ONE-STEP BACKSTEPPING DESIGN FOR ADAPTIVE OUTPUT FEEDBACK CONTROL OF UNCERTAIN NONLINEAR SYSTEMS

Ikuro Mizumoto * **Ryuji Michino** * **Makoto Kumon** *
Zenta Iwai *

* *Department of Mechanical Engineering, Kumamoto University, 2-29-1 Kurokami, Kumamoto, 860-8555, Japan*

Abstract: In this paper, a novel one-step backstepping design scheme for an adaptive output feedback control of uncertain nonlinear system with a higher order relative degree and nonparametric uncertainties is proposed. The proposed method can design an output feedback based adaptive controller through a backstepping of only one step even when the controlled system has a higher order relative degree.

Keywords: adaptive control, output feedback, uncertain nonlinear system, backstepping, parallel feedforward compensator

1. INTRODUCTION

A nonlinear system is said to be OFEP (output feedback exponentially passive) if there exists an output feedback such that the resulting closed-loop system is exponentially passive (Fradkov and Hill, 1998). The sufficient conditions for a nonlinear system to be OFEP have been provided (Fradkov and Hill, 1998) such as (1) the system has a relative degree of one, (2) the system be globally exponential minimum-phase and (3) the nonlinearities of the system satisfy the Lipschitz condition. It has been shown that, under these conditions, one can easily stabilize uncertain nonlinear systems with a simple high-gain output feedback based adaptive controller (Allgower *et al.*, 1997; Fradkov, 1996; Fradkov *et al.*, 1999). These adaptive control methods utilize only the output signal without an observer to design the output feedback controller, as a result the structure of the controllers is very simple. Further it has been shown that the methods have a strong robustness with respect to bounded disturbances in spite of its simple structure. Therefore the control methods based on the OFEP property of the controlled system are considered one of the powerful control tools for uncertain nonlinear systems. Unfortunately however, since most practical systems do not satisfy the OFEP conditions mentioned above, the OFEP conditions have imposed very severe restrictions to practical application of OFEP based adaptive output feedback controls.

In order to improve the applicability of the above-mentioned OFEP based adaptive control to practical systems, some alleviation methods to OFEP conditions have been proposed (Fradkov, 1996; Fradkov *et al.*, 1999; Michino *et al.*, 2003; Mizumoto *et al.*, 2003). The introduction of a parallel feedforward compensator (PFC) in parallel with the controlled non-OFEP system is a simple and innovative method to alleviate the restrictions imposed by the relative degree and/or the minimum-phase property (Fradkov, 1996; Fradkov *et al.*, 1999). However since the controller is designed for a system augmented with a PFC, which is rendered OFEP by adding the PFC, the bias error from the PFC output may remain in the actual output even though one could attain a control objective for the augmented OFEP system with the

PFC (Kaufman *et al.*, 1998; Iwai and Mizumoto, 1994). A robust design for a non-OFEP system, in which there exist uncertain nonlinearities that do not satisfy the Lipschitz condition and that can not be represented by parametric form, has also been presented in Michino *et al.* (2003). However, this method is suitable for systems having a relative degree one. Recently, the method proposed in Michino *et al.* (2003) was extended to systems of triangular form with higher order relative degrees by utilizing the backstepping strategy (Mizumoto *et al.*, 2003). Unlike the previous works (Polycarpou and Ioannou, 1996; Yao and Tomizuka, 1997; Jiang and Praly, 1998; Arslan and Basar, 1999; Lin and Qian, 2002) on robust adaptive control of such an uncertain triangular system, the method proposed in Mizumoto *et al.* (2003) can design an adaptive controller without the use of state variables and/or a state observer by introducing a virtual filter for the control input since the actual control input is designed through a backstepping strategy applied to the virtual filter. The introduction of a virtual input filter was initiated by Marino and Tomei (1993). One can solve a problem on the relative degree by using a virtual input filter. In method by Mizumoto *et al.* (2003), the virtual input filter method has been applied in order to overcome the restriction of the relative degree in the OFEP conditions. However, the structure of the controller might become complex for a system with a higher order relative degree because the number of steps in the recursive design of the controller through backstepping depends on the order of the relative degree of the controlled system.

In this paper, we will propose a novel one step backstepping design method for an adaptive controller based on high-gain output feedback for uncertain nonlinear systems. We introduce a virtual input filter augmented by a PFC to design an adaptive controller through a backstepping strategy of only one step even if the controlled system has a higher order relative degree. In the proposed method, a virtual filter is first introduced in order to solve a problem imposed by relative degree and in order to make the virtual OFEP system as well as previous methods by Marino and Tomei (1993) and Mizumoto *et al.* (2003). Further, a PFC will be added in parallel with the virtual filter in order to create an augmented virtual filter with a relative degree of one. This augmentation allows us to design an adaptive controller through a backstepping of only one step even when the controlled system has a higher order relative degree. Unlike the conventional method with a PFC only (Fradkov, 1996; Fradkov *et al.*, 1999; Kaufman *et al.*, 1998; Iwai and Mizumoto, 1994), when the PFC is put in parallel with the virtual filter, the bias effect from the PFC output does not directly appear in the output of the controlled system. Therefore, we can show that the tracking error of the control system with proposed robust adaptive controller converges to any given bound. Further unlike the general method using a backstepping strategy, we can obtain a very simple adaptive controller through backstepping of only one step even when the controlled system has a higher order and a higher order of relative degree.

