) \right]$$

 F_n is the normal force exerted on the clutch disks. Friction is modeled through a nonlinear springdamper dynamic system whose internal state zis analogous to the bristle flection in the bristle friction model. Detailed explication of the model and its parameters is can be found in (Olsson *et al.*, 1997).

Right and left tire longitudinal friction forces are defined by a similar averaged lumped LuGre model (Canudas de Wit *et al.*, 2003):

$$v_{ri} = v - R_w \omega_{wi}$$

$$g_i(v_{ri}) = \mu_{ci} + (\mu_{si} - \mu_{ci})e^{-|v_{ri}/v_{si}|^{1/2}}$$

$$\dot{z}_i = v_{ri} - \sigma_{0i} \frac{|v_{ri}|}{g_i(v_{ri})} - \kappa |\omega_{wi}R_w|z_i$$

$$F_{xi} = F_z \left[\sigma_{0i}z_i + \sigma_{1i}\dot{z}_l + \sigma_{2i}(v_{ri})\right]$$

The differential torque Γ_d is defined by imposing $\omega_g = 1/2(\omega_{tr} + \omega_{tl}).$

2.2 Simplified model

In order to reduce the complexity of the model the following simplifications are made:

- only the sliding phase is considered so that $\Gamma_c \simeq \gamma F_n$ since $\omega_e > \omega_g$ during a standing start
- the two sides of the transmission behave symmetrically
- perfect adherence of the tire with no dynamic effects, effectively collapsing the Lu-Gre model in $F_x = R_w \Gamma_w$
- vehicle mass is transformed in the equivalent rotational inertia
- all the elements after the gearbox are transformed in their engine-side equivalent

the resulting simplified system is thus:

$$\begin{cases} \bar{J}_e \dot{\bar{\omega}}_e = \bar{\Gamma}_e - \gamma F_n \\ \bar{J}_g \dot{\bar{\omega}}_g = \gamma F_n - \bar{k}_t \bar{\theta} - \bar{\beta}_t (\bar{\omega}_g - \bar{\omega}_v) \\ \bar{J}_v \dot{\bar{\omega}}_v = \bar{k}_t \bar{\theta} + \bar{\beta}_t (\bar{\omega}_g - \bar{\omega}_v) \\ \dot{\bar{\theta}} = \bar{\omega}_g - \bar{\omega}_v \end{cases}$$
(2)

where:

$$\bar{J}_{e} = J_{e} + J_{d} \bar{J}_{g} = J_{g} + 1/\alpha^{2} (J_{tr} + J_{tl}) \bar{J}_{v} = 1/(\alpha R_{w})^{2} M + 1/\alpha^{2} (J_{wr} + J_{wl}) \bar{k}_{t} = 1/(2\alpha^{2}) (k_{tr} + k_{tl}) \bar{\beta}_{t} = 1/(2\alpha^{2}) (\beta_{tr} + \beta_{tl})$$



Fig. 1. Powertrain scheme



Fig. 2. Simplified powertrain scheme

 \bar{J}_v is the equivalent vehicle inertia measured on the engine side of the gearbox and $\gamma = 2\mu_d R_c$ where μ_d is the dynamic clutch fiction coefficient and R_c is the mean radius of the clutch disks. This model is sufficiently simple to be used for designing the controller while still capturing the main dynamic of the the driveline.

3. CONTROLLER DESIGN

3.1 Control objectives

The control objective is reaching the clutch's engagement, i.e. $\bar{\omega}_e = \bar{\omega}_g$, while assuring the comfort and minimising the dissipated energy and actuator activity. A finite-time optimal control approach with prescribed final states has been chosen since it assures the engagement to take place in a limited time window. The optimal controller is designed for the simplified system 2 with F_n as controlled input and Γ_e as known non-controlled input.

Oscillations induced in the powertrain by the sudden change of torque due to the action of the clutch are an important element of the engagement comfort. In order to suppress, or at least reduce, these oscillations it is necessary that the torque $\Gamma_c(t_s^-)$ transmitted by the clutch just before the engagement equals the engine torque $\Gamma_e(t_s^+)$ minus the engine inertia torque. Expressing this equivalence in terms of the speeds at the sides of the clutch gives the so called *no-lurch condition* (Garofalo *et al.*, 2001) which requires that $\dot{\omega}_e(t_s^-) - \dot{\omega}_g(t_s^-) = 0$. This condition does not guarantee the driveline equilibrium after the engagement; numerical simulations, in fact, show

that even when this condition is met the surplus elastic energy in the transmission can cause residual oscillations.

