REDUCED-ORDER H_∞ CONTROLLERS BASED ON IMPROVED GENETIC ALGORITHM

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Abstract: An approach of reduced-order H_∞ controller for a class of linear continuous dynamic systems is presented based on Genetic Algorithm. Necessary and sufficient conditions are given, in terms of linear matrix inequality, for the existence of controller. A rank condition is changed to object function of genetic algorithm(GA). The minimum order n_k of the controller and a corresponding parameter pair (X, Y) of positive definite matrix are obtained by means of searching the object function. And then the reduced-order H_∞ controllers are constructed. In this paper, the code is float, the selection operator is rank-based fitness assignment and elitist model, the crossover operator is improved real cross, the mutation operator is real mutation. The simulation results show that the controller has the same control effect as all-order H_∞ controllers based on MATLAB.

Keywords: Improved Genetic Algorithm, Linear Matrix Inequality (LMI), Reduced-order H_∞ controllers, All-order H_∞ controllers, float code

1. INTRODUCTION

It is not easy to come true in engineering because the order of H_∞ controller is higher or equal to the order of the object commonly. So it is significant in engineering to discuss the reduced-order H_∞ controller. Design problems of it can be expressed as a set of linear matrix inequality adding a rank-restricted condition of matrix (Jwasaki *et al.*, 1994), (Gahinet *et al.*, 1994), (Pan *et al.*, 2004), while the added rank-restricted condition is not a protruding restriction. Therefore, reduced-order H_∞ controller is a non-linear and non- protruded optimization problem. At present, there are not ordinary methods to solve it except for numerical value methods, such as eliminate method and variable substitute method (Grigoriadis *et al.*, 1995). But, many restriction conditions about object in eliminate method, and substitute formula is difficult to find in substitute method because of complex non-linear relation among H_∞ controller variables.

GA is a kind of process searching the optimized solution algorithm that simulates nature genetic mechanism and biological evolutionism. The characteristic of GA is that it can find the optimized solution even if any information of the solved question almost unneeded while only the information of object function needed. And also it is not restricted whether the searching space is continuous or differentiable. GA is an effective optimized searching algorithm which depends on colony searching strategy and the individual information exchanging and has not relation with the grads information. It is suitable to deal with complex non-linear searching optimization question which is difficult to solve for traditional searching optimization method. So GA provides a kind of an effective numerical value method for solving the above non-linear and non-protruding optimization questions.

A kind of reduced-order H_∞ controllers design methods based on GA is put forward aiming at linear time-invariable continuous dynamic systems. The method makes the rank of the matrix as small as possible by optimizing the matrix so that the orders of the H_∞ controllers are reduced.

2. PROBLEM DESCRIPTION

Consider a singular discrete system. The state-space description is

$$\begin{aligned} \dot{x} &= \quad Ax + B_1 w + B_2 u \\ z &= C_1 x + D_{11} w + D_{12} u \\ y &= C_2 x + D_{21} w + D_{22} u \end{aligned} \tag{1}$$

where $X \in R^n, u \in R^m, y \in R^p, z \in R^r$ and $w \in R^q$ are state, control input, measure output, controlled output and outer disturbance respectively.

Design an H_∞ output feedback controller for system (1):

$$u(s) = K(s)y(s) \tag{2}$$

then,

$$\begin{aligned} \dot{\widetilde{x}} &= A_k \widetilde{x} + B_k y \\ u &= C_k \widetilde{x} + D_k y \end{aligned} \tag{3}$$

which makes the closed loop system stable and the closed loop function $\parallel Twz \parallel_\infty$ from disturbance input ω to controlled output z less than the given positive number γ.

Reference (Xin *et al.*, 1996) gives the sufficient and necessary condition that the H_∞ controller exists, if and only if symmetry positive matrix X and Y can be found to satisfy the following inequality.

$$\begin{bmatrix} N_O & 0 \\ 0 & I \end{bmatrix}^T M_O \begin{bmatrix} N_O & 0 \\ 0 & I \end{bmatrix} < 0 \tag{4}$$

$$\begin{bmatrix} N_C & 0 \\ 0 & I \end{bmatrix}^T M_C \begin{bmatrix} N_C & 0 \\ 0 & I \end{bmatrix} < 0 \tag{5}$$

$$\begin{bmatrix} X & I \\ I & Y \end{bmatrix} \geq 0 \tag{6}$$

where,

$$\begin{bmatrix} A^T X + XA & XB_1 & C_1^T \\ B_1^T X & -I & D_{11}^T \\ C_1 & D_{11} & -I \end{bmatrix} = M_O,$$

$$\begin{bmatrix} AY + YA^T & YC_1^T & B_1 \\ C_1 Y & -I & D_{11} \\ B_1^T C_1 & D_{11}^T & -I \end{bmatrix} = M_C$$

Note N_O and N_C are matrixes constructed by any set of fundamental vector as column vectors in the subspace $ker([B_2^T \ D_{12}^T])$ and $ker([C_2 \ D_{21}])$ respectively.