2. PROBLEM STATEMENT

We consider the following nth order uncertain time varying nonlinear system with a relative degree of γ.

$$\begin{aligned} \dot{x}_i &= f_i(\boldsymbol{x},t) + g_i(t)x_{i+1} \ (1 \le i \le \gamma - 1) \\ \dot{x}_r &= f_r(\boldsymbol{x},t) + g_r(t)u(t) + \boldsymbol{b}(t)^T\boldsymbol{\eta} \\ \dot{\boldsymbol{\eta}} &= \boldsymbol{f}_\eta(\boldsymbol{x},t) + \boldsymbol{q}(y,\boldsymbol{\eta}) \\ y &= x_1 \end{aligned} \quad (1)$$

where $\boldsymbol{x} = [x_1, \cdots, x_n]^T \in R^n$, $\boldsymbol{\eta} = [x_{\gamma+1}, \cdots, x_n]^T \in R^{n-\gamma}$ are the state variables, $u, y \in R$ are the control input and output, respectively. $g_i(t)$ and $\boldsymbol{b}(t)$ are unknown time-varying functions, $f_i(\boldsymbol{x},t), \boldsymbol{f}_\eta(\boldsymbol{x},t)$ and $\boldsymbol{q}(y,\boldsymbol{\eta})$ are uncertain nonlinear functions. Here we impose the following assumptions to the system (1).

Assumption 1. The uncertain nonlinear functions $f_i(\boldsymbol{x},t)$ and the vector function $\boldsymbol{f}_\eta(\boldsymbol{x},t)$ can be evaluated for all $\boldsymbol{x} \in R^n$ and $t \in R^+$ by

$$\begin{aligned} |f_i(\boldsymbol{x},t)| &\le d_{1i}|\psi_i(y)| + d_{0i} \ (1 \le i \le \gamma) \\ \|\boldsymbol{f}_\eta(\boldsymbol{x},t)\| &\le d_{1\eta}|\psi_\eta(y)| + d_{0\eta} \end{aligned} \quad (2)$$

with unknown positive constants $d_{1i}, d_{1\eta}, d_{0i}, d_{0\eta}$ and known smooth functions $\psi_i(y)$ and $\psi_\eta(y)$ that have the following properties for any variables y_1 and y_2:

$$\begin{aligned} |\psi_i(y_1+y_2)| &\le |\psi_{1i}(y_1,y_2)||y_1| + |\psi_{2i}(y_2)| \\ |\psi_\eta(y_1+y_2)| &\le |\psi_{1\eta}(y_1,y_2)||y_1| + |\psi_{2\eta}(y_2)| \end{aligned} \quad (3)$$

with known smooth functions ψ_{1i} and $\psi_{1\eta}$ and with functions ψ_{2i} and $\psi_{2\eta}$ which are bounded for all bounded y_2.

Assumption 2. Unknown functions $g_i(t)$ $(1 \le i \le r)$ are smooth and bounded with bounded derivative for any $t \ge 0$ and there exists an unknown positive constant g_m such that

$$g_{1,r}(t) := \prod_{i=1}^{r} g_i(t) \ge g_m > 0. \quad (4)$$

Assumption 3. Unknown vector function $\boldsymbol{b}(t)$ is bounded for all $t > 0$.

Assumption 4. The uncertain nonlinear function $\boldsymbol{q}(y,\boldsymbol{\eta})$ is globally Lipschitz with respect to $(y,\boldsymbol{\eta})$, i.e., there exists a positive constant L_1 such that

$$\|\boldsymbol{q}(y_1,\boldsymbol{\eta}_1) - \boldsymbol{q}(y_2,\boldsymbol{\eta}_2)\| \le L_1(|y_1 - y_2| + \|\boldsymbol{\eta}_1 - \boldsymbol{\eta}_2\|). \quad (5)$$

for any variables y_1, y_2 and $\boldsymbol{\eta}_1, \boldsymbol{\eta}_2$.

Assumption 5. Nominal part of the system (1) is exponentially minimum-phase. That is, the zero dynamics of the nominal system:

$$\dot{\boldsymbol{\eta}}(t) = \boldsymbol{q}(0,\boldsymbol{\eta}) \quad (6)$$

is exponentially stable.

Under these assumptions the control objective is to achieve the goal

$$\lim_{t\to\infty} |y(t) - y^*(t)| \le \delta \tag{7}$$

for a given positive constant δ and a smooth reference signal $y^*(t)$ such as $|y^*(t)| \le \sigma_0$ and $|\dot{y}^*(t)| \le \sigma_1$ with positive constants σ_0 and σ_1.

3. CONTROLLER DESIGN

3.1 Virtual System

For the controlled system (1) we introduce the following $(\gamma-1)$th order stable virtual input filter:

$$\begin{aligned} \dot{\boldsymbol{u}}_f &= A_{u_f}\boldsymbol{u}_f + \boldsymbol{b}_{u_f} u \\ u_{f1} &= c_{u_f}^T \boldsymbol{u}_f \end{aligned} \tag{8}$$

where $\boldsymbol{u}_f = [u_{f_1}, \cdots, u_{f_{\gamma-1}}]^T$ and

$$A_{u_f} = \begin{bmatrix} \mathbf{0} & I_{\gamma-2,\gamma-2} \\ -\beta_1 \cdots - \beta_{\gamma-1} \end{bmatrix},\ \boldsymbol{b}_{u_f} = \begin{bmatrix} \mathbf{0} \\ b_u \end{bmatrix},\ \boldsymbol{c}_{u_f} = \begin{bmatrix} 1 \\ \mathbf{0} \end{bmatrix}$$

The virtual system, which is obtained by considering u_{f_1} given from a virtual input filter as the control input, can be expressed by the following form with an appropriate variable transformation using the filtered signal u_{f_i} given in (8).

