VISION-BASED DOCKING FOR BIOMIMETIC WHEELED ROBOTS [1]

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Abstract: We present a new control law for the problem of docking a wheeled robot at a certain location with a desired heading. Recent research into insect navigation has inspired a solution which uses just one environment sensor: a video camera. The control law is of the "behavioral" or "reactive" type, in that no attempt is made to observe the relative pose of robot and target, all control actions are based on immediate visual information. Knowledge of the distance to the target is not required. Docking success under certain conditions is proved mathematically, and simulation studies show the control law to be robust to camera calibration errors.

Keywords: Mobile Robots, Robot Vision, Biocybernetics

1. INTRODUCTION

It is currently very popular among roboticists to draw inspiration from the animal kingdom. In particular, from the ability of the simpler life-forms to navigate successfully through complex environments with simple reactions to "sense data", and without a detailed environmental model (Arkin, 1998; Bar-Cohen and Breazeal, 2003; Franz and Mallot, 2000).

The trend of mimicking this with robots is termed "biomimetics". Robot navigation strategies thus derived, often categorized as "behavioral" or "reactive", are important when robots must operate in a complex environment using simple sensors. Frequently, a complete environment model would be difficult or impossible to reconstruct with the sensed information.

In this paper we propose one such strategy for the problem of positioning a wheeled robot at a certain location with a certain heading, i.e. *docking*, using information provided by a video camera. The kinematics of the robot are non-holonomic, so standard techniques of visual servoing (see, e.g., (Hutchinson *et al.*, 1996)) cannot be directly applied. We introduce a change of variables and a camera-space regulation condition which allow solution of the problem via a relatively simple nonlinear control law. Our approach was directly inspired by the work of Srinivasan and his colleagues, studying the optical navigation systems of honeybees, see (Srinivasan *et al.*, 2000) and references therein.

This research has previously prompted work on helicopter navigation (Barrows *et al.*, 2003) and missile-guidance systems (Manchester *et al.*, 2003). In that work we studied guidance with an impact-angle constraint, using a combination of geometrical considerations, and recent results in robust control and filtering theory (Manchester

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and Savkin, 2004; Manchester and Savkin, 2002; Savkin *et al.*, 2003; Petersen *et al.*, 2000; Petersen and Savkin, 1999).

We note a few some facts about honeybees which are of particular interest to researchers in control and robotics. A bee's eyes are immobile and fixed focus, and are not sufficiently separated for stereopsis to be of any real use. With these sensors, and such minute brains for processing, it seems that accurate estimation of distances is quite beyond them. However, they still manage to make smooth landings on surfaces, and find their way to and from the hive, for example. Recent studies by Srinivasan and others have indicated that reacting to optical flow in their visual field is one of their most useful navigation tools.

Many studies have been done, but one particularly striking example is the method a bee uses to land on a flat surface. The bee looks down at the ground, and measures its optical flow, or angle-rate. It is straightforward to show that, if it keeps this optical flow constant, and keeps its vertical velocity a constant proportion of its horizontal velocity, it will make a smooth landing on the surface. We refer the reader to (Srinivasan *et al.*, 2000) for details.

Throughout nature, and even in human behavior, we see many such simple "vision-space" strategies, keeping certain angles and optical flows constant, which lead to effective behaviour in physical space (or configuration space). The docking strategy we present is directly inspired by these.

Studies of the docking problem can be roughly grouped into two approaches. One focuses on the robot's "configuration space", i.e. the relative positions and angles of the robot and target, and perhaps obstacles, in the plane. All these relations are assumed to be available to the control law, and from them it chooses some desirable path. Examples are found in (de Wit and Sørdalen, 1992; Laumond, 1998; Souères and Laumond, 1996; Kelly and B.Nagy, 2003) and references therein.

The method described in (de Wit and Sørdalen, 1992) is similar in its approach to the method presented in this paper, in that the aim is to follow to a circular path. The main differences are that, firstly, they assume a slightly simpler kinematic model (often termed the *unicycle* model), and secondly, they are able to prove exponential stabilization to the desired final location, but at the expense of a control law which is more complicated and requires more information.

