STATIC AND DYNAMIC ATTITUDE DECOMPOSITION FOR ESTIMATION WITH MAGNETOMETER SENSOR

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Abstract: A priori information given by the complete modelling of the ballistic behavior of a projectile is simplified to give a pertinent reduced evolution model. This model is composed of quasi-static and dynamic models. An extended Kalman filter is designed to estimate dynamic part of the 3 attitude angles (roll in [0, 2π], angle of attack and side-slip in the range of few milliradians) from measures of the magnetic field of the earth given by a three-axis magnetometer sensor embedded on the projectile.
The algorithm has been tested in simulation, using realistic evolution of attitude data with measurement noise.

Keywords: Kalman filters, Attitude, Magnetic fields, Models, Estimation algorithms, Nonlinear filters

1. INTRODUCTION

A three-axis magnetometer sensor is embedded on a projectile to measure the projection of the Earth magnetic field. The figure 1 shows the three-axis sensor orientation and the direction of the Earth magnetic field. According to Euler's

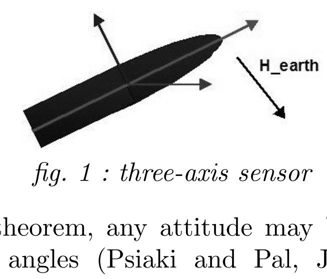


fig. 1 : three-axis sensor

rotation theorem, any attitude may be describe by three angles (Psiaki and Pal, June 1990). By this global representation, it is impossible to separate the orientation of the velocity vector from the attitude of the projectile around this vector. Therefore a new representation with 5 angles is necessary *(figure 2)* :

- η and θ define the orientation of the velocity vector in the *"Earth Frame"* $\left(\overrightarrow{i}, \overrightarrow{j}, \overrightarrow{k}\right)$;
- α, β and φ_2 define attitude of the *"projectile frame"* $\left(\overrightarrow{c}, \overrightarrow{a}, \overrightarrow{b}\right)$ in the *"velocity frame"* $\left(\overrightarrow{t}, \overrightarrow{s}, \overrightarrow{h}\right)$.

This paper is focused on the attitude estimation of the projectile around its velocity vector : only the three last angles α, β, φ_2 are to be estimated. As angles of attack α and side-slip β are in the range of few milliradians, angles and *sinus* can be

$$\underset{Earth}{(\vec{i},\vec{j},\vec{k})} \xrightarrow{\eta\,,\,\theta} \underset{Velocity}{(\vec{t},\vec{s},\vec{h})} \xrightarrow{\alpha\,,\,\beta\,,\,\varphi_2=\varphi_1+\psi} \underset{Projectile}{(\vec{c},\vec{a},\vec{b})}$$

fig. 2 : 5 angles are useful

approximated : $sin(\alpha) \simeq \alpha$ and $sin(\beta) \simeq \beta$. The roll angle $\varphi_2 = \varphi_1 + \psi$ defines the rotation around $\overrightarrow{t}$.

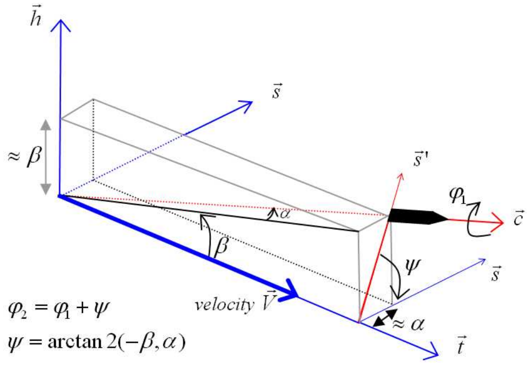


fig. 3 : attitude definition

Figure 5 describes the evolution of the attitude for a shot with a 155 mm, 45 kg rotating projectile over a distance of 16 km, with measurement noise. The whole shot lasts 50 seconds from initial conditions defined as $V_0 = 684\ m.s^{-1}$, $\eta_0 = 1,35^o$ and $\theta_0 = 25^o$. The oscillations of the projectile in a nutation and precession movement around its velocity vector are clearly apparent on figure 5.b. Temporal evolution of attitude angles α and β are specified on 5.c. These two angles are composed of static ($\alpha_s,\ \beta_s$) and dynamic ($\alpha_d,\ \beta_d$) parts (5.d). An embedded magnetometer sensor measures the projection of the Earth magnetic field along the axes of the projectile : one sensor is along the axial direction $\overrightarrow{c}$ and the two others measure magnetic field in radial directions $\overrightarrow{a}$, $\overrightarrow{b}$. The Earth magnetic field direction is known in the *"Earth frame"* $\left(\overrightarrow{i}, \overrightarrow{j}, \overrightarrow{k}\right)$ and it is measured in the *"projectile frame"* $\left(\overrightarrow{c}, \overrightarrow{a}, \overrightarrow{b}\right)$. The rotation matrix *(figure 4)* between these two frames represents attitude information. Its exact expression, defining dependance with the 5 angles used to describe attitude, is given in (Fleck, 1998).