$$\begin{aligned} \dot{y} &= a(y, \boldsymbol{\xi}, t) + g'_{1,r}(t)u_{f_1} + f_1(y, \boldsymbol{\xi}, \boldsymbol{\eta}, t) \\ \dot{\boldsymbol{\xi}} &= A_{u_f}\boldsymbol{\xi} + \boldsymbol{a}_\xi(t)y + B_\xi(t)\boldsymbol{\eta} + \boldsymbol{F}(y, \boldsymbol{\xi}, \boldsymbol{\eta}, t) \\ \dot{\boldsymbol{\eta}} &= \boldsymbol{q}(y, \boldsymbol{\eta}) + \boldsymbol{f}_\eta(y, \boldsymbol{\xi}, \boldsymbol{\eta}, t) \end{aligned} \tag{9}$$

where $\boldsymbol{\xi} = [\xi_2, \cdots, \xi_k, \cdots, \xi_\gamma]^T$,

$$\xi_k = b_u \bar{g}_{k,r} x_k - u_{f_{k-1}} - \sum_{d=1}^{k-1} \chi_{k,d} x_{k-d} \tag{10}$$

with $g_0(t) := 1,\ \bar{g}_m(t) := \frac{1}{g_m(t)}$ and

$$g_{m,n}(t) := \prod_{i=m}^{n} g_i(t),\ \bar{g}_{m,n}(t) := \frac{1}{g_{m,n}(t)},$$

$$\chi_{r,1} = b_u \bar{g}_{r-1}(\beta_{r-1}\bar{g}_r + \dot{\bar{g}}_r)$$

$$\chi_{r,k} = \bar{g}_{r-k}(-\sum_{d=1}^{k-1} \beta_{r+d-k}\chi_{r+d-k+1,d} - \dot{\chi}_{r,k-1} + b_u \beta_{r-k}\bar{g}_{r-k+1,r}),\ (2 \le k \le r-1)$$

$$\chi_{r,r} = -\sum_{d=1}^{r-1} \beta_d \chi_{d+1,d} - \dot{\chi}_{r,r-1}$$

$$\chi_{k,1} = \bar{g}_{k-1}(\chi_{k+1,1} + b_u \dot{\bar{g}}_{k,r}),\ (2 \le k \le r-1)$$

$$\chi_{k,d+1} = \bar{g}_{k-d-1}(\chi_{k+1,d+1} - \dot{\chi}_{k,d}),\ (2 \le d \le k-1)$$

Further, $a, \boldsymbol{a}_\xi, B_\xi$ and $\boldsymbol{F}$ in (9) are given by the following form:

$$a(y, \boldsymbol{\xi}, t) = (\xi_2 + \chi_{2,1}y)g'_{1,r},\ g'_{1,r} = g_{1,r}/b_u$$

$$\boldsymbol{a}_\xi(t) = \begin{bmatrix} \chi_{2,2} \\ \vdots \\ \chi_{r,r} \end{bmatrix},\ B_\xi = \begin{bmatrix} \mathbf{0} \\ b_u \bar{g}_r \boldsymbol{b}^T \end{bmatrix}$$

$$\boldsymbol{F}(y, \boldsymbol{\xi}, \boldsymbol{\eta}, t) = [f_{\xi_2}, \cdots, f_{\xi_k}, \cdots, f_{\xi_\gamma}]^T$$

$$f_{\xi_k} = b_u \bar{g}_{k,r} f_k - \sum_{d=1}^{k-1} \chi_{k,d} f_{k-d}$$

For the obtained virtual system, it is easy to confirm from assumption 2 that $a(y, \boldsymbol{\xi}, t)$ is Lipschitz with respect to $(y, \boldsymbol{\xi})$ so that there exists a positive constant L_2 such that

$$|a(y_1, \boldsymbol{\xi}_1) - a(y_2, \boldsymbol{\xi}_2)| \le L_2(|y_1 - y_2| + ||\boldsymbol{\xi}_1 - \boldsymbol{\xi}_2||). \tag{11}$$

The uncertain vector function $\boldsymbol{F}(y, \boldsymbol{\xi}, \boldsymbol{\eta}, t)$ can be evaluated from assumption 1 by

$$||\boldsymbol{F}(y, \boldsymbol{\xi}, \boldsymbol{\eta}, t)|| \le p_1|\phi(y)| + p_0 \tag{12}$$

with unknown positive constants p_0, p_1 and a known function $\phi(y)$, which has the following property for any variables y_1 and y_2:

$$|\phi(y_1 + y_2)| \le |\phi_1(y_1, y_2)||y_1| + |\phi_2(y_2)| \tag{13}$$

with a known smooth function $\phi_1(y_1, y_2)$ and a function $\phi_2(y_2)$ that is bounded for all bounded y_2. Furthermore, since A_{u_f} is a stable matrix, there exists a positive symmetric matrix P_ξ for any positive matrix Q_ξ such as

$$P_\xi A_{u_f} + A_{u_f}^T P_\xi = -Q_\xi. \tag{14}$$

Moreover, since the system (1) is exponentially minimum-phase from assumption 5, there exist a positive definite function $W(\boldsymbol{\eta})$ and positive constants κ_1 to κ_4 from the converse theorem of Lyapunov(Khalil, 1996) such that

$$\begin{aligned} &\frac{\partial W(\boldsymbol{\eta})}{\partial \boldsymbol{\eta}} \boldsymbol{q}(0, \boldsymbol{\eta}) \le -\kappa_1 \|\boldsymbol{\eta}(t)\|^2, \left\| \frac{\partial W(\boldsymbol{\eta})}{\partial \boldsymbol{\eta}} \right\| \le \kappa_2 \|\boldsymbol{\eta}(t)\| \\ &\kappa_4 \|\boldsymbol{\eta}(t)\|^2 \le \|W(\boldsymbol{\eta})\| \le \kappa_3 \|\boldsymbol{\eta}(t)\|^2 \end{aligned} \tag{15}$$