The other main approach focuses on "camera space" or "visual space". It is no longer assumed that the robot has knowledge of the full configuration, but only of the target's image (and obstacles) as the camera sees them. Typically it also knows how they ought to look if the goal is achieved. From this information a control law is assigned which drives the appearance of the target towards its goal. That is, dynamics are examined in camera space. Examples of this approach are found in the papers (Santos-Victor and Sandini, 1997; Hashimoto and Noritsugu, 1997; Lee *et al.*, 1999; Conticelli *et al.*, 1999; Zhang and Ostrowski, 2002; Cárdenas *et al.*, 2003) and references therein.

Our paper can be seen as a blend of the two approaches. A simple camera-space condition is defined which, if kept, leads to desirable configuration-space trajectories.

2. PROBLEM STATEMENT

Our aim is to design a control law by which a car-like vehicle may dock to a target point. The information available to the control law is consistent with the use of a video camera as the main sensor.

In this section we describe the kinematic model of the robot and the measurements available to it, and finally give a mathematically precise problem statement.

The relative position of vehicle and target is given in polar form (see Figure 1). The vehicle's position is an extension-less point in the plane, and is identified in a physical system with the mid-point of the rear axle. The scalar quantity r is the range between the vehicle and the target, and the angle ε is the angle between the desired heading, and the line-of-sight from the car to the target. These two quantities can be thought of as polar coordinates, placing the vehicle with respect to the fixed target frame.

Two more angles are required to completely characterize the state of the system. These are the heading of the vehicle, and the angle of its steering wheels. The angle λ is the angle between the vehicles current heading and the line-of-sight. The angle ϕ is the angle of the steering wheels, with respect to the centerline of the car, and is controlled with the input u. The forward speed is controlled with the input v.

The reason for this unusual representation of the state will become clear later in the paper, when the *CNG Principle* is described, and the control law derived.

The state-space of the car-target system is then the manifold $\mathbb{R} \times \mathbb{T}^3$ of states $(r, \lambda, \varepsilon, \phi)$, where $\mathbb{T}$ is the circle group: $\mathbb{R} \mod 2\pi\mathbb{Z}$.

We give the kinematic equations of motion on this manifold with respect to distance travelled

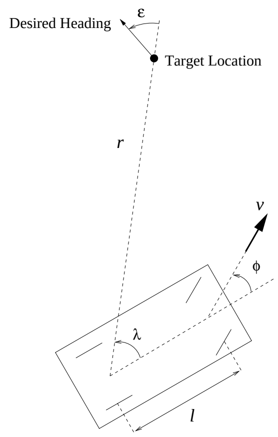


Fig. 1. System geometry

(ds), rather than time (dt), so as to make the resulting control law independent of the forward speed profile chosen. The change of variables $ds = v\cos\phi dt$ allows us to pass from one representation to another

Hereafter, x' denotes derivative of a variable x with respect to path length s. The dynamics of the states in this form are given below:

$$\lambda' = \frac{\sin\lambda}{r} - \frac{\tan\phi}{l}, \quad \varepsilon' = \frac{-\sin\lambda}{r}, \tag{1}$$

$$r' = -\cos\lambda, \quad \phi' = \frac{u}{v\cos\phi}. \tag{2}$$

Where l is the distance between the front wheels and rear wheels.

We use a measurement model consistent with a single video camera mounted on the robot, and a simple optical flow algorithm.

The main restriction felt with this model is that the range to the the target, r, is not directly measurable. Furthermore, in certain situations it is unobservable, or weakly observable, from the measurements we do have. For this reason we do not use this quantity in our control law.

The angular position of the dock-target in the field of view is the angle λ. The derivative of this variable is the optical flow of the image. A simple algorithm such as (Srinivasan, 1994), as was used in (Manchester *et al.*, 2003), can calculate this value with very little computation.