Ideally after the engagement all the powertrain elements should rotate with same speed and acceleration:

$$\begin{split} \bar{\omega}_e &= \bar{\omega}_g = \bar{\omega}_v \\ \dot{\bar{\omega}}_e &= \dot{\bar{\omega}}_g = \dot{\bar{\omega}}_v = \frac{\Gamma_e}{(\bar{J}_e + \bar{J}_q + \bar{J}_v)} \end{split}$$

Defining $y_1 = \bar{\omega}_e - \bar{\omega}_g$ and $y_2 = \bar{\omega}_g - \bar{\omega}_v$, through simple algebraic manipulation the simplified system (2) can be written as:

$$\begin{cases} \dot{y}_1 = \frac{\bar{\beta}_t}{\bar{J}_g} y_2 + \frac{\bar{k}_t}{\bar{J}_g} \bar{\theta} + \frac{1}{\bar{J}_e} \Gamma_e - \frac{1}{J_{t1}} \Gamma_c \\ \dot{y}_2 = -\frac{\bar{\beta}_t}{J_{t2}} y_2 - \frac{\bar{k}_t}{J_{t2}} \bar{\theta} + \frac{1}{\bar{J}_g} \Gamma_c \\ \dot{\bar{\theta}} = y_2 \end{cases}$$
(3)

where

$$J_{t1} = \frac{\bar{J}_e \bar{J}_g}{\bar{J}_e + \bar{J}_q} \quad J_{t2} = \frac{\bar{J}_g \bar{J}_v}{\bar{J}_q + \bar{J}_v}$$

The final conditions that assure a perfect equilibrium at the end of the engagement for this model are:

$$y_1(t_f) = 0 \quad y_2(t_f) = 0 \tag{4}$$

$$\theta(t_f) = \frac{1}{k_t} \frac{\bar{J}_v \Gamma_e(t_f)}{\bar{J}_e + \bar{J}_g + \bar{J}_v} \quad \Gamma_c(t_f) = \frac{(\bar{J}_g + \bar{J}_v) \Gamma_e(t_f)}{\bar{J}_e + \bar{J}_g + \bar{J}_v}$$

Due to the physical structure of the clutch the main limitation of the actuator is the slew rate of the normal force F_n . Adding the extra state equation $\dot{\Gamma}_c = u$ to the simplified system and weighting the new input u allows to limit this aspect of the actuator activity since $\Gamma_c \propto F_n$.

3.2 Optimisation problem

All the control objectives defined in the previous subsection define the following optimisation problem: finding u(t) on $T = [t_0, t_f]$ such that minimises:

$$J = \int_{t_0}^{t_f} \left[y_1^2(t) + a \, y_2^2(t) + b \, u^2(t) \right] \mathrm{d}t$$

under the following constraints:

$$\begin{cases} \dot{y}_1 = \frac{\beta_t}{\bar{J}_g} y_2 + \frac{k_t}{\bar{J}_g} \theta + \frac{1}{\bar{J}_e} \Gamma_e - \frac{1}{J_{t1}} \Gamma_c \\ \dot{y}_2 = -\frac{\beta_t}{J_{t2}} y_2 - \frac{k_t}{J_{t2}} \theta + \frac{1}{\bar{J}_g} \Gamma_c \\ \dot{\bar{\theta}} = y_2 \\ \dot{\Gamma}_c = \gamma u \end{cases}$$
(5)

with prescribed initial and final states:

$$\begin{split} y_1(t_0) &= y_{10} \quad y_2(t_0) = y_{20} \\ \bar{\theta} &= \bar{\theta}_0 \quad \Gamma_c(t_0) = 0 \end{split}$$
$$\begin{split} y_1(t_f) &= 0 \quad y_2(t_f) = 0 \\ \bar{\theta}(t_f) &= \frac{1}{k_t} \frac{J_v \Gamma_e}{\bar{J}_e + \bar{J}_g + \bar{J}_v} \quad \Gamma_c(t_f) = \frac{(\bar{J}_g + \bar{J}_v) \Gamma_e}{\bar{J}_e + \bar{J}_g + \bar{J}_v} \end{split}$$

under the assumption that Γ_e is a measured noncontrolled input. Even dropping the hard final state constraints in favour of an heavy weighting of the final states in the cost function does not allow the use of a LQ controller since (5) is not stabilisable under the same assumptions made in subsection 3.3 for Γ_e .