	$\overrightarrow{c}$	$\overrightarrow{a}$	$\overrightarrow{b}$
$\overrightarrow{i}$	R_1	R_2	R_3
$\overrightarrow{j}$	R_4	R_5	R_6
$\overrightarrow{k}$	R_7	R_8	R_9

$R_i = f_i(\eta,\, \theta,\, \alpha,\, \beta,\, \varphi_2)$

fig. 4 : rotation matrix

2. MECHANICAL MODELLING

To estimate the attitude of the projectile from the magnetometer sensor measurements, a Kalman filter is designed. This algorithm needs an evolution model of the attitude for the projectile.

2.1 Evolution

An extended Kalman filter designed as in (Psiaki and Pal, June 1990), with a model based on kinematic equations in quaternion representation and a model of the torque applied to the spacecraft, would not succeed in accurately estimating attitude of our projectile attitude because of various uncertainties on the used model.
The reduced model, presented in this paper, is designed from a priori analysis of the behavior of the projectile given by the non-linear complex equations of ballistics (Fleck, 1998) :

$$\ddot{\varphi_2} \ = \ -k\,\dot{\varphi_2} \tag{1}$$

$$\xi'' + (a_1 - i\,b_1)\,\xi' + (a_2 - i\,b_2)\,\xi = a_3 - i\,b_3 \tag{2}$$

$$\text{with } \xi' = \frac{D}{V}\dot{\xi}$$

where :

- D is the constant diameter of the projectile ;
- V is the velocity ;
- $a_i,\ b_i$ *and* k are functions of velocity V, altitude y, angular rotation w_c, angles η and θ, depending on mechanical parameters ;
- ξ is a complex variable, describing attitude, which can be approximated by $\alpha - i\,\beta$ because of the range (milliradians) of theses angles.

The non-linear complex equation of attitude can be solved by separating *static* and *dynamic* behavior in the attitude movement :

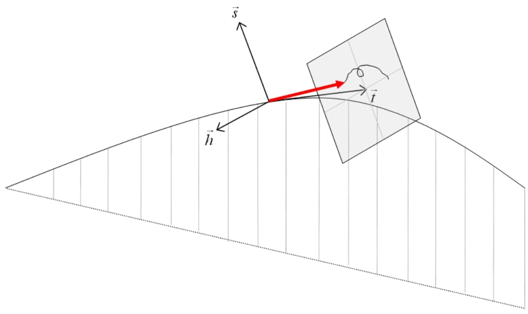
$$\begin{gathered} \xi = \xi_s + \xi_d \\ with\ \ \xi_s = \frac{a_3 - i\,b_3}{a_2 - i\,b_2}. \end{gathered} \tag{3}$$

$\dot{\xi}$ could be approximate by $\dot{\xi} = \dot{\xi_s} + \dot{\xi_d} \simeq \dot{\xi_d}$ because of the very slow variation of ξ_s in front of the variation of ξ_d.
With theses new notations and definitions, the equation of attitude is written :

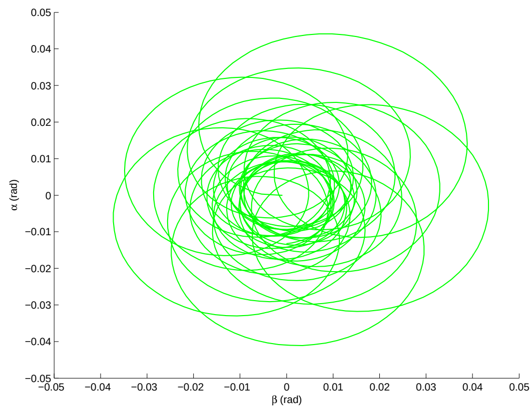
$$\begin{cases} \dfrac{D^2}{V^2}\,\ddot{\xi}_d + (a_1 - i\,b_1)\,\dfrac{D}{V}\,\dot{\xi}_d + (a_2 - i\,b_2)\,\xi_d = 0 \\ (a_2 - i\,b_2)\,\xi_s = a_3 - i\,b_3 \end{cases} \tag{4}$$