The angle ε must be known, as it is not an environmental variable, but part of the problem statement. Two possibilities of how it might be calculated are: (a) Visual analysis of the dock-target image may allow us to judge the angle between the line-of-sight and the target-heading. Or (b) If the target-heading is defined as an abstract bearing, the heading of the vehicle could be dead-reckoned and from this and the angle λ, ε could be calculated. Detailed considerations of this issue are beyond the scope of this paper.

Further to the information from the video camera, we need some knowledge of the internal state of the vehicle. Specifically, we assume knowledge of the forward speed v, the angle of the steering wheels ϕ and the distance between the axles l.

2.1 Complete Problem Statement

Our complete problem statement is this. To find a control law of the form

$$u = f(l, \phi, v, \varepsilon, \lambda, \dot{\lambda}) \tag{3}$$

such that range and angle error at final time, i.e. $r(T)$ and $\varepsilon(T)$, are minimized Corresponding to this, we make the following definition:

Definition 1. A docking manoeuvre is considered perfect if there exists some finite time T such that

$$r(T) = 0,$$
$$\lim_{t\to T} \varepsilon(t) = 0.$$

A limit is used in the above definition because if $r = 0$ the angle ε is undefined.

3. CONTROL LAW

From the optical flow measurements, we can cancel the component due to the robot's rotation ($= -v\sin\phi/l$), and retain only the component due to the relative motion of robot and dock-target. We denote this remaining flow $\mathcal{O}_f$, so:

$$\mathcal{O}_f := \dot{\lambda} + \frac{v\sin\phi}{l} \tag{4}$$

The control input u is then chosen as:

$$e_h := \lambda - \varepsilon, \quad e_c := \frac{2\mathcal{O}_f}{v\cos\phi} - \frac{\tan\phi}{l}, \tag{5}$$

$$u := lv\cos^3\phi(ae_c + be_h). \tag{6}$$

Here we can think of e_h as the heading error, and e_c as the curvature error, as the car describes a path toward the target.

The gains a and b should both be positive, and can be chosen with the following guidelines:

- The dynamics of the linear system $e_h'' + ae_h' + be_h = 0$ should represent suitable regulation to the desired path,
- The range $r_0 := 2/a$ should be small enough that divergence from the desired path within this region of the target is acceptable.

A discussion of the reasoning behind this control law, and the tuning guidelines, is presented over the next two sections.

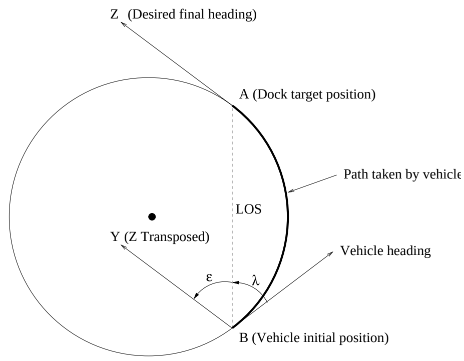


Fig. 2. Geometry for Theorem 1

4. CONTROL LAW DERIVATION

The method with which we arrived at the above control law is slightly different than most previous approaches. The control objective is to reach some final state, but rather than trying to derive a controller which provides some type of stability to this state, our approach has two stages.

Firstly, simple geometry allows us to pass from the terminal condition to a condition on the instantaneous configuration of the vehicle, this is what we call the *CNG Principle*. Secondly, from this instantaneous condition we derive a feedback-control law using methods similar to feedback linearization.

The following theorem forms the basis of our control law, and was proved in (Manchester and Savkin, 2002). It is significant because it can guide a robot to a target with a desired approach angle, without any knowledge of the distance to the target.

Theorem 1. (Circular-Navigation-Guidance Principle) Introduce the circle uniquely defined by the following properties: The initial and final positions of the vehicle lies on the circle, and the desired final-heading vector at the target's position is a tangent to the circle.