The obtained virtual system (9) with the input u_{f_1} has a relative degree of one and the zero dynamics of the nominal part of (9), which is obtained by neglecting f_1, F and f_n as external disturbances, is exponentially stable because A_{u_f} is a stable matrix and the zero dynamics of (1) given in (6) is exponentially stable. Thus the nominal part of the virtual system (9) has the OFEP properties so that we can attain the control objective (7) by applying the robust adaptive control method provided in Michino *et al.* (2003) to the virtual system (9) with the control input u_{f_1}. However, unfortunately since the input u_{f_1} is not the actual control input, one can not directly design u_{f_1} using the controller design method given in Michino *et al.* (2003). The method given in Mizumoto *et al.* (2003) gives us a solution for designing an actual control input by adopting the *Backstepping* strategy to the virtual input filter. However, this method might require a complex controller structure for a nonlinear system with a higher order relative degree.

Here we propose a novel one-step backstepping design scheme utilizing a PFC so that one can design an adaptive controller through a backstepping of only one step for uncertain nonlinear systems even when the nonlinear systems have higher order relative degrees.

3.2 Augmented Virtual System

Consider a stable PFC with relative degree of 1 and minimum phase:

$$\begin{aligned} \dot{y}_f &= -a_{f_1} y_f + \boldsymbol{a}_{f_2}^T \boldsymbol{\eta}_f + b_a u \\ \dot{\boldsymbol{\eta}}_f &= A_f \boldsymbol{\eta}_f + \boldsymbol{b}_f y_f \end{aligned} \tag{16}$$

where $y_f \in R$ is the PFC output and $\boldsymbol{\eta}_f \in R^{n_f - 1}$ is the state variables of PFC. a_f is any positive constant and A_f is a stable matrix.

Suppose that the PFC (16) is designed such that the augmented virtual filter, which is costituted by the virtual filter and the PFC, has a relative degree of one and is minimum-phase. The augmented virtual filter can be expressed as follows(Isidori, 1995):

$$\begin{aligned} \dot{u}_{a_f} &= a_{a1} u_{af} + \boldsymbol{a}_{a2}^T \boldsymbol{\eta}_a + b_a u \\ \dot{\boldsymbol{\eta}}_a &= A_a \boldsymbol{\eta}_a + \begin{bmatrix} \boldsymbol{0} \\ 1 \end{bmatrix} u_{a_f} \end{aligned} \tag{17}$$

where $u_{a_f} = u_{f_1} + y_f, b_a = \boldsymbol{c}_f^T \boldsymbol{b}_f$ and A_a is the system matrix corresponding to the zero dynamics of the augmented virtual filter. Since the augmented virtual filter is minimum-phase, A_a is a stable matrix.

The virtual system (9) with the augmented virtual filter output u_{a_f} as the control input can be represented by

$$\begin{aligned} \dot{y} &= a(y, \boldsymbol{\xi}, t) + g'_{1,r}(t)(u_{a_f} - y_f) + f_1(y, \boldsymbol{\xi}, \boldsymbol{\eta}, t) \\ \dot{\boldsymbol{\xi}} &= A_{u_f} \boldsymbol{\xi} + \boldsymbol{a}_\xi(t) y + B_\xi(t) \boldsymbol{\eta} + \boldsymbol{F}(y, \boldsymbol{\xi}, \boldsymbol{\eta}, t) \\ \dot{\boldsymbol{\eta}} &= \boldsymbol{q}(y, \boldsymbol{\eta}) + \boldsymbol{f}_\eta(y, \boldsymbol{\xi}, \boldsymbol{\eta}, t) \end{aligned} \tag{18}$$

3.3 Adaptive Controller Design

Consider the tracking error $\nu(t) = y(t) - y^*(t)$, the resulting error system is given from (18) by

$$\begin{aligned} \dot{\nu} &= a(\nu + y^*, \boldsymbol{\xi}) + g'_{1,r}(u_{a_f} - y_f) + f_1(\nu + y^*, \boldsymbol{\xi}, \boldsymbol{\eta}) - \dot{y}^* \\ \dot{\boldsymbol{\xi}} &= A_{u_f} \boldsymbol{\xi} + \boldsymbol{a}_\xi y + B_\xi \boldsymbol{\eta} + \boldsymbol{F}(\nu + y^*, \boldsymbol{\xi}, \boldsymbol{\eta}) \\ \dot{\boldsymbol{\eta}} &= \boldsymbol{q}(\nu + y^*, \boldsymbol{\eta}) + \boldsymbol{f}_\eta(\nu + y^*, \boldsymbol{\xi}, \boldsymbol{\eta}) \end{aligned} \tag{19}$$

Pre-step: For the error system (19), We first design a virtual control input α_1 for the augmented virtual filtered signal u_{a_f} in the error system as follows by using a robust adaptive high gain feedback control:

$$\alpha_1(t) = -k(t)\nu(t) + \Psi_0(t) \tag{20}$$

$$k(t) = k_I(t) + k_P(t) + k_R(t) \tag{21}$$

$$\dot{k}_I(t) = \gamma_I D(\nu)\nu(t)^2, \ k_I(0) \geq 0 \tag{22}$$

$$k_P(t) = \gamma_P[\phi_1(\nu, y^*)^4 + \psi_{1\eta}(\nu, y^*)^4]\nu(t)^2 \tag{23}$$

$$k_R(t) = \gamma_R \psi_1(y)^2 \tag{24}$$

$$\dot{\Psi}_0(t) = D(y_f)[-a_{f_1}\Psi_0 + b_a u] \tag{25}$$

where $\gamma_I, \gamma_P, \gamma_R$ are arbitrary positive constants and $D(x)$ is defined such as

$$D(x) = \begin{cases} 0, & \text{for } \ x \in \Omega_{x_0} \\ 1, & \text{for } \ x \in \Omega_{x_1} \end{cases}$$

$$\Omega_{x_0} = \{x \in R \mid |x| \leq \delta_x\}, \ \Omega_{x_1} = \{x \in R \mid |x| > \delta_x\}$$

for any given positive constants δ_x.