By this way, real and imaginary parts of the mechanical equation of ballistics can be written to obtain state space evolution equations :

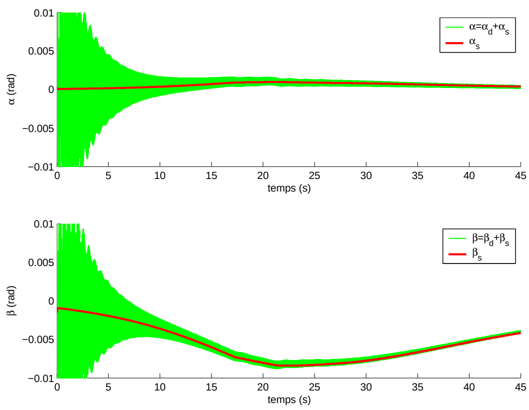
$$\dot{X} = A(X,t)\,X(t) \tag{5}$$



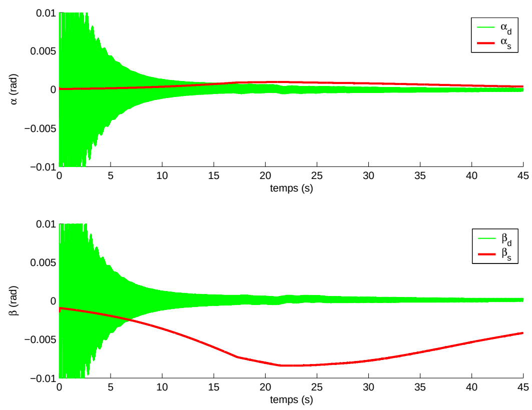
(a) schema



(b) incidence (2 first secondes)



(c) α and β



(d) α_s, α_d and β_s, β_d

fig. 5 : evolution of the projectile's attitude

with :

$$X(t) = \begin{bmatrix} \alpha_d(t) & \beta_d(t) & \dot{\alpha_d}(t) & \dot{\beta_d}(t) & \varphi_2(t) & \dot{\varphi_2}(t) \end{bmatrix}^T$$

$$A(X,t) = \begin{bmatrix} 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ -a_2\,\frac{V^2}{D^2} & b_2\,\frac{V^2}{D^2} & -a_1\,\frac{V}{D} & b_1\,\frac{V}{D} & 0 & 0 \\ -b_2\,\frac{V^2}{D^2} & -a_2\,\frac{V^2}{D^2} & -b_1\,\frac{V}{D} & -a_1\,\frac{V}{D} & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & -k \end{bmatrix}$$

and

$$\begin{cases} \alpha = \alpha_d + \alpha_s \\ \beta = \beta_d + \beta_s \end{cases}$$

where α_s and β_s are solution of 4.

$$\begin{cases} \alpha_s = \dfrac{a_2\,a_3 + b_2\,b_3}{a_2^2 + b_2^2} \\ \beta_s = \dfrac{a_2\,b_3 - b_2\,a_3}{a_2^2 + b_2^2} \end{cases} \tag{6}$$

2.2 Observation

As the sensor measures global attitude, the exact expressions of the observations, written from the rotation matrix *(figure 4)*, are composed by trigonometric expressions of the five considered angles. Because of the low amplitude of the attitude angles (α, β) and because of the low amplitude of η (by definition of frames), these expressions are simplified to give :

$$Y = [H_c \;\; H_a \;\; H_b]^T \tag{7}$$

$$\begin{aligned} Hc &= \alpha\,\{cos(\theta)\,H_j - sin(\theta)\,H_i)\} - \beta\,H_k \\ &\quad +\{-\eta\,cos(\theta)\,H_k + cos(\theta)\,H_i + sin(\theta)\,H_j\} \\ Ha &= H_1\,cos(\varphi_2) + H_2\,sin(\varphi_2) \\ Hb &= H_2\,cos(\varphi_2) - H_1\,sin(\varphi_2) \end{aligned}$$

with

$$\begin{cases} H_1 = -\alpha\,\{cos(\theta)\,H_i + sin(\theta)\,H_j)\} - sin(\theta)\,H_i \\ \qquad +cos(\theta)\,H_j + \eta\,sin(\theta)\,H_k \\ H_2 = \beta\,\{cos(\theta)\,H_i + sin(\theta)\,H_j\} \\ \qquad +\eta\,H_i + H_k \end{cases}$$

where H_i, H_j and H_k are the known components of the normalized Earth Magnetic Field at the location of the shot.