Suppose that a controller of the form (3) is designed such that the angles λ and ε are kept exactly equal over the full docking manoeuvre, then the vehicle's trajectory will be an arc on this circle. Furthermore, this will result in a perfect docking manoeuvre, as defined in Definition 1 [2]
□

This is visualized in Figure 2, where the point A is the dock target position, and B the vehicle's initial position. BA, then, is the line-of-sight, and let AZ (equivalently BY) be the desired final-heading vector.

[2] In (Manchester and Savkin, 2002) the definition of perfect intercept was slightly different. However, in the case we consider here it is equivalent to Definition 1.

Note that in the case where $\lambda = \varepsilon = \pi$ and $\phi = 0$, the car is heading away from the target, and will continue to do so forever. In a sense, the car is following a circle of infinite radius: a straight line.

It is only in this case, corresponding to just a one-dimensional line in a four-dimensional manifold, where a perfect docking manoeuvre will not occur. Since this is a "thin" set, and would be simple to overcome in practice, we do not consider it further.

In order to regulate λ to be equal to ε, we consider two errors: $\lambda - \varepsilon$ and $\lambda' - \varepsilon'$. The second of these can be expanded as follows, from Equations (1,2, 4):

$$\lambda' - \varepsilon' = \frac{2\sin\lambda}{r} - \frac{\tan\phi}{l}, \quad (7)$$

$$= \frac{2\mathcal{O}_f}{v\cos\phi} - \frac{\tan\phi}{l}, \quad (8)$$

giving us Equation (5).

This can also be interpreted in the following way: Given any position of the car in the plane, relative to the dock target, there exists a unique circle it should follow. To follow this circle, it must have a certain instantaneous heading and curvature. There are then two errors worth considering: heading error and curvature error. e_h is obviously the heading error, and e_c is the curvature error.

This follows, since the curvature of the circle defined in Theorem 1 is given by the function $2\sin\lambda/r$, and the instantaneous curvature of the vehicle is given by the function $\tan\phi/l$.

If both of these errors are zero, then the vehicle will follow a circular path to the dock target. We can think of these error functions as describing a two-dimensional *target sub-manifold* of the four-dimensional state-space:

$$M := \{(r, \lambda, \varepsilon, \phi) : e_h = 0 \text{ and } e_c = 0)\}.$$

Viewed like this, our objective is similar to that of sliding-mode control: to regulate the system to a particular sub-manifold on which it is known to behave well.

So we have transformed the terminal-state control problem into an instantaneous-state control problem, i.e. the regulation of e_h and e_c. This is reminiscent of the way a honeybee can land on a surface by regulating certain visual cues. We now tackle this regulation problem in a way similar to input-output linearization (see, e.g., (Khalil, 1993), Chapter 13), and analyze the resulting control law using Lyapunov theory.

Let us choose the heading error, $e_h = \lambda - \varepsilon$, as an output function, and attempt to regulate it using input-output linearization.

Differentiating e_h with respect to path-length, we obtain:

$$e_h' = \frac{2\sin\lambda}{r} - \frac{\tan\phi}{l} = e_c,$$

We differentiate this again, obtaining

$$e_h'' = \frac{2\cos\lambda}{r}e_c - \frac{\sec^3\phi}{lv}u. \tag{9}$$

In this equation we note that the control appears explicitly, so a natural approach would be introduce a fictional control input $\bar{u}$ and set

$$u = lv\cos^3\phi\left(-\bar{u} + \frac{2\cos\lambda}{r}e_c\right) \tag{10}$$

rendering the dynamics from $\bar{u}$ to e_h linear, in fact just a double integrator. However, since the range r is unknown to the controller, we cannot do this.

We then "almost feedback linearize" the system, and treat the first term in (9) like an uncertainty. The second term is canceled with the nonlinear control law:

$$u = lv\cos^3\phi(ae_c + be_h)$$

as given in Section 3, then we have

$$e_h'' + \left(a - \frac{2\cos\lambda}{r}\right)e_h' + be_h = 0. \tag{11}$$

If r is large, this is "almost" like the linear system

$$e_h'' + ae_h' + be_h = 0,$$

and it is clear that, by choosing a and b, both the errors e_h and $e_c = e_h'$ can be made to converge in any desired fashion.