Now consider the following positive definite function for $\nu \in \Omega_{\nu_1}$

$$V_0 = \frac{1}{2}\nu^2 + \mu_0 \boldsymbol{\xi}^T P_\xi \boldsymbol{\xi} + \mu_1 W(\boldsymbol{\eta}) + \frac{g'_m}{2\gamma_I}[k_I - k^*]^2 \tag{26}$$

where μ_0 and μ_1 are any positive constants, k^* is an ideal feedback gain to be determined later and $g'_m = g_m/b_u$. The time derivative of V_0 can be evaluated by

$$\begin{aligned} \dot{V}_0 \leq &- (g'_m k^* - v_0)\nu^2 - (\mu_0 \lambda_{min}[Q_\xi] - v_1)\|\boldsymbol{\xi}\|^2 \\ &- (\mu_1 \kappa_1 - v_2)\|\boldsymbol{\eta}\|^2 + g'_{1,r}\nu\omega_1 \\ &- g'_{1,r}(y_f - \Psi_0)\nu + R_0 \end{aligned} \tag{27}$$

where $\omega_1 = u_{a_f} - \alpha_1$ and v_0 to v_2 and R_0 are given as follows:

$$v_0 = L_2 + \frac{(\mu_1 \kappa_2 L_1)^2}{4\rho_1} + \rho_2 + \frac{(L_2 + 2\mu_0 \boldsymbol{a}_{\xi M}\|P_\xi\|)^2}{4\rho_3}$$

$$v_1 = \rho_3 + \rho_4 + \rho_6 + \frac{(\mu_0 B_{\xi M}\|P_\xi\|)^2}{\rho_8}$$

$$v_2 = \rho_1 + \rho_5 + \rho_7 + \rho_8$$

$$\begin{aligned} R_0 = &\frac{d_{11}^2}{4\gamma_R g'_m} + \frac{1}{4\gamma_P g'_m}\left[\frac{(\mu_0 p_1 \|P_\xi\|)^2}{\rho_4^2} + \frac{(\mu_1 \kappa_2 d_{1\eta})^2}{16\rho_5^2}\right] \\ &+ \frac{(\sigma_0 L_2 + d_{01} + \sigma_1)^2}{4\rho_2} \\ &+ \frac{[\mu_0 \|P_\xi\|(2\sigma_0\|\boldsymbol{a}_\xi\| + p_1\phi_{2M} + p_0)]^2}{\rho_6} \\ &+ \frac{[\mu_1 \kappa_1(\sigma_0 L_1 + d_{1\eta}\psi_{2\eta M} + d_{0\eta})]^2}{4\rho_7} \end{aligned}$$

with any positive constants ρ_1 to ρ_8 and positive constants $\boldsymbol{a}_{\xi M}, B_{\xi M}$ which satsfy $\|\boldsymbol{a}_\xi(t)\| \leq \boldsymbol{a}_{\xi M}, \|B_\xi(t)\| \leq B_{\xi M}$ from assumption 2. ϕ_{2M}, $\psi_{2\eta M}$ are positive constants such that $|\phi_2(y^*)| \leq \phi_{2M}$, $|\psi_{2\eta}(y^*)| \leq \psi_{2\eta M}$. Since y^* is bounded, such a constant exists from Assumption 1 that $\phi_{2i}(y_2)$ is bounded for all bounded y_2.

Step 1: Consider the error system, ω_1-system, between u_{a_f} and α_1. ω_1-system is given from (17) by

$$\dot{\omega}_1 = a_{a1}u_{a_f} + \boldsymbol{a}_{a2}^T \boldsymbol{\eta}_a + b_a u - \dot{\alpha}_1 \tag{28}$$

The time derivative of α_1 is given by

$$\begin{aligned} \dot{\alpha}_1 = &\frac{\partial \alpha_1}{\partial y}[a(y, \boldsymbol{\xi}) + g_{1,r}u_{f_1} + f_1(y, \boldsymbol{\xi}, \boldsymbol{\eta}, t)] + \frac{\partial \alpha_1}{\partial y^*}\dot{y}^* \\ &+ \frac{\partial \alpha_1}{\partial k_I}\gamma_I \nu^2 + \frac{\partial \alpha_1}{\partial \Psi_0} D(y_f)[-a_f \Psi_0 + b_a u] \end{aligned} \tag{29}$$

Taking this into consideration, the actual control input is designed as follows:

$$u = \begin{cases} -\dfrac{1}{b_a}[c_1\omega_1 + \epsilon_0(u_{a_f}^2 + \|\boldsymbol{\eta}_a\|^2)\omega_1 + \epsilon_1 \Psi_1 \omega_1] \\ \hfill \text{if } y_f \in \Omega_{y_{f0}} \\ -\dfrac{\omega_1}{b_a y_f}[c_1\omega_1 + \epsilon_0(u_{a_f}^2 + \|\boldsymbol{\eta}_a\|^2)\omega_1 + \epsilon_1 \Psi_1 \omega_1] \\ \quad -\dfrac{1}{b_a}[\gamma_f y_f + \epsilon_2 \|\boldsymbol{\eta}_f\|^2 y_f] - \dfrac{\epsilon_3}{b_a y_f}\Psi_0^2 \\ \hfill \text{if } y_f \in \Omega_{y_{f1}} \end{cases} \tag{30}$$

where ϵ_0 to ϵ_3 and c_1 are any positive constants, γ_f is a positive constant such that

$$\gamma_f \geq \frac{\|\boldsymbol{a}_{f_2}\|^2}{4\epsilon_2 \delta_f^2},\ c_1 > \frac{a_{f_1}^2}{2\epsilon_3} \tag{31}$$

and Ψ_1 is given by

$$\Psi_1 = (l_1 + u_{f_1}^2 + \psi_1^2)\left(\frac{\partial \alpha_1}{\partial y}\right)^2 + \left(\frac{\partial \alpha_1}{\partial y^*}\right)^2 + \left(\frac{\partial \alpha_1}{\partial k_I}\right)^2 \nu^4 \tag{32}$$

with any positive constant l_1.