2.3 Discretization and design model

In view of real implementation, as roll rate is about $w_c = 1500\,rad.s^{-1}$, a discretization step is chosen according to Shannon's theorem : $T_e =$

0.001 s. For Kalman filter design, state and measurements noise must be also introduced :

$$\begin{cases} X[k+1] = \{I + A(X[k],k)\,T_e\}\,X[k] \\ \qquad\qquad +T_e\,B(X[k],k) + V[k] \\ Y[k+1] = C(X[k],k) + W[k] \end{cases} \tag{8}$$

$$\begin{cases} X[k+1] = X[k] + T_e\,\{A(X[k],k)\,X[k] \\ \qquad\qquad +B(X[k],k)\} + V[k] \\ Y[k+1] = C(X[k],k) + W[k] \end{cases} \tag{9}$$

$W[k]$ is a white noise of variance matrix R, introduced on the measured magnetic components. The variance matrix is chosen $R = 2.10^{-7}\,I_{3X3}$ to match with the noise characteristics observed on the real sensor outputs.
$V[k]$ is a discrete white noise of variance matrix Q introduced to take into account model errors on $X[k+1]$. Q is chosen as a diagonal matrix, as if model error on $(\alpha,\,\beta,\dot{\alpha},\,\dot{\beta},\,\varphi_2,\,\dot{\varphi_2})$ were independent from each other. $Q = diag(0\,,\,0\,,\,q\,,\,q\,,\,0\,,\,q_2)$.
On the one hand, the 3 zeros represent a perfect knowledge of the evolution model of $\alpha,\ \beta,\ \varphi_2$; for example :

$$\begin{aligned} X_1[k+1] = \alpha[k+1] &= \alpha[k] + T_e\,\dot{\alpha}[k] \\ &= X_1[k] + T_e\,X_3[k] \end{aligned} \tag{10}$$

On the other hand, the evolution model of $\dot{\alpha}$, $\dot{\beta}$ and $\dot{\varphi_2}$ are approximated : q is chosen to compensate a 2% error on $\ddot{\alpha}$ and $\ddot{\beta}$ ($\simeq$ 20 $rad.s^{-2}$), owing to a gaussian distribution of that error :

$$\begin{aligned} &3\,\sqrt{q} = 2\%\ 20\,T_e\,rad.s^{-1} \\ &\Rightarrow q = 1.7\,10^{-8}\,rad^2.s^{-2} \end{aligned} \tag{11}$$

By the same way q_2 is chosen to compensate a 2% error on $\ddot{\varphi_2}$ ($\simeq$ 10 $rad.s^{-2}$) :

$$\begin{aligned} &3\,\sqrt{q_2} = 2\%\ 10\,T_e\,rad.s^{-1} \\ &\Rightarrow q_2 = 4.45\,10^{-9}\,rad^2.s^{-2} \end{aligned} \tag{12}$$

So Q and R can be written :

$$Q = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 2\,10^{-8} & 0 & 0 & 0 \\ 0 & 0 & 0 & 2\,10^{-8} & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 5\,10^{-9} \end{bmatrix} \tag{13}$$

$$R = \begin{bmatrix} 2\,10^{-7} & 0 & 0 \\ 0 & 2\,10^{-7} & 0 \\ 0 & 0 & 2\,10^{-7} \end{bmatrix} \tag{14}$$

3. FILTERING

The filter computed in this paper is an extended Kalman filter because of non-linearity of evolution and observation model. As the filter focuses on α, β and φ_2 estimation, η, θ, v and y evolution are supposed to be known or well estimated by another system.

3.1 Algorithm

In the state space representation, the evolution matrix A depends on six mechanical coefficients a_i, b_i and the velocity V. All of them have very slow variations. According to the attitude dynamics, the evolution matrix A could be considerate as constant during the sample time : at each step of the algorithm, A is computed from *velocity* V and parameters *(a_i, b_i)*. Theses mechanical values are calculated before from velocity V, altitude y, angular motion w_c and aerodynamic coefficients (drag, etc ...).
The algorithm used to estimate attitude is presented on *figure 6*, with 2 subparts :

- the mechanical computation subsystem calculates in line the coefficients a_i and b_i at each step of the algorithm ; it also computes α_s and β_s ;
- The Kalman filter algorithm ((Brown and Hwang, 1997),(Beauvois, 1997)) is used in its extended form because of the non linearity of observation expression.