For the reduced-order H_∞ controller, it is necessary to add a fourth inequality, that is

$$rank \begin{bmatrix} X & I \\ I & Y \end{bmatrix} \leq n_k + n \tag{7}$$

where n, n_k are the ranks of system model and controller respectively.

The added rank restricting condition is not a protruding restriction which is difficult to solve the symmetry positive matrix X and Y. Therefore, we convert the problem to solve the minimal rank problem in satisfied LMI of formula (4)-(6).

Define object function for GA

$$\psi(X,Y) = \sum_{i=1}^{n-n_k} \lambda_i \tag{8}$$

where, $\lambda_1 \leq \cdots \leq \lambda_{n-n_k}$ express the minimal eigenvalue of matrix $\begin{bmatrix} X & I \\ I & Y \end{bmatrix}$. Then, the design of the reduced-order H_∞ controller is converted to looking for the minimal $\varphi(X,Y)$ satisfied LMI of formula (4)-(6). So, the existence of reduced-order H_∞ controller is equivalent to the existence of the minimal value of $\varphi(X,Y)$ with the value 0.

3. SOLUTION PROCESS OF GA

Combining the discussed problems in this paper, the operating process of GA is as follows.

3.1 Code

Code(Fogel *et al.*, 1994) converts the parameters in question space to chromosomes or individuals which formed by genes in some structure in genetic space, that is mapping from question space to GA space.

This paper adopts real number coding project, because it is high-precision, natural and intuitionistic. All items in the two optimized matrixes parameters code straightly and make up of P.

3.2 Enactment of original colony

The individual in original colony is generated at random, which is the start of GA.

Suppose that system(1) does not exist control input and measure output. Then in system model, assume that $B_2 = 0, C_2 = 0, D_{12} = 0, D_{21} = 0$, so, we have $N_O = I, N_C = I$.

It is easily to know, under the condition, the formula (4)-(6) can be simplified as

$$\begin{bmatrix} A^T X + XA & XB_1 & C_1^T \\ B_1^T X & -I & D_{11}^T \\ C_1 & D_{11} & -I \end{bmatrix} < 0 \tag{9}$$

$$\begin{bmatrix} AY + YA^T & YC_1^T & B_1 \\ C_1 Y & -I & D_{11} \\ B_1^T C_1 & D_{11}^T & -I \end{bmatrix} < 0 \tag{10}$$

$$\begin{bmatrix} X & I \\ I & Y \end{bmatrix} \geq 0 \tag{11}$$

where, (9) and (10) are equal, and $Y = X^{-1}$

N individuals P_j can be achieved from inequality (9) and inverse matrix can construct the original colony.

3.3 The solution of fitness function

The evaluation of fitness function is the basis of the choice of operation, and the design of fitness function influences the performance of GA straightly.

For the minimal rank optimized question in this paper, define fitness function as follows:

$$f = \frac{1}{\varphi(X,Y) + eps} \tag{12}$$

where, eps is a small positive value .

3.4 The design of GA operation

i. Selection

Selection is to choose superior individuals and eliminate through selection or contest the inferior individuals from colony. The aim of it is to inherit the optimized individuals to the next generation. This paper adopts rank-based fitness assignment and protection of the optimized individuals of elitist model.

After calculating the fitness degree of every individual, the rank-based fitness assignment arranges individual order according to fitness degree, and then distributes probability form which is designed in advance for the individuals orderly as each selecting probability.

Elitist model does not carry out inheritance operation while copy to the next the generation straightly for the individual with the highest fitness degree, which can guarantee the optimized solution of some generation undestroyed.

ii. Crossover

Real cross includes discrete recompose, midst recompose and linear recompose etc. But the above methods have an obvious limitation, that is, when new individuals come into being, if the two bodies of father generation have the same symbols(equal in great probability), then filial generation make for increscent direction of coordinate absolute value, which go against fast convergence of optimization. So, the improved real cross(Srinivas *et al.*, 1994) operator is constructed.