3.4 Boundedness and Convergence Analysis

Theorem 1. Under assumptions 1 to 5 on the control system (1), all the signals in the resulting closed-loop system with the controller (30) are bounded. Further, the tracking error ν converges to any given bound

$$\lim_{t\to\infty} |\nu| \leq \delta \tag{33}$$

Proof : Consider the following positive and continuous function V:

$$V = \begin{cases} \frac{1}{2}\delta_\nu^2 + V_a,\ \nu \in \Omega_{\nu_0} \\ \frac{1}{2}\nu^2 + V_a,\ \nu \in \Omega_{\nu_1} \end{cases} \tag{34}$$

where

$$V_a = \begin{cases} \frac{1}{2}\delta_{y_f}^2 + \delta_{V_v}^2 + \frac{g_m'}{2\gamma_I}\Delta k_I^2, y_f \in \Omega_{y_{f0}}, (\boldsymbol{\xi}, \boldsymbol{\eta}, \omega_1) \in \Omega_{v_0} \\ \frac{1}{2}\delta_{y_f}^2 + V_v + \frac{g_m'}{2\gamma_I}\Delta k_I^2,\ y_f \in \Omega_{y_{f0}}, (\boldsymbol{\xi}, \boldsymbol{\eta}, \omega_1) \in \Omega_{v_1} \\ \frac{1}{2}y_f^2 + \delta_{V_v}^2 + \frac{g_m'}{2\gamma_I}\Delta k_I^2,\ y_f \in \Omega_{y_{f1}}, (\boldsymbol{\xi}, \boldsymbol{\eta}, \omega_1) \in \Omega_{v_0} \\ \frac{1}{2}y_f^2 + V_v + \frac{g_m'}{2\gamma_I}\Delta k_I^2,\ \ y_f \in \Omega_{y_{f1}}, (\boldsymbol{\xi}, \boldsymbol{\eta}, \omega_1) \in \Omega_{v_1} \end{cases}$$

$$V_v = \mu_0 \boldsymbol{\xi}^T P_\xi \boldsymbol{\xi} + \mu_1 W(\boldsymbol{\eta}) + \frac{1}{2}\omega_1^2,\ \Delta k_I = k_I - k^*$$

and Ω_{v_0} and Ω_{v_1} are defined by

$$\Omega_{v_0} = \{\boldsymbol{\xi} \in R^{r-1}, \boldsymbol{\eta} \in R^{n-r}, \omega_1 \in R \mid V_v \leq \delta_{V_v}^2\}$$
$$\Omega_{v_1} = \{\boldsymbol{\xi} \in R^{r-1}, \boldsymbol{\eta} \in R^{n-r}, \omega_1 \in R \mid V_v > \delta_{V_v}^2\}$$

with a positive constant δ_{V_v} which is determined such as $\delta_{V_v}^2 \geq \bar{R}/\bar{\alpha}_v$. Where $\bar{\alpha}_v$ is defined by

$$\bar{\alpha}_v = \min\left[\frac{\lambda_{min}[Q_\xi] - v_1'/\mu_0}{\lambda_{max}[P_\xi]}, \frac{\kappa_1 - v_2/\mu_1}{\kappa_3}, 2\bar{c}_1\right]$$

for positive constants μ_0, μ_1 and $\bar{\rho}_1$ that satisfy

$$\mu_0 \lambda_{min}[Q_\xi] - v_1' > 0,\ \mu_1\kappa_1 - v_2 > 0,$$
$$\bar{c}_1 = c_1 - \bar{\rho}_1 - \frac{a_{f_1}^2}{2\epsilon_3} > 0,\ v_1' = v_1 + \frac{L_2^2}{\epsilon_1 l_1}$$

and $\bar{R}$ is given by

$$\bar{R} = R_0 + \frac{1}{4\epsilon_0}(|a_{a1}|^2 + \|\boldsymbol{a}_{a2}\|^2) + \frac{\|\boldsymbol{a}_{f_2}\|^2}{4\epsilon_2} + \frac{1}{4\epsilon_1}\left(\frac{4(L_2\sigma_0)^2}{l_1} + \frac{4d_{01}^2}{l_1} + g_M^2 + d_{11}^2 + \sigma_1^2 + \gamma_I^2\right) + \frac{[\mu_0\|P_\xi\|\sigma_3]^2}{\rho_6} + \frac{[\mu_1\kappa_1\sigma_4]^2}{4\rho_7} + \frac{(L_2\delta_\nu)^2}{\epsilon_1 l_1}$$

where

$$\sigma_3 = \|\boldsymbol{a}_\xi\|(\sigma_0 + \delta_\nu) + p_1(\phi_{1M}\delta_\nu + \phi_{2M}) + p_0$$
$$\sigma_4 = L_1(\sigma_0 + \delta_\nu) + d_{1\eta}(\psi_{1\eta M}\delta_\nu + \phi_{2\eta M}) + d_{0\eta}$$

$\phi_{1M}, \psi_{1\eta M}$ are positive constants that satisfy $|\phi_1(y)| \leq \phi_{1M}, |\psi_{1\eta}(y)| \leq \psi_{1\eta M}$ for y such that $|y| \leq \delta_\nu + \sigma_0$ and g_M is a positive constant which satisfies $g_{1,r}'(t) \leq g_M$ for all t.