1. $(a_i,\,b_i)$ and $(\alpha_s,\,\beta_s)$ computation
2. $A_d[k] = I + A(X_{est}[k],k)\,T_e$ and $B_d[k] = T_e\,B(X_{est}[k],k)$
3. $X_{pred} = A_d\,X_{est} + B_d$
4. $\hat{\alpha_p} = \alpha_s + X_{pred}[1]$; $\hat{\beta_p} = \beta_s + X_{pred}[2]$; $\hat{\varphi 2_p} = X_{pred}[5]$
5. $\Sigma_{pred} = A_d\,\Sigma_{est}\,A_d^T + Q$
6. $C = \frac{\partial Y}{\partial X}(\hat{\alpha_p},\,\hat{\beta_p},\,\hat{\varphi 2_p})$
7. $Y_{pred} = f(\hat{\alpha_p},\,\hat{\beta_p},\,\hat{\varphi 2_p})$
8. $K = \Sigma_{pred}\,C^T\,(C\,\Sigma_{pred}\,C^T + R)^{-1}$
9. $X_{est} = X_{pred} + K\,(Y_{obs} - Y_{pred})$
10. $\Sigma_{est} = (I - K\,C)\,\Sigma_{pred}$

$X_{est}[0]$ and $\Sigma_{est}[0]$ are computed assuming that incidence is a centered random variable of standard deviation about 1^o and roll a centered uniform variable in the range $[0; 2\pi]$. By this way :

$$X_{est}[0] = \begin{bmatrix} 0 \\ 0 \\ 1 \\ 1 \\ \pi \\ w_{c0} \end{bmatrix} \tag{15}$$

$$\Sigma_{est}[0] = \begin{bmatrix} 10^{-4} & 0 & 0 & 0 & 0 & 0 \\ 0 & 10^{-4} & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 3 & 0 \\ 0 & 0 & 0 & 0 & 0 & 100 \end{bmatrix} \tag{16}$$

3.2 Results

As it is very hard to obtain, by direct measures, the real evolution of the attitude of the projec-

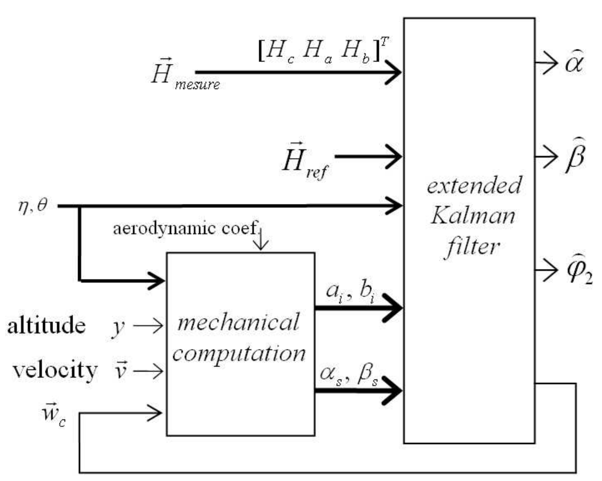


fig. 6 : Filter

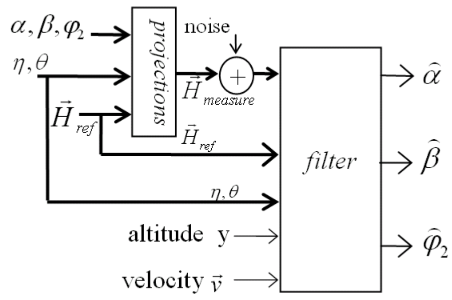
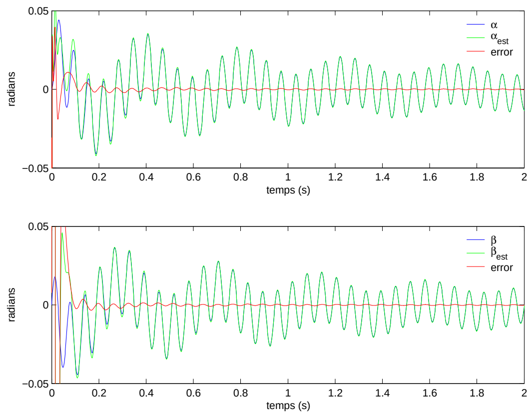
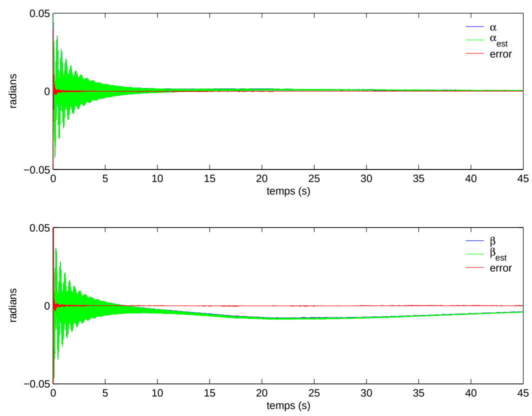