The improved real cross operator is described as follows:

$$\begin{aligned} X_1' &= b_1 X_1 + b_2 X_2 \\ X_2' &= b_2 X_1 + b_1 X_2 \end{aligned} \tag{13}$$

where, $b_1 = 0.5 + b$, $b_2 = 0.5 - b$, $(b = 0.95\beta$ or $b = \beta)$, β is random in [0,1].

iii. Mutation

The essence of mutation(Zitzler *et al.*, 1999) is to look for individuals' diversity in colony and improve algorithm local random searching ability.

It is difficult to choose the step of mutation, the optimized step is determined under the exact condition. Mutation step will change automatically in optimizing process according to colony evolvement process.

Mutation step is given in the following formula:

$$X' = X \pm 0.5L\Delta \tag{14}$$

where, X and X' is individual before mutation and after mutation respectively, $\Delta = \sum_{i=0}^{m} \frac{a(i)}{2^i}$, $a(i)$ is 1 as the probability of $\frac{1}{m}$, and gets 0 as the probability of $1 - \frac{1}{m}$. In the paper, m=20, L express difference between the maximal value and the minimal value of individuals in generations.

The designing process of reduced-order H_∞ controller is as follows. Searching is stated from $n_k = n-1$, fixing n_k and looking for parameters (X, Y) of object function $\varphi(X, Y) \leq \epsilon$. If it is found, then let $n_k = n - 1$, and repeat the previous process. Otherwise if it still can not be found after generations, it shows that there is not the optimized solution satisfied object function $\varphi(X, Y) \leq \epsilon$. Then algorithm is end. The flow chart is showed in Fig. 1.

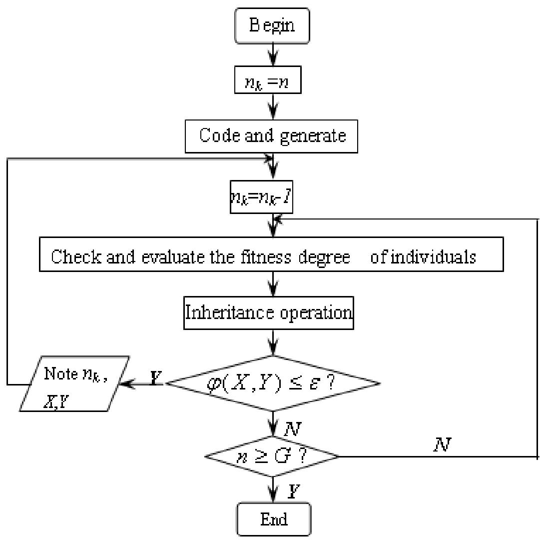

Fig. 1. Flow char of GA

The optimized solution and the order of controller are got by using the above algorithm, then the controller is designed with the method in reference (Jwasaki *et al.*, 1994).

4. EXAMPLE

Consider the following linear time invariable continuous dynamic system, in which the parameters of every object is as follows.

$$A = \begin{pmatrix} 0 & -0.2 & -0.25 & -1.0 \\ -1.0 & -2.0 & 1.0 & 0 \\ -1.0 & -0.1 & 0.85 & -1.0 \\ -0.25 & -0.5 & 0 & -0.25 \end{pmatrix},$$

$$B_1 = \begin{pmatrix} 0.25 & 0 & 0 & 0 \\ 0 & 0.25 & 0 & 0 \\ 0 & 0 & 0.25 & 0 \\ 0 & 0 & 0 & 0.25 \end{pmatrix},$$

$$B_2 = \begin{pmatrix} 1.0 & 0 \\ 1.0 & 0 \\ 0 & 1.0 \\ 0 & 1.0 \end{pmatrix},$$

$$C_1 = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 \end{pmatrix}, C_2 = \begin{pmatrix} 20 & 0 & 0 & 0 \\ 1.2 & 1.8 & 0 & 0 \\ 0 & 0.25 & 27 & 0 \\ 0 & 0 & 0 & 10 \end{pmatrix},$$

$$D_{11} = \begin{pmatrix} 0.1 & 0 & 0 & 0 \\ 0 & 0.1 & 0 & 0 \\ 0 & 0 & 0.1 & 0 \\ 0 & 0 & 0 & 0.1 \end{pmatrix}, D_{12} = \begin{pmatrix} 0 & 0 \\ 0 & 0 \\ 1 & 0 \\ 0 & 1 \end{pmatrix}$$

$$D_{21} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0.1 & 0.1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}, D_{22} = \begin{pmatrix} 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \end{pmatrix}.$$

The system exists a pair of unstable poles $0.9857 \pm 0.4587i$ distributing at right-half virtual axis of plural plane.