5. CONTROL LAW ANALYSIS

Since our control law only "almost" linearized the system, we need some further analysis to understand how the system will behave.

The following simple theorem says this: if we start with zero errors, we will continue to have zero errors and achieve a perfect docking manoeuvre. Another way to put this is that if, at any time, the state $(r, \lambda, \varepsilon, \phi) \in M$ then it will stay in M.

Theorem 2. Suppose the vehicle system (1, 2) has the desired heading and curvature, i.e. $e_h(0) = 0$ and $e_c(0) = 0$, then the vehicle will perform a perfect docking manoeuvre, as per Definition 1. □

The proof is straightforward from Equation (11) and Theorem 1.

Now suppose the state starts outside M, that is, with incorrect heading and curvature. Now we'd like to know something about convergence to the target sub-manifold. The dynamics of (11) are those of a linear system with time-varying coefficients, and can be analysed with Lyapunov theory.

Theorem 3. Consider the function

$$V(e_h, e_c) := be_h^2 + e_c^2. \tag{12}$$

This is a positive-definite quadratic form in the heading and curvature errors, and may be considered as the distance to the target sub-manifold.

Let $[s_1, s_2], s_2 > s_1$ be any path interval over which $V(e_h, e_c, s) \neq 0$ and the following inequality holds:

$$a - 2\cos\lambda/r > 0. \tag{13}$$

Then $V(e_h(s_2), e_c(s_2)) < V(e_h(s_1), e_c(s_1))$. That is, over any interval of non-zero length, the norm of the errors strictly decreases. □

The proof is omitted due to space restrictions.

This theorem reflects the following physically meaningful problem: When the vehicle is very close to the desired target location, large gains are required to make it swing around and track the correct path.

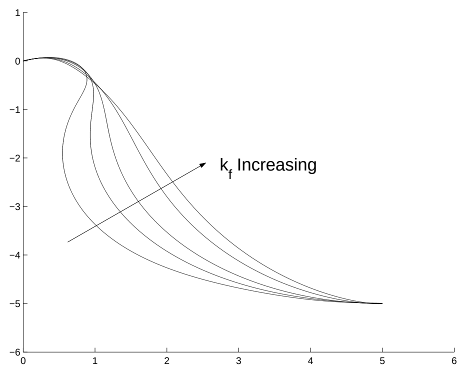
6. ROBUSTNESS

It has been mentioned in the literature that a particularly important test of a docking algorithm is the robustness of its terminal positioning precision to imperfect modeling of the kinematics and camera calibration (Cárdenas *et al.*, 2003), (Laumond, 1998).

The parameters chosen for the simulation were: l = 1m, v = 1m/s, a = 4, b = 4.04. The initial conditions were $r(0)$ = 7m, $\lambda(0) = \pi/4$ rad, $\varepsilon = \pi/4$ rad, $\phi = \pi/8$ rad.

We simulate the effect of incorrect camera calibration. We skew the measurement of λ and the optical flow in a way consistent with an incorrect assumption on the focal length of the camera. We introduce the ratio k_f as the true focal length divided by the assumed focal length.

This parameter was varied from 0.6 to 1.8. In Figure 3 we see graphical plots of trajectories, and numerical data for the final range and final-angle error. It is clear that, although the trajectories throughout the middle stage of the docking manoeuvre vary widely, in all cases the robot docked with less than 1cm positioning error, and less than 10° angle error.



k_f	0.6	0.8	1	1.4	1.8
r(T)($\times 10^{-2}$)	0.44	0.04	0.06	0.23	0.36
$\varepsilon(T)$ (deg)	7.93	2.42	0.12	7.25	9.05

Fig. 3. The effect of incorrect camera calibration

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