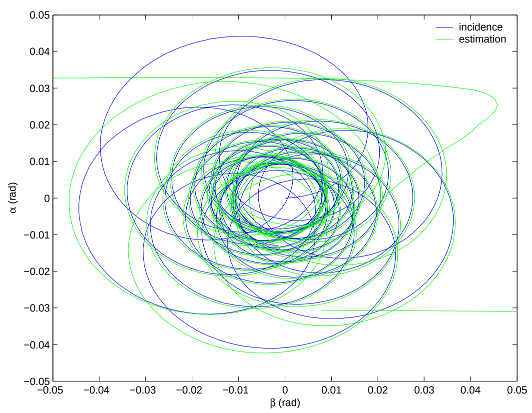
fig. 7 : Simulation process



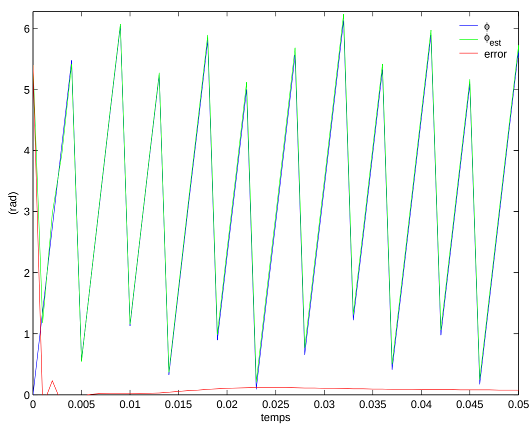
(e) begin of the shot
upper : α, $\hat{\alpha}$ and $(\alpha - \hat{\alpha})$
lower : β, $\hat{\beta}$ and $(\beta - \hat{\beta})$



(f) complete shot



(g) incidence



(h) φ_2

fig. 8 : estimation of the projectile's attitude

tile, we can only use simulation to validate the algorithm. So a complete model generates various signals to be provided to the filter (V, y, η, θ, H_a, H_b, H_c) and attitude angles for comparison with estimated variables.
The various signals obtained from the simulation model can be considerated as a realistic evolution because they have been validated by comparaison with an indirect measure of the attitude during a real shot disturbed by wind.
On *figure 8* simulation results can be studied all along the shot *(f)* and only at the beginning of the shot *(e,g)*. It is interesting to focus on the beginning of the shot because of the convergence of the algorithm : time to achieve a zero level error is about 0.1 seconds. The attitude incidence is observed in a polar diagram, during the first 2 seconds of the shot *(g)* : estimation and real incidence are very closed to each other. On figure *8.h* the estimation of roll angle φ_2 is swifter than those of α and β ; only the first 50 ms are presented there, because of the very high frequency of roll. There is no interest in presenting φ_2 estimation all along the shot because of the very low estimation error : after 50 ms, φ_2 estimation error keeps a very low level error.
The proposed algorithm has a very good accuracy all along the shot. At the beginning of the shot, attitude is well estimated due to high level of incidence oscillations. As the projectile goes ahead *(figure 8.f)*, the amplitude of the oscillations are estimated around static values. These results are sufficient to control the projectile.

4. PERSPECTIVES

The results presented in this paper could probably be adjusted by a better evaluation of the state covariance matrix Q. This matrix has been here assumed to be a diagonal matrix. To make a better estimation, it could be interesting to take into account the dependencies between α and β model error.
In this paper, the estimation algorithm is implemented in simulation with a unique sample time $T_e = 0.001\,s$. Futur works will consider a multirate implementation in order to take into account the diversity of the dynamics of the various angles and so decrease the computation time.
It will also be interesting to take into account effects of the wind on the attitude of the projectile.

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