The more general differential analysis theory (Agrawal and Fabien, 1994) defines the optimal input u(t) as:

$$u = -\frac{1}{2b}\lambda_4$$

where λ_4 is defined by the following Two Point Boundary Value Problem (TPBVP):

$$\dot{x} = \mathbf{A}_L x + \mathbf{B}_L \Gamma_e \tag{6}$$

where Γ_e is a known non-controllable input and:

$$x = \begin{bmatrix} y_1 \ y_2 \ \bar{\theta} \ \Gamma_c \ \lambda_1 \ \lambda_2 \ \lambda_3 \ \lambda_4 \end{bmatrix}^T$$

with the following boundary conditions:

$$\begin{array}{ll} y_1(t_0) = y_{10} & y_1(t_f) = y_{1f} \\ y_2(t_0) = y_{20} & y_2(t_f) = y_{2f} \\ \bar{\theta}(t_0) = \bar{\theta}_0 & \bar{\theta}(t_f) = \bar{\theta}_f \\ \Gamma_c(t_0) = \Gamma_{c0} & \Gamma_c(t_f) = \Gamma_{cf} \end{array}$$

3.3 Analytic solution

Solving the TPBVP implies finding $\lambda_1(t_0) \dots \lambda_4(t_0)$ that satisfy the boundary conditions in t_f effectively transforming (6) in an initial value problem (IVP). Standard TPBVP numerical resolution methods, such as the shooting method, are not an interesting option for online implementation of the controller due to their calculation cost.

Assuming Γ_e constant² over the interval T, (6) can be written as an homogeneous linear system with a non-controllable constant state Γ_e :

$$\dot{x} = \mathbf{A}_L x + \mathbf{B}_L \Gamma_e \quad \Rightarrow \quad \dot{z} = \bar{\mathbf{A}}_L z$$

where

$$z = \left[y_1 \ y_2 \ \bar{\theta} \ \Gamma_c \ \lambda_1 \ \lambda_2 \ \lambda_3 \ \lambda_4 \ \Gamma_e \right]^T$$

Since the system is linear:

$$z(t_f) = e^{\mathbf{A}_L t_f} z(t_0) = \Phi_{t_f} z(t_0)$$

Defining:

$$\bar{z}(t_0) = \left[\underbrace{\frac{y_{10} \ y_{20} \ \bar{\theta}_0 \ \Gamma_{c0}}{\bar{y}_0}}_{\bar{y}_0} \underbrace{\frac{\lambda_{10} \ \lambda_{20} \ \lambda_{30} \ \lambda_{40}}{\bar{\lambda}_0}}_{\bar{\lambda}_0} \Gamma_e \right]^T$$
$$\bar{z}(t_f) = \left[\underbrace{\frac{y_{1f} \ y_{2f} \ \bar{\theta}_f \ \Gamma_{cf}}{\bar{y}_f}}_{\bar{y}_f} \underbrace{\frac{\lambda_{1f} \ \lambda_{2f} \ \lambda_{3f} \ \lambda_{4f}}{\bar{\lambda}_f}}_{\bar{\lambda}_f} \Gamma_e \right]^T$$

 $^{^2\,}$ The same line of reasoning holds for Γ_e linearly time-dependant.