H_∞ controller is get making use of hinflmi() function in LMI control toolbox of MATLAB, the order of controller and model are 4. Performance of system optimization full-order H_∞ controller is γ_{opt}=0.6019, step response curve of closed loop is showed in Fig. 2.

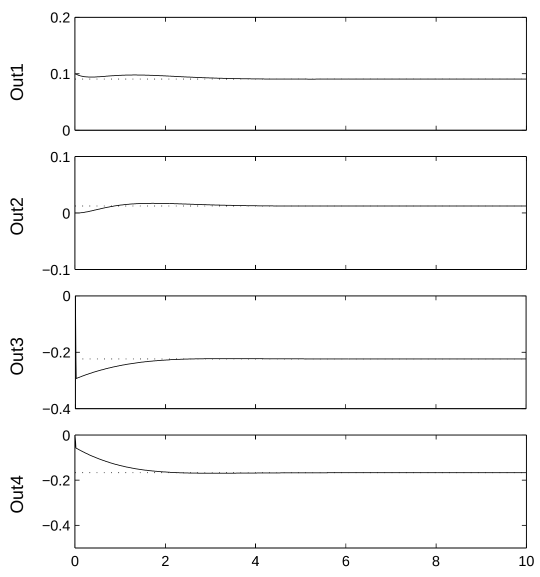

Fig. 2. Step response of all-order H_∞ controllers

Reduced-order H_∞ controller is solved by making use of the algorithm proposed in the paper. Parameters are designed as follows: N=12 −− colony model, G=100 −− the maximal value of evaluation generation, P_c=0.65 −− cross probability, P_m=0.001 −− mutation probability, $\varepsilon = 10^{-10}$ −− ending condition.

In the process of searching the minimal order of controller, when evolution has happened 19 times, that is n_k=3, object functions of the optimized individuals can be achieved 2.8×10^{-11}, while evolution has happened 100 times, that is $n_k = 2$, object functions of the optimized individual only get 1.8842. So, the minimal order of controller is 3 in the example, and 3-order H_∞ controller is designed according to corresponding value of (X, Y). The optimized H_∞ controller performance of the system is γ_{opt}=0.7854, then step responding curve of the closed loop system is shown as Fig. 3.

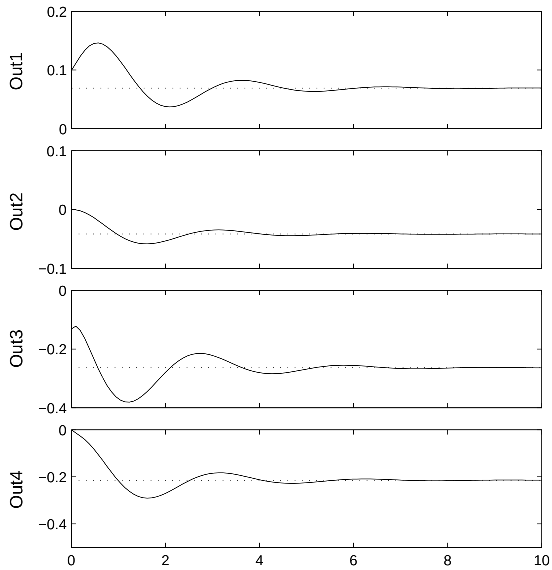

Fig. 3. Step response of all-order H_∞ controllers

Surge frequency of designed reduced-order H_∞ controller is high and stable time is long, but order of controller is reduced which makes engineering realization possible. With comparison in Fig. 2 and Fig. 3, reduced-order H_∞ controller and full-order H_∞ controller designed with MATLAB get the same control purpose.

5. CONCLUSIONS

The design method of reduced-order H_∞ controller based on GA is put forward in the paper. The minimal rank of a set of matrixes with LMI restriction is solved by GA, and then the minimal order n_k of reduced-order H_∞ controller and the corresponding parameter (X, Y) are obtained, thereby the reduced-order H_∞ controller is designed.

GA is an effective optimized searching algorithm, which regards object function value as searching information while it does not depend on grads information, it deals with coding set of decision-making variable straightly not rather than the real value of its own, which guarantees the applicability not only for control problems of singular object but also for non-singular.

The results in example show that the designed controller not only reduces the order but also stabilizes the system preferably.

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