Further, in the function V, we consider an ideal feedback gain k^* which satisfies the following inequality:

$$-(g_m' k^* - v_0')\delta_v^2 + R_2 \leq -\gamma_\nu < 0 \tag{35}$$

for

$$v_0' = v_0 + \frac{g_M^2}{4\bar{\rho}_1} + \bar{\rho}_2 + \frac{g_M^2}{4a_{f_1}} + \frac{g_M^2}{4\epsilon_3} + \frac{L_2^2}{\epsilon_1 l_1}$$
$$R_2 = \bar{R} + \frac{2\delta_{V_v}^2 g_M^2}{4\bar{\rho}_2}$$

where γ_ν, $\bar{\rho}_2$ are any positive constants.

From (34), the time derivative of V for $\nu \in \Omega_{\nu_0}$ is given by

$$\dot{V} = 0 \tag{36}$$

for $y_f \in \Omega_{y_{f0}}$ and $(\boldsymbol{\xi}, \boldsymbol{\eta}, \omega_1) \in \Omega_{v_0}$, and for $y_f \in \Omega_{y_{f0}}$ and $(\boldsymbol{\xi}, \boldsymbol{\eta}, \omega_1) \in \Omega_{v_1}$, it can be evaluated by

$$\dot{V} = \dot{V}_v \leq -\bar{\alpha}_v V_v + \bar{R} \leq 0, \tag{37}$$

since $V_v > \delta_{V_v}^2$ for $(\boldsymbol{\xi}, \boldsymbol{\eta}, \omega_1) \in \Omega_{v_1}$. Furthermore, for $y_f \in \Omega_{y_{f1}}$ and $(\boldsymbol{\xi}, \boldsymbol{\eta}, \omega_1) \in \Omega_{v_0}$, $\dot{V}$ for $\nu \in \Omega_{\nu_0}$ is evaluated by

$$\dot{V} \leq -\gamma_f y_f^2 + \frac{\|\boldsymbol{a}_{f_2}\|^2}{4\epsilon_2} \leq 0 \tag{38}$$

from (31), and for $y_f \in \Omega_{y_{f1}}$ and $(\boldsymbol{\xi}, \boldsymbol{\eta}, \omega_1) \in \Omega_{v_1}$, it follows that

$$\dot{V} \leq -\bar{\alpha}_v V_v - \gamma_f y_f^2 + \bar{R} \leq 0. \tag{39}$$

As for the time derivative of V for $\nu \in \Omega_{\nu_1}$, $\dot{V}$ is evaluated by

$$\dot{V} \leq -(g_m' k^* - v_0')\nu^2 - \gamma_f y_f^2 + R_2 \leq -\gamma_\nu \tag{40}$$

for $y_f \in \Omega_{y_{f1}}$ and $(\boldsymbol{\xi}, \boldsymbol{\eta}, \omega_1) \in \Omega_{v_0}$. For $y_f \in \Omega_{y_{f1}}$ and $(\boldsymbol{\xi}, \boldsymbol{\eta}, \omega_1) \in \Omega_{v_1}$, we have

$$\dot{V} \leq -(g_m' k^* - v_0')\nu^2 - \bar{\alpha}_v V_v - \gamma_f y_f^2 + \bar{R} \leq -\gamma_\nu \tag{41}$$

from (35). We can see from (38) to (41) that the PFC output y_f is bounded. Furthermore it follows form (16) that the PFC states $\boldsymbol{\eta}_f$ are bounded. As a consequence, since the signal $y_f - \Psi_0$ is given by

$$\frac{d}{dt}(y_f - \Psi_0) = -a_f(y_f - \Psi_0) + \boldsymbol{a}_{f_2}^T \boldsymbol{\eta}_f \tag{42}$$

for $y_f \in \Omega_{y_{f1}}$, $y_f - \Psi_0$ is also bounded. Thus there exists a positive constant Ψ_{0M} such that

$$|y_f - \Psi_0| \leq \Psi_{0M} \tag{43}$$

for the both regions $\Omega_{y_{f0}}$ and $\Omega_{y_{f1}}$. Here we consider the ideal feedback gain k^* again such that the following inequality is satisfied.

$$-(g_m' k^* - v_0')\delta_v^2 + \max(R_2, R_3) \leq -\gamma_\nu < 0 \tag{44}$$

where

$$R_3 = \frac{\sigma_5^2}{4\rho_2} + \frac{d_{11}}{4g'_m \gamma_R}$$
$$\sigma_5 = L_2\sigma_0 + d_{01} + \sigma_1 + \frac{L_2\delta_{V_v}}{\sqrt{\mu_0\lambda_{min}[P_\xi]}} + \sqrt{2}g_M\delta_{V_v} + g_M\Psi_{0M}$$

This k^* must satisfy (35). For such a k^*, the time derivative of V for $y_f \in \Omega_{yf0}$ and $(\boldsymbol{\xi}, \boldsymbol{\eta}, \omega_1) \in \Omega_{v_0}$ can be evaluated by

$$\dot{V} \le -(g'_m k^* - v'_0)\nu^2 + R_3 \le -\gamma_\nu \quad (45)$$

and for $y_f \in \Omega_{yf0}$ and $(\boldsymbol{\xi}, \boldsymbol{\eta}, \omega_1) \in \Omega_{v_1}$, we have

$$\dot{V} \le -(g'_m k^* - v'_0)\nu^2 - \bar{\alpha}_v V_v + \bar{R} + R_3 \le -\gamma_\nu \quad (46)$$

from $\delta_{V_v}^2 \ge \bar{R}/\bar{\alpha}_v$ and (44).