we get:

$$\begin{bmatrix} \bar{y}_f \\ \bar{\lambda}_f \\ \Gamma_e \end{bmatrix} = \begin{bmatrix} \varphi_{11} & \varphi_{12} & \varphi_{13} \\ \varphi_{21} & \varphi_{22} & \varphi_{23} \\ \varphi_{31} & \varphi_{32} & \varphi_{33} \end{bmatrix} \begin{bmatrix} \bar{y}_0 \\ \bar{\lambda}_0 \\ \Gamma_e \end{bmatrix}$$
(7)

whose first line defines the linear system:

$$\varphi_{12}\lambda_0 = \bar{y}_f - \varphi_{11}\bar{y}_0 - \varphi_{13}\Gamma_e$$

which, since φ_{12} is invertible, defines $\bar{\lambda}_0$ as a function of \bar{y}_0 , \bar{y}_f and Γ_e . Once this initial value is known the feedback controller is defined by an opportune partition of \mathbf{A}_L :

$$\dot{\bar{\lambda}} = \mathbf{A}_{\lambda}\bar{\lambda} + \mathbf{B}_{\lambda}\begin{bmatrix}y_1\\y_2\end{bmatrix}$$
$$u = \begin{bmatrix}0 \ 0 \ 0 \ -1/(2b)\end{bmatrix}\bar{\lambda}$$

3.4 Resulting controller

The normal force

$$F_n(t) = \int_{t_0}^t u(\tau) \mathrm{d}t$$

where u is defined by the dynamic feedback:

$$\dot{\bar{\lambda}} = \mathbf{A}_{\lambda}\bar{\lambda} + \mathbf{B}_{\lambda}\begin{bmatrix} y_1\\ y_2 \end{bmatrix}$$
$$u = \begin{bmatrix} 0 \ 0 \ 0 \ -1/(2b) \end{bmatrix} \bar{\lambda}$$

is the optimal clutch engagement control relative to the weight function:

$$J = \int_{t_0}^{t_f} \left[y_1^2(t) + a \, y_2^2(t) + b \, u^2(t) \right] \mathrm{d}t$$

for the system 2 under the assumption that Γ_e is a known constant or linearly time dependant input. The initial state of the controller is defined by the a linear combination of the initial and final states:

$$\bar{\lambda}_0 = \varphi_{12}^{-1} \left(\bar{y}_f - \varphi_{11} \bar{y}_0 - \varphi_{13} \Gamma_e \right)$$

4. NUMERICAL RESULTS

Simulations different optimal engagement for the same standing start scenario with various a and b parameter settings are shown in Fig. 3, 4 and 5. The a parameter weights the transmission torsion derivative. High values of a reduce the transmission torsion even if his effect is quite limited, Fig. 3, due to the finite-time nature of the controller that imposes the engagement to be completed at time t_f .



Fig. 3. Detail of the optimal engagement speed profiles for different values of a. (Dotted line a = 1)



Fig. 4. Optimal engagement speed profiles for different values of b. (Dotted line b = 1)

The *b* parameter weights the slew rate of the clutch actuator. High values, thus, give a slower clutch movement thus allowing a higher engine speed. Again, due to the finite-time nature of the controller, the engagement must be complete at time t_f ; in order to compensate the slower initial acceleration and higher engine speeds, controllers with high values of *b* have to dissipate more energy in the clutch in the second half of the engagement process.

Numerical simulations of an optimal engagement using the perviously derived controlled applied to the nonlinear model 1 have been carried out. Fig. 6 and 7 show the speed profiles and acceleration during a standing start simulation for a midsized car (Megane Scénic II) equipped with a 2.0 litres atmospheric petrol engine for $t_f = 0.8$, a = 1 and b = 10. It can be noticed that the optimal control performs remarkably well in spite of the model errors introduced by the complete system. Residual oscillations of the driveline are



Fig. 5. Optimal engagement clutch torque profiles for different values of b. (Dotted line b = 1)



Fig. 6. Engine, gearbox and vehicle speeds for the optimal controller applied to the complete model

less than $0.005m/s^2$, at least six times lower than the human perception threshold thus assuring a high level of comfort.

5. CONCLUSIONS

The engagement of a dry clutch for an automated manual transmission vehicle under realistic control and resources constraints has been considered. The finite-time optimal control problem obtained from this specifications has been addressed using the dynamic lagrangian method with analytical solution of the resulting Two Point Boundary Value Problem. The control law thus obtained has the structure of a dynamic feedback with initial states defined by a linear combination of initial and target driveline states. The proposed controller has been tested using a novel, highly realistic, nonlinear model on the standing start scenario with very good results.



Fig. 7. Vehicle acceleration for the optimal controller applied to the complete model

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