Consequently we have

$$\begin{aligned} \dot{V} &\le 0, && \text{for } \nu \in \Omega_{\nu_0} \\ \dot{V} &\le -\gamma_\nu < 0, && \text{for } \nu \in \Omega_{\nu_1} \end{aligned} \quad (47)$$

Finally the time derivative of V can be evaluated as $\dot{V} \le 0$ for all $t \ge 0$, so we can conclude that all the signals in the control system are bounded.

Next, we analyze the convergence of the tracking error ν. Suppose that there exists a time t_0 such that $\nu^2 > \delta_\nu^2$ for all $t \ge t_0$. This implies that $V > \frac{1}{2}\delta_\nu^2, \forall t \ge t_0$. Further, in this case it follows from (47) that

$$V(t) = V(t_0) + \int_{t_0}^{t} \dot{V}(\tau)d\tau \le V(t_0) - \gamma_\nu(t - t_0) \quad (48)$$

Since the right-hand side of (48) will eventually become negative as $t \to \infty$, the inequality contradicts the fact that $V > \frac{1}{2}\delta_\nu^2, \forall t \ge t_0$. This means that the interval (t_0, t_1) in which $\nu \in \Omega_{\nu_1}$ is finite. Let (t_2, t_3) be a finite interval during which $\nu^2 \le \delta_v$, i.e. $\nu \in \Omega_{\nu_0}$ and (t_3, t_4) be a finite interval during which $\nu^2 > \delta_v$, i.e. $\nu \in \Omega_{\nu_1}$. Since $\dot{V} \le 0$ for $\nu \in \Omega_{\nu_0}$ and $\dot{V} \le -\gamma_\nu < 0$ for $\nu \in \Omega_{\nu_1}$, it follows that $V(t_3) \le V(t_2)$ for the interval (t_2, t_3) and that $V(t_4) < V(t_3)$ for the interval (t_3, t_4).

Thus the function V decreases a finite amount every time ν leaves Ω_{ν_0} and re-enters in Ω_{ν_0} and V does not increase during that $\nu \in \Omega_{\nu_0}$. Finally we can conclude that there exists a finite time T such that V converges to a constant for all $t \ge T$, i.e. $\nu \in \Omega_{\nu_0}$ for all $t \ge T$. Thus we obtain that

$$\lim_{t\to\infty} |\nu| \le \delta_\nu \quad (49)$$

and we can attain the control objective (7) by setting the positive constant δ_ν as $\delta_\nu = \delta$. ■

4. CONCLUSIONS

In this paper, we proposed a novel one-step backstepping design scheme for a robust adaptive tracking control of uncertain nonlinear systems. The proposed method can be applied to the uncertain nonlinear systems with any order of relative degree and has a relatively simple controller structure.

REFERENCES

Allgower, F., J. Ashman and A. Ilchmann (1997). High-gain adaptive λ-tracking for nonlinear systems. *Automatica* **33**(5), 881–888.

Arslan, G. and T. Basar (1999). Robust output-feedback control of strict-feedback systems with unknown nonlinearities. *Proc. of the 38th IEEE CDC, Arizona* pp. 4748–4753.

Fradkov, A. L. (1996). Shunt output feedback adaptive controllers for nonlinear plants. *Proc. of 13th IFAC World Congress, San Francisco* **K**, 367–372.

Fradkov, A. L. and D. J. Hill (1998). Exponential feedback passivity and stabilizability of nonlinear systems. *Automatica* **34**(6), 697–703.

Fradkov, A. L., I. Mizumoto and Z. Iwai (1999). Shunt output feedback adaptive tracking for nonlinear plants. *Proc. of the SPAS'99, St. Peterburg, Russia* **1**, 79–85.

Isidori, A. (1995). *Nonlinear Control Systems.* 3rd ed.. Springer-Verlag. New YorkLondon.

Iwai, Z. and I. Mizumoto (1994). Realization of simple adaptive control by using parallel feedforward compensator. *Int. J. Control* **59**(6), 1543–1565.

Jiang, Z-P. and L. Praly (1998). Design of robust adaptive controllers for nonlinear systems with dynamic uncertainties. *Automatica* **34**(7), 825–840.

Kaufman, H., I. Bar-Kana and K. Sobel (1998). *Direct Adaptive Control Algorithms.* 2nd ed.. Springer-Verlag. New York.

Khalil, H. K. (1996). *Nonlinear Systems.* 2nd ed.. Prentice Hall.

Lin, W. and C. Qian (2002). Adaptive control of nonlinearly parameterized systems: The smooth feedback case. *IEEE Trans. Automatic Control* **47**(8), 1249–1266.

Marino, R. and P. Tomei (1993). Global adaptive output-feedback control of nonlinear systems, part I and part II. *IEEE Trans. Automatic Control* **38**(1), 17–48.

Michino, R., I. Mizumoto, Z. Iwai and M. Kumon (2003). Robust high gain adaptive output feedback control for nonlinear systems with uncertain nonlinearities in control input term. *Int. J. of Control, Automation and Systems* **1**(1), 19–27.

Mizumoto, I., R. Michino, Y. Tao and Z. Iwai (2003). Robust adaptive tracking control for time-varying nonlinear systems with higher order relative degree. *Proc. of the 42nd IEEE CDC, Hawaii* pp. 4303–4308.

Polycarpou, M.M. and P.A. Ioannou (1996). A robust adaptive nonlinear control design. *Automatica* **32**(3), 423–427.

Yao, B. and M. Tomizuka (1997). Adaptive robust control of siso nonlinear systems in a semi-strict feedback form. *Automatica* **33**(5), 